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# SOME COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACES

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**Abstract.** The aim of this paper is to prove some common fixed point theorems in (GV)-fuzzy metric spaces.While proving our results, we employed the idea of compatibility due to Jungck [14] together with subsequentially continuity due to Bouhadjera and Godet-Thobie [4] respectively (also alternately reciprocal continuity due to Pant [28] together with subcompatibility due to Bouhadjera and Godet-Thobie [4] as in Imdad et al. [12] wherein conditions on completeness of the underlying space (or subspaces) together with conditions on continuity in respect of any one of the involved maps are relaxed. Our results substantially generalize and improve a multitude of relevant common fixed point theorems of the existing literature in metric as well as fuzzy metric spaces which include some relevant results due to Imdad et al.[10], Mihet [18], Mishra [19], Singh [28] and several others.

**Keywords**: Fuzzy metric space,Common fixed point,Compatible mappings,Occasionally weakly compatible mappings, Sub-compatible mappings,Reciprocal continuity, Subsequentially continuous mappings, Implicit relation.

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## 1. Introduction and Preliminaries

With a view to improve contractive (commutativity) conditions in common fixed point theorems, Sessa [27] introduced the notion of weakly commuting pair. Inspired by this definition of Sessa [27], researchers of this domain introduced several definitions of weak commutativity such as: compatible mappings, compatible of type (A), (B), (C), (P) and several others whose systematic comparisons and illustrations (up to 2001) are available inMurthy [20]. Most recently, Bouhadjera et al. [4] (see also [12]) introduced the notion of subcompatible pair which is weaker than most of the earlier definitions including occasionally weakly commuting pair (in short O.W.C.) (cf. [1,15,21]). In the following lines, we state some of these relevant definitions. The first common fixed point theorem (respectively fixed point theorem) without any continuity requirement were established by Pant [22,24] (see also [29]) when he introduced and utilized the ideas of non-compatible and reciprocally continuous maps. In fact, Pant [22,24] has shown that mappings may be discontinuous even at their respective fixed points. Most recently, Bouhadjera et al. [4] (see also [12]) introduced two new notions namely:subsequential continuity and sub compatibility which are weaker than reciprocal continuity and compatibility respectively which are to be utilized to prove our results in this paper.Recently the procedure deviced by Bouhadjera et al. [4] is novel in the sense that their procedure never requiresspace to be complete or closed prior to this paper this was essential.Fixed point theory in fuzzy metric metric spaces was initiated by Grabiec [9].Subrahmanyam [30] gave a generalization of Jungck [13] common fixed point theorem for commuting mappings in the setting of fuzzy metric spaces, whereas Vasuki [31] gave a fuzzy version of a result contained in Pant [21]. Thereafter, many authors established fuzzy versions of a host of classical metrical common fixed point theorems (e.g.[2,28,31]). Recently, Mihet [18], in his paper, emphasised the role of property (E. A.) in KM as well as GV-fuzzy metric spaces which is in fact an extension of a result of Imdad et al. [10] to fuzzy metric spaces. These observations motivated us to some common fixed point theorems for two pair of compatible and subsequentially continuous (alternately subcompatible and reciprocally continuous) mappings satisfying an implicit relation containing rational terms in fuzzy metric spaces.

Consequently,our results improve and sharpen many known common fixed point theorems available in the existing literature.

Hereby we give some preliminary definitions and notations

**Definition 1.1.** [26] *A continuous t-norm is a binary operation* $\Delta : [0,1] \times [0,1] \to [0,1]$ *satisfying the following conditions:*

*(1)* $\Delta(a,1) = a$,*for all* $a \in [0,1]$,
*(2)* $\Delta(a,b) = \Delta(b,a)$,
*(3)* $\Delta(c,d) \geq \Delta(a,b) for c \geq a, d \geq b$,
*(4)* $\Delta((a,b),c) = \Delta(a,\Delta(b,c))$ *for all* $a,b,c \in [0,1]$.

**Example 1.1.** *The following four t-norms are most basic:*
*(i) The minimum t-norm:* $\Delta M(a,b) = min(a,b)$.
*(ii) The product t-norm:* $\Delta P(a,b) = a.b$.
*(iii)The Lukasiewicz t-norm:* $\Delta L(a,b) = max(a+b-1,0)$.
*(iv) The weakest t-norm, the drastic product:*

$$\Delta D(a,b) = \begin{cases} min\{a,b\} \ if \ max\{a,b\} = 1 \\ \\ 0 \ otherwise \end{cases}$$

**Definition 1.2.** [7] *A fuzzy metric space in the sense of George and Veeramani [7] is a triplet* $(X, M, \Delta)$ *wherein* $X$ *is a nonempty set,*$\Delta$ *is a continuous t-norm,* $M$ *is a fuzzy set on* $X \times X \times (0,\infty)$ *which also satisfy the following conditions (for all* $x,y,z \in X$ *and* $s,t > 0$*):*
*(GV-1)* $M(x,y,t) > 0$,
*(GV-2)*$M(x,y,t) = 1 iff x = y$,
*(GV-3)* $M(x,y,t) = M(y,x,t)$,
*(GV-4)*$M(x,y,t)\Delta M(y,z,s) \leq M(x,z,t+s)$,

*(GV-5)* $M(x, y, .) : [0, \infty) \to [0, 1]$ *is continuous.*

*It is worth pointing out that, in the presence of (GV-1) and (GV-2), it follows that* $0 < M(x, y, t) < 1$ *for all* $t > 0$ *provided* $x \neq y$ *(cf. [7]). Also, it is well known that* $M(x, y, .)$ *is non decreasing for every* $x, y \in X$.

*In what follows, we denote a fuzzy metric spaces in George and Veeramani sense as* $(GV)$*-fuzzy metric space whereas the same in the sense of Kramosil and Michalek as* $(KM)$*-fuzzy metric space.*

**Example 1.2.** [7] *Let* $(X, d)$ *be a metric space, where* $\Delta M(a, b) = min(a, b)$ *with minimum t-norm, and* $M(x, y, t) = \frac{t}{t+|x-y|}$ *for all* $t > 0$ *and for all* $x, y \in X$.

*Then* $(X, M, \Delta)$ *is a* $GV$*-fuzzy metric space often referred as standard fuzzy metric space induced by* $(X, d)$.

**Definition 1.3.** [8] *Let* $(X, M, \Delta)$ *be a GV-fuzzy metric space, A sequence* $(x_n)_{n\in N}$ *in* $X$ *is said to be convergent to* $x \in X$ *if* $\lim_{n\to\infty} M(x_n, x, t) = 1$*, for all* $t > 0$.

**Definition 1.4.** [9] *Let* $(X, M, \Delta)$ *be a fuzzy metric space. Then M is said to be continuous on* $X \times X \times (0, \infty)$ *if* $\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$*, whenever* $\{(x_n, y_n, t_n)\}$ *is a sequence in* $X \times X \times (0, \infty)$ *which converges to a point* $(x, y, t) \in X \times X \times (0, \infty)$*; i.e.* $\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$ *and* $\lim_{n\to\infty} M(x, y, t_n) = M(x, y, t)$

**Lemma 1.1.** [9] $M$ *is a continuous function on* $X \times X \times (0, \infty)$.

**Definition 1.5.** [11, 22] *A pair of self-mappings* $(A, S)$ *of a fuzzy metric space* $(X, M, \Delta)$ *is said to be compatible (or asymptotically commuting) if* $\forall t > 0$, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$*, whenever* $\{x_n\}$ *is a sequence in* $X$ *such that,* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ *for some* $z$ *in* $X$*. Also, the pair* $(A, S)$ *is called non-compatible if there exists a sequence* $\{x_n\}$ *in* $X$ *with* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$*, for some* $z$ *in* $X$ *but either* $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$ *or the limit does not exists.*

**Definition 1.6.** *A pair* $(A, S)$ *of self mappings of a nonempty set* $X$ *is said to be weakly compatible if the pair commutes on the set of coincidence points i.e.* $Ax = Sx$, $(x \in X)$ *implies* $ASx = SAx$.

**Remark:** In metric space, the notion of weak compatibility (or coincidently commuting property or partially commuting property) coincide with pointwise R-weak commutativity.

**Definition 1.7.** [1, 15, 25] *A pair* $(A, S)$ *of self mappings of a nonempty set* $X$ *is said to be occasionally weakly compatible (O.W.C.) iff the pair* $(A, S)$ *commutes at least on one coincidence point (of the pair); i. e. there exits at least one point* $x$ *in* $X$ *such that* $Ax = Sx$ *and* $ASx = SAx$

**Definition 1.8.** [4] *A pair of self mappings* $(A, S)$ *defined on a fuzzy metric space* $(X, M, \Delta)$ *is said to be subcompatible iff there exists a sequence* $\{x_n\}$ *in* $X$ *with* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$*, for some* $z \in X$ *and*
$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \forall t > 0$
*Obviously, every OWC pair is subcompatible but not conversely (see [4]).*

**Definition 1.9.** [22] *A pair of self mappings* $(A, S)$ *defined on a fuzzy metric space* $(X, M, \Delta)$ *is called reciprocally continuous if for sequences* $\{x_n\}$ *in* $X$*,* $\lim_{n\to\infty} ASx_n = Az$ *and* $\lim_{n\to\infty} SAx_n = Sz$ *whenever* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$*, for some* $z \in X$
*Clearly, every pair of continuous mappings is reciprocally continuous but not conversely.*

**Definition 1.10.** [4] *A pair of self mappings* $(A, S)$ *of a fuzzy metric space* $(X, M, \Delta)$ *is called subsequentially continuous iff there exists a sequence* $\{x_n\}$ *in* $X$*, such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$*, for some* $z \in X$ *and* $\lim_{n\to\infty} ASx_n = Az$ *and* $\lim_{n\to\infty} SAx_n = Sz$
*In general, if the maps* $A$ *and* $S$ *are continuous or reciprocally continuous, then they are naturally subsequentially continuous. However, there exist subsequentially continuous pair ofmaps which are neither continuous nor reciprocally continuous can be seen in example (see Example 3.1 ).*

## 2. Implicit Relation

Following Ali and Imdad [3], let $F_6$ be the set of all continuous functions $F(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ satisfying the following condition:
$(F_1) : F(u, 1, u, 1, u, u) < 0$, for all $u \in (0, 1)$.

**Example 2.1.** *Define* $F(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ *as*

$F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi(min\{t_2, t_3, t_4, t_5, t_6\})$ *where* $\psi : [0, 1] \to [0, 1]$ *is increasing and continuous function such that* $\psi(t) > t$ *for all* $t \in (0, 1)$*.*

*Notice that* $(F_1) : F(u, 1, u, 1, u, u) < u - \psi(u) < 0$*, for all* $u \in (0, 1)$*.*

**Example 2.2.** *Define* $F(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ *as*

$F(t_1, t_2, t_3, t_4, t_5, t_6) = \int_0^{t_1} \phi(t)dt - \psi(\int_0^{min(t_2,t_3,t_4,t_5,t_6)} \phi(t)dt)$

*where* $\psi : [0, 1] \to [0, 1]$ *is increasing and continuous function such that* $\psi(t) > t$ *for all* $t \in (0, 1)$*.and* $\phi : R_+ \to R_+$ *is a Lebesgue integrable function which is summable and satisfies:* $0 < \int_0^{\epsilon} \phi(s)ds < 1$ *for all* $0 < \epsilon < 1$*.*

*Observe that* $(F_1) : F(u, 1, u, 1, u, u) = \int_0^u \phi(t)dt - \psi(\int_0^u \phi(t)dt) < 0$*, for all* $u \in (0, 1)$

**Example 2.3.** *Define* $F(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ *as*

$F(t_1, t_2, t_3, t_4, t_5, t_6) =$

$\psi(min\{\int_0^{t_2} \phi(t)dt, \int_0^{t_3} \phi(t)dt, \int_0^{t_4} \phi(t)dt, \int_0^{t_5} \phi(t)dt, \int_0^{t_6} \phi(t)dt\})$

*where* $\psi : [0, 1] \to [0, 1]$ *is increasing and continuous function such that* $\psi(t) > t$ *for all* $t \in (0, 1)$*.and* $\phi : R_+ \to R_+$ *is a Lebesgue integrable function which is summable and satisfies:* $0 < \int_0^{\epsilon} \phi(s)ds < 1$ *for all* $0 < \epsilon < 1$*.*

*Observe that* $(F_1) : F(u, 1, u, 1, u, u) = \int_0^u \phi(t)dt - \psi(\int_0^u \phi(t)dt) < 0$*, for all* $u \in (0, 1)^3$

## 3. Main Result

We prove our main result as follows.

**Theorem 3.1.** *Let* $A, B, S$ *and* $T$ *be four self mappings of a (GV)-fuzzy metric space* $(X, M, \Delta)$*. If the pairs* $(A, S)4$ *and* $(B, T)$ *are compatible and subsequentially continuous mappings, then*

*(i) the pair* $(A, S)$ *has a coincidence point,*

*(ii) the pair* $(B, T)$ *has a coincidence point.*

*(iii) Further, $A, B, S$ and $T$ have a unique common fixed point provided the involved maps satisfy the following inequality.*

$$\begin{aligned}&F(M(Ax,By,t),M(Ax,Sx,t),M(Ax,Ty,t)*\\ &M(Sx,By,t),\frac{M(By,Sx,t)}{M(By,Sx,t)*M(Sx,Ty,t)},M(By,Ty,t),M(Sx,By,t))\geq 0.\end{aligned} \tag{3.1}$$

*for all $x, y \in X$, $F \in F_6$ and $t > 0$*

**Proof.** Since the pair $(A, S)$ (also $(B, T)$) is sub sequentially continuous and compatible mappings, therefore there exists a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$. And $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1$ (for all $t > 0$), whereas in respect of the pair $(B, T)$, there exists a sequence $\{y_n\}$ in $X$ such that $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$, for some $w \in X$. And $\lim_{n\to\infty} M(BTy_n, TBy_n, t) = M(Bw, Tw, t) = 1$ (for all $t > 0$), so that $Az = Sz$ and $Bw = Tw$ i.e. z is a coincidence point of the pair $(A, S)$ whereas w is a coincidence point of the pair $(B, T)$. Now, we prove that $z = w$. Indeed using (3.1), we can have

$$F\begin{pmatrix} M(Ax_n,By_n,t),M(Ax_n,Sx_n,t),M(Ax_n,Ty_n,t)*M(Sx_n,By_n,t)\\ \frac{M(Sx_n,By_n,t)}{M(Sx_n,By_n,t)*M(Sx_n,Ty_n,t)},M(By_n,Ty_n,t),M(Sx_n,By_n,t)\end{pmatrix}\geq 0$$

which on making $n \to \infty$, reduces

$F(M(z, w, t), 1, M(z, w, t), M(z, w, t), M(w, z, t), M(z, w, t)) \geq 0$ a contradiction to $(F_1)$ so that $z = w$. Now, we assert that $Az = z$. If (on contrary) $Az \neq z$, then using (3.1), we get

$$F\begin{pmatrix} M(Az,By_n,t),M(Az,Sz,t),M(Az,Ty_n,t)*M(Sz,By_n,t)\\ \frac{M(Sz,By_n,t)}{M(Sz,By_n,t)*M(Sz,Ty_n,t)},M(By_n,Ty_n,t),M(Sz,By_n,t)\end{pmatrix}\geq 0$$

which on making $n \to \infty$, reduces

$F(M(Az, Bz, t), 1, M(Az, z, t), M(Az, z, t), 1, M(Az, z, t)) \geq 0$ Which is a contradiction to $(F_1)$ implying thereby $Az = Sz = z$. Next, suppose that $Bz = z$, then using (3.1), we get

$$F\begin{pmatrix} M(Az,Bz,t),M(Az,Sz,t),M(Az,Tz,t)*M(Sz,Bz,t)\\ \frac{M(Sz,Bz,t)}{M(Sz,Bz,t)*M(Sz,Tz,t)},M(Bz,Tz,t),M(Sz,Bz,t)\end{pmatrix}\geq 0$$

Or $F(M(z,Bz,t),1,M(z,Bz,t),M(z,Bz,t),1,M(z,Bz,t)) \geq 0$ which is again a contradiction to $(F_1)$. Thus $z = Bz = Tz$ Therefore in all, $z = Az = Bz = Sz = Tz$ i.e. z is common fixed point of $A$, $B$, $S$ and $T$ . The uniqueness of common fixed point can be established easily using the inequality (3.1). This completes the proof of the theorem. Alternately, using reciprocal continuity (due to Pant [22]) together with sub compatibility (due to Bouhadjera and Godet-Thobie [4]), we motivate to prove the following result:

**Theorem 3.2.** *Let $A, B, S$ and $T$ be four self mappings of a (GV)-fuzzy metric space $(X, M, \Delta)$. If the pairs $(A, S)4$ and $(B, T)$ are Subcompatible and reciprocally continuous mappings, then*
*(i) the pair $(A, S)$ has a coincidence point,*
*(ii) the pair $(B, T)$ has a coincidence point.*
*(iii) Further, $A, B, S$ and $T$ have a unique common fixed point provided the involved maps satisfy the following inequality.*

$$F(M(Ax,By,t),M(Ax,Sx,t),M(Ax,Ty,t)*$$
$$M(Sx,By,t),\frac{M(By,Sx,t)}{M(By,Sx,t)*M(Sx,Ty,t)},M(By,Ty,t),M(Sx,By,t)) \geq 0.$$

*for all $x, y \in X$, $F \in F_6$ and $t > 0$*

**Proof.** Since the pair $(A, S)$ (also $(B, T)$) is Subcompatible and reciprocally continuous mappings, therefore there exists a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ , for some $z \in X$. And $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1$ (for all $t > 0$), whereas in respect of the pair $(B, T)$, there exists a sequence $\{y_n\}$ in $X$ such that $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$, for some $w \in X$. And $\lim_{n\to\infty} M(BTy_n, TBy_n, t) = M(Bw, Tw, t) = 1$ (for all $t > 0$), so that $Az = Sz$ and $Bw = Tw$ i.e. z is a coincidence point of the pair $(A, S)$ whereas w is a coincidence point of the pair $(B, T)$. The rest of the proof can be completed on the lines of above Theorem 3.1.This concludes the proof.

The earlier defined implicit relation enables us to derive a multitude of fixed point theorems as carried out in Ali and Imdad [3]. But, here we limit ourselves to only those conditions which correspond to Examples 2.1, 2.2 and 2.3 yielding thereby the refined and sharpened versions of some fixed point theorems contained in Singh and Chauhan [28], Imdad et al. [11], Mishra et al. [19] besides some other ones which can be stated as under:

**Corollary 3.1** The conclusions of Theorem 3.1 remain true if we replace the inequality (3.1) (of Theorem 3.1) by any one of the following (besides retaining rest of the hypotheses):

$$(i) M(Ax, By, t) \geq \psi(min\{M(Ax, Sx, t), M(Ax, Ty, t) * M(Sx, By, t), \frac{M(By, Sx, t)}{M(By, Sx, t) * M(Sx, Ty, t)}, M(By, Ty, t), M(Sx, By, t)\})$$

where $\psi : [0, 1] \to [0, 1]$ is increasing and continuous function such that $\psi(t) > t$ for all $t \in (0, 1)$

$$(ii) \int_0^{M(Ax,By,t)} \phi(t)dt \geq \psi(\int_0^{min\{M(Ax,Sx,t),M(Ax,Ty,t)*M(Sx,By,t),\frac{M(By,Sx,t)}{M(By,Sx,t)*M(Sx,Ty,t)},M(By,Ty,t),M(Sx,By,t)\}} \phi(t)dt)$$

where $\psi : [0, 1] \to [0, 1]$ is increasing and continuous function such that $\psi(t) > t$ for all $t \in (0, 1)$
and $\Re_+ \to \Re_+$ is a Lebesgue integrable function which is summable and satisfies $0 < \int_0^{\in} \phi(s)ds < 1$, for all $0 <\in< 1$

$$(iii) \int_0^{M(Ax,By,t)} \phi(t)dt \geq \psi(\int_0^{M(Ax,Sx,t)} \phi(t)dt, \int_0^{M(Ax,Ty,t)} \phi(t)dt, \int_0^{M(Sx,By,t)}, \int_0^{\frac{M(By,Sx,t)}{M(By,Sx,t)*M(Sx,Ty,t)}} \phi(t)dt, \int_0^{M(By,Ty,t)} \phi(t)dt, \int_0^{M(Sx,By,t)} \phi(t)dt).$$

where $\psi : [0, 1] \to [0, 1]$ is increasing and continuous function such that $\psi(t) > t$ for all $t \in (0, 1)$
and $\Re_+ \to \Re_+$ is a Lebesgue integrable function which is summable and satisfies $0 <$

$\int_0^{\in} \phi(s)ds < 1$, for all $0 <\in< 1$

**Proof.** Proof follows from Theorem 3.1 .In view of Examples 2.12.3.

**Remark 3.1** A corollary similar to Corollary 3.1 can be outlined in respect of Theorem 3.2. By setting $A = B$ in Theorem 3.1, we derive the following corollary for three mappings.

**Corollary 3.2** Let $A$, $S$ and $T$ be self mappings defined on a (GV)-fuzzy metric space $(X, M, \Delta)$. If $(A, S)$ and $(A, T)$ are two pairs of compatible and subsequentially continuous self-mappings of a fuzzy metric space $(X, M, \Delta)$, then

$(i)$ the pair $(A, S)$ has a coincidence point,

$(ii)$ the pair $(A, T)$ has a coincidence point.

$(iii)$ Further, $A, S$ and $T$ have a unique common fixed point provided the involved maps satisfy the following inequality.

$$\begin{aligned} & F(M(Ax, Ay, t), M(Ax, Sx, t), M(Ax, Ay, t)* \\ (3.2) \qquad & M(Sx, Ay, t), \frac{M(Ay, Sx, t)}{M(Ay, Sx, t) * M(Sx, Ty, t)}, M(Ay, Ty, t), M(Sx, Ay, t)) \geq 0. \end{aligned}$$

all $x, y \in X$, $F \in F_6$ and for all $t > 0$

Alternately, by setting $S = T$ in Theorem 3.1, we can also derive yet another corollary for three mappings which runs as follows.

**Corollary 3.3** Let $A$, $B$, and $S$ be self mappings defined on a (GV)-fuzzy metric space $(X, M, \Delta)$. If $(A, S)$ and $(B, S)$ are two pairs of compatible and subsequentially continuous self-mappings of a fuzzy metric space $(X, M, \Delta)$, then

$(i)$ the pair $(A, S)$ has a coincidence point,

$(ii)$ the pair $(B, S)$ has a coincidence point.

$(iii)$ Further, $A, B$ and $S$ have a unique common fixed point provided the involved maps satisfy the following inequality.

$$\begin{aligned} &F(M(Ax, By, t), M(Ax, Sx, t), M(Ax, Sy, t)* \\ &M(Sx, By, t), \frac{M(By, Sx, t)}{M(By, Sx, t) * M(Sx, Sy, t)}, M(By, Sy, t), M(Sx, By, t)) \geq 0. \end{aligned} \tag{3.3}$$

all $x, y \in X$, $F \in F_6$ and for all $t > 0$

Finally, by setting $A = B$ and $S = T$ in Theorem 3.1, we can also derive the following corollary for a pair of maps.

**Corollary 3.4** Let A and S be self mappings defined on a (GV)-fuzzy metric space . If the pair (A, S) is compatible and sub sequentially continuous, then

$(i)$ the pair $(A, S)$ has a coincidence point,

$(ii)$ Further, $A$ and $S$ have a unique common fixed point provided the involved maps satisfy the following inequality.

$$\begin{aligned} &F(M(Ax, Ay, t), M(Ax, Sx, t), M(Ax, Ay, t)* \\ &M(Sx, Ay, t), \frac{M(Ay, Sx, t)}{M(Ay, Sx, t) * M(Sx, Sy, t)}, M(Ay, Ty, t), M(Sx, Ay, t)) \geq 0. \end{aligned} \tag{3.4}$$

all $x, y \in X$, $F \in F_6$ and for all $t > 0$

**Remark 3.3** Corollaries analogous to Corollary 3.1 can be outlined in respect of Corollaries 3.2, 3.3 and 3.4 which will give rise several common fixed point theorems for three as well as two mappings. We conclude this paper with two illustrative examples which demonstrate the validity of the hypotheses of Theorems 3.1 and 3.2.

**Example 3.1.** *Let $(X, M, \Delta)$ be a (GV)-fuzzy metric space as defined in Example 1.2, where in $X = (-3, \infty)$. Set $A = B$ and $S = T$. Define $A, S : X \to X$ as follows:*

$$Ax = \begin{cases} 0 & if\ x \in [0, 1] \\ \frac{x}{3} & if\ 1 < x \leq 2 \\ 2x - 1 & if x \in (1, \infty) \end{cases}$$

Consider the sequence $\{x_n\} = \frac{1}{n}$ in $X$

Then $\lim_{n\to\infty} A\{x_n\} = \lim_{n\to\infty} \frac{1}{3n} = 0 = \lim_{n\to\infty} \frac{1}{2n} = \lim_{n\to\infty} S\{x_n\}$
Next,
$\lim_{n\to\infty} AS\{x_n\} = \lim_{n\to\infty} A(\frac{1}{2n}) == \lim_{n\to\infty} \frac{1}{6n} = 0 = A(0)$

$\lim_{n\to\infty} SA\{x_n\} = \lim_{n\to\infty} S(\frac{1}{3n}) == \lim_{n\to\infty} \frac{1}{6n} = 0 = S(0)$

and $\lim_{n\to\infty} M(AS\{x_n\}, SA\{x_n\}, t) = 1$, for all $t > 0$

Consider another sequence $\{x_n\} = 1 + \frac{1}{n}$,

then $\lim_{n\to\infty} A\{x_n\} = \lim_{n\to\infty} 2 + \frac{2}{n} - 1 = 1$

and $\lim_{n\to\infty} S\{x_n\} = \lim_{n\to\infty} 3 + \frac{3}{n} - 1 = 1$
Also
$\lim_{n\to\infty} AS\{x_n\} = \lim_{n\to\infty} A(1 + \frac{3}{n}) = \lim_{n\to\infty} 2 + \frac{6}{n} - 1 = 1 \neq A(1)$

$\lim_{n\to\infty} SA\{x_n\} = \lim_{n\to\infty} A(1 + \frac{2}{n}) = \lim_{n\to\infty} 3 + \frac{6}{n} - 2 = 1 \neq S(1)$

but $\lim_{n\to\infty} M(AS\{x_n\}, SA\{x_n\}, t) = 1$. Thus, the pair $(A, S)$ is compatible as well as subsequentially continuous but not reciprocally continuous. Further, one can easily verify inequality (3.1) by defining $F$ as in Example 2.1 and choosing $\phi(t) = \sqrt{t}$ for all $t \in (0, 1)$. Therefore all the conditions of Theorem 3.1 are satisfied. Evidently, 0 is a coincidence as well as unique common fixed point of the pair $(A, S)$. Notice that this example cannot be covered by earlier fixed point theorems involving compatibility and reciprocal continuity both or by the ones involving conditions on completeness (or closedness) of underlying space (or subspaces). Notice that in this example neither $X$ is complete nor the subspaces $A(X) = [0, \frac{1}{3}] \bigcup (1, \infty)$ and $S(X) = [\frac{-3}{2}, \frac{1}{2}]$ are closed. (e.g. [19,22,29]).

**Example 3.2.** *Let $(X, M, \Delta)$ be a fuzzy metric space as defined in Example 1.2, where in $X = \mathcal{R}$. Set $A = B$ and $S = T$. Define $A, S : X \to X$ as follows:*

$$Ax = \begin{cases} x+1 & if\ x \in (-\infty, 1) \\ \\ 2x-1 & if x \in [1, \infty) \end{cases}$$

$$Sx = \begin{cases} \frac{x}{2} & if\ x \in (-\infty, 1) \\ \\ 3x-2 & if x \in (1, \infty) \end{cases}$$

Consider a sequence $\{x_n\} = 1 + \frac{1}{n}$,

then $\lim_{n\to\infty} A\{x_n\} = \lim_{n\to\infty} 2 + \frac{2}{n} - 1 = 1$

and $\lim_{n\to\infty} S\{x_n\} = \lim_{n\to\infty} 3 + \frac{3}{n} - 2 = 1$
Also

$\lim_{n\to\infty} AS\{x_n\} = \lim_{n\to\infty} A(1 + \frac{3}{n}) = \lim_{n\to\infty} 2 + \frac{6}{n} - 1 = 1 = A(1)$

$\lim_{n\to\infty} SA\{x_n\} = \lim_{n\to\infty} S(1 + \frac{2}{n}) = \lim_{n\to\infty} 3 + \frac{6}{n} - 2 = 1 = S(1)$

and $\lim_{n\to\infty} M(AS\{x_n\}, SA\{x_n\}, t) = 0$

Consider a sequence $\{x_n\} = \frac{1}{n} - 2$,

then $\lim_{n\to\infty} A\{x_n\} = \lim_{n\to\infty} \frac{2}{n} - 2 + 1 = -1$

and $\lim_{n\to\infty} S\{x_n\} = \lim_{n\to\infty} \frac{1}{2n} - 1 = -1$
Also

$\lim_{n\to\infty} AS\{x_n\} = \lim_{n\to\infty} A(\frac{1}{2n} - 1) = \lim_{n\to\infty} \frac{1}{2n} - 1 + 1 = 0 = A(-1)$

$$\lim_{n\to\infty} SA\{x_n\} = \lim_{n\to\infty} S(\tfrac{1}{n} - 1) = \lim_{n\to\infty} \tfrac{1}{2n} - \tfrac{1}{2} - \tfrac{1}{2} = -\tfrac{1}{2} = S(-1)$$

and $\lim_{n\to\infty} M(AS\{x_n\}, SA\{x_n\}, s) = 0$

Thus, the pair $(A, S)$ is reciprocally continuous as well as subcompatible but not compatible. Further, one can easily verify inequality (3.1) by defining $F$ as in Example 2.1 and choosing $\phi(t) = \sqrt{t}$ for all $t \in (0, 1)$. Thus, all the conditions of Theorem 3.2 are satisfied. Here, 1 is a coincidence as well as unique common fixed point of the pair . As noticed earlier, this example too cannot be covered by those fixed point theoremswhich involve compatibility and reciprocal continuity both (e.g. [19,22,29]).

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