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THREE-STEP ITERATIVE SCHEME FOR APPROXIMATING FIXED POINTS OF MULTIVALUED NONEXPANSIVE MAPPINGS

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Abstract. In this paper, we introduce a new three-step iterative scheme to approximate a common fixed point of multivalued nonexpansive mappings in a uniformly convex real Banach space and establish strong and weak convergence theorems for the proposed process. Our results extend important results.

Keywords: Multivalued nonexpansive mappings; Strong and weak convergence.

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1. Introduction and Preliminaries

Throughout this paper, let E be a Banach space with the norm $\| \ \|$ and $\mathbb{N}$ denote the set of all positive integers.

Let K be a nonempty subset of E. The set K is said to be proximinal if for each $x \in E$, there exists an element $y \in K$ such that $\|x - y\| = d(x, K)$, where $d(x, K) = \inf\{\|x - z\| : z \in K\}$. We shall denote $CB(K)$, $C(K)$ and $P(K)$ by the families of nonempty closed and bounded subsets, nonempty compact subsets and nonempty proximinal bounded subsets of K, respectively. Let H be the Hausdorff metric induced by the

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metric d of E and given by

$$H\left(A,B\right)=\max\left\{\sup_{x\in A}d\left(x,B\right),\ \sup_{y\in B}d(y,A)\right\}$$

for $A,\ B\in CB\left(E\right).$ A multivalued mapping $T:K\to P(K)$ is said to be contraction if there exists a constant $k\in[0,1)$ such that for all $x,y\in K$,

$$H\left(Tx,Ty\right)\leq k\left\|x-y\right\|,$$

and nonexpansive if

$$H\left(Tx,Ty\right)\leq\left\|x-y\right\|$$

for all $x,\ y\in K$.A point $x\in K$ is called a fixed point of a multivalued mapping T if $x\in Tx$. Denote by $F\left(T\right)$ the set of fixed points of T, that is, $F\left(T\right)=\{x\in K:Tx=x\}$.

A multivalued nonexpansive mapping $T:K\to CB\left(K\right)$ where K a subset of E, is said to satisfy condition (I) if there exists a nondecreasing function $f:(0,\infty)\to 0,\infty)$ with $f\left(0\right)=0,\ \ f\left(r\right)>0$ for all $r\in(0,\infty)$ such that $d(x,Tx)\geq f(d\left(x,F\left(T\right)\right)$ for all $x\in K$ (see [9]).

A multivalued mapping $T:K\to P(E)$ is called demiclosed at $y\in K$ if for any sequence $\{x_n\}$ in K weakly convergent to an element x and $y_n\in Tx_n$ strongly convergent to y, we have $y\in Tx$.

A Banach space E is said to satisfy Opial's condition [6] if for any sequence $\{x_n\}$ in E, $x_n\rightharpoonup x$ implies that

$$\limsup_{n\to\infty}\left\|x_n-x\right\|<\limsup_{n\to\infty}\left\|x_n-y\right\|$$

for all $y\in E$ with $y\neq x$. Examples of Banach spaces satisfying this condition are Hilbert spaces and l^p spaces $(1<p<\infty)$. On the other hand, $L^p\left[0,2\pi\right]$ with $1<p\neq 2$ fail to satisfy Opail's condition.

The study of fixed points for multivalued contractions and nonexpansive mappings using the Hausdorff metric was initiated by Markin [2] (see also [1]). Later, an interesting and rich fixed point theory for such maps was developed which has applications in control theory, convex optimization, differential inclusion and economics (see [3]). Morever, the existence of fixed points for multivalued nonexpansive mappings in uniformly convex Banach spaces was proved by Lim [4].

The theory of multivalued nonexpansive mappings is harder than the corresponding theory of single valued nonexpansive mappings. Different iterative processes have been used to approximate fixed points of multivalued nonexpansive mappings; in particular, Sastry and Babu [5] considered the following:

Let K be a nonempty convex subset of E, $T : K \to P(K)$ be a multivalued mapping with $p \in Tp$.

(i) The sequences of Mann iterates is defined by $x_1 \in K$,

$$x_{n+1} = (1 - a_n) x_n + a_n y_n, \quad n \in \mathbb{N}$$

where $y_n \in Tx_n$ is such that $\|y_n - p\| = d(p, Tx_n)$, and $\{a_n\}$ is sequence in (0,1) satisfying $\sum a_n = \infty$.

(ii) The sequence of Ishikawa iterates is defined by $x_1 \in K$

$$\begin{cases} y_n = (1 - b_n) x_n + b_n z_n \\ x_{n+1} = (1 - a_n) x_n + a_n u_n, \quad n \in \mathbb{N} \end{cases}$$

where $z_n \in Tx_n$, $u_n \in Ty_n$ are such that $\|z_n - p\| = d(p, Tx_n)$ and $\|u_n - p\| = d(p, Ty_n)$, and $\{a_n\}$, $\{b_n\}$ are real sequences of numbers with $0 \le a_n,\ b_n < 1$ satisying $\lim_{n\to\infty} b_n = 0$ and $\sum a_n b_n = \infty$.

Recently, Panyanak [13] obtained the following theorem.

Theorem 1. *Let K be a nonempty compact convex subset of a uniformly convex Banach space E. Suppose that a nonexpansivemap $T : K \to P(K)$ has a fixed point p. Let $\{x_n\}$ be the sequence of Ishikawa iterates defined by (ii). Then $\{x_n\}$ converges to a fixed point of T.*

The following is a useful Lemma due to Nadler [1].

Lemma 1. *Let A,$B \in CB(E)$ and $a \in A$. If $\eta > 0$, then there exists $b \in B$ such that $d(a, b) \le H(A, B) + \eta$.*

Based on the above lemma, Song and Wang [12] modified the iteration scheme used in [13] and improved the results presented therein. This scheme reads as follows:

(iii) The sequence of Ishikawa iterates is defined $x_1 \in K$

$$\begin{cases} y_n = (1-b_n)\, x_n + b_n z_n \\ x_{n+1} = (1-a_n)\, x_n + a_n u_n, \;\; n \in \mathbb{N} \end{cases}$$

where $z_n \in Tx_n,\ u_n \in Ty_n$ are such that $\|z_n - u_n\| \leq H(Tx_n, Ty_n) + \eta_n$ and $\|z_{n+1} - u_n\| \leq H(Tx_{n+1}, Ty_n) + \eta_n$, $\eta_n \in (0, \infty)$ and $\{a_n\}$, $\{b_n\}$ are real sequences of numbers with $0 \leq a_n,\ b_n \leq 1$ satisfying $\lim_{n\to\infty} b_n = 0$ and $\sum a_n b_n = \infty$.

It is to be noted that Song and Wang [12] need the condition $Tp = \{p\}$ in order to prove their Theorem 1. Actually, Panyanak [13] proved some results using Ishikawa type iteration process without this condition. Song and Wang [12] showed that without this condition his process was not well-defined. They reconstructed the process using the condition $Tp = \{p\}$ which made it well-defined. Such a condition was also used by Jung [14]. They defined $P_T(x) = \{y \in Tx : ||x - y|| = d(x, Tx)\}$ for a multivalued mapping $T : K \to P(K)$. They also proved a couple of strong convergence results using Ishikawa type iteration process

On the other hand, Agarwal et al. [10] introduced the following iteration scheme for single-valued mappings:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = (1-\alpha_n)\, Tx_n + \alpha_n T y_n \\ y_n = (1-\beta_n)\, x_n + \beta_n T x_n, \;\; n \in \mathbb{N} \end{cases} \tag{1.1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequances in $(0, 1)$. This scheme is independent of both Mann and Ishikawa schemes. They proved that this scheme converges at a rate faster than both Picard iteration scheme $x_{n+1} = Tx_n$ and Mann iteration scheme for contractions. Following their method, it was observed in [11] that this scheme also converges faster than Ishikawa iteration scheme.

Very recently, Khan and Yildirim [7] gave a mutlivalued version of the iteration scheme (1.1) of Agarwal et al. [10] to approximate fixed points of a multivalued nonexpansive mapping T. They defined iteration scheme as follows:

$$\left\{\begin{array}{c} x_1 \in K, \\ x_{n+1} = (1-\alpha_n)\, v_n + \alpha_n u_n \\ y_n = (1-\beta_n)\, x_n + \beta_n v_n, \;\; n \in \mathbb{N} \end{array}\right. \tag{1.2}$$

where $v_n \in P_T(x_n)$, $u_n \in P_T(y_n)$ and $\{\alpha_n\}$, $\{\beta_n\}$ are sequances in $(0,1)$. İn this way, they approximate fixed points of a multivalued nonexpansive mapping by an iteration scheme which is independent of but faster than Ishikawa scheme.

In this paper, we give three-step iteration scheme version of the iteration scheme (1.2) of Khan and Yildirim [7] to approximate fixed points of a multivalued nonexpansive mappings. We define our iteration scheme as follows:

$$\left\{\begin{array}{c} x_1 \in K, \\ x_{n+1} = (1-\alpha_n)\, v_n + \alpha_n u_n \\ y_n = (1-\beta_n)\, v_n + \beta_n w_n \\ z_n = (1-\gamma_n)\, x_n + \gamma_n v_n, \;\; n \in \mathbb{N} \end{array}\right. \tag{1.3}$$

where $v_n \in P_T(x_n)$, $u_n \in P_T(y_n)$, $w_n \in P_T(z_n)$ and $\alpha_n, \beta_n, \gamma_n \in [a,b] \subset (0,1)$.

Lemma 2. [8] *Let E be a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$ for all $n \in \mathbb{N}$. Suppose that $\{x_n\}$ and $\{y_n\}$ are two sequences of E such that*

$$\limsup_{n\to\infty} \|x_n\| \leq r, \;\; \limsup_{n\to\infty} \|y_n\| \leq r \;\textit{and}\; \lim_{n\to\infty} \|t_n x_n + (1-t_n)\, y_n\| = r$$

hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

Lemma 3. [7] *Let $T : K \to P(K)$ be a multivalued mapping and $P_T(x) = \{y \in Tx : ||x-y|| = d(x,Tx)\}$. Then the following are equivalent.*

(1) $x \in F(T)$

(2) $P_T(x) = \{x\}$

(3) $x \in F(P_T)$. *Morever, $F(T) = F(P_T)$.*

2. Main results

We start with the following couple of important lemmas.

Lemma 4. [7] *Let E be a normed space and K a nonempty closed convex subset of E . Let $T : K \to P(K)$ be a multivalued mapping such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping. Let $\{x_n\}$ be the sequence as defined in (1.3). Then $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T)$.*

Proof. Let $p \in F(T)$. Then $p \in P_T(p) = \{p\}$ by Lemma 3. It follows from (1.3) that

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1-\alpha_n) v_n + \alpha_n u_n - p\| \\ &\leq (1-\alpha_n)\|v_n - p\| + \alpha_n \|u_n - p\| \\ &\leq (1-\alpha_n) H(P_T(x_n), P_T(p)) + \alpha_n H(P_T(y_n), P_T(p)) \\ &\leq (1-\alpha_n)\|x_n - p\| + \alpha_n \|y_n - p\|. \end{aligned} \tag{2.1}$$

Also,

$$\begin{aligned} \|y_n - p\| &= \|(1-\beta_n) v_n + \beta_n w_n - p\| \\ &\leq (1-\beta_n)\|v_n - p\| + \beta_n \|w_n - p\| \\ &\leq (1-\beta_n) H(P_T(x_n), P_T(p)) + \beta_n H(P_T(z_n), P_T(p)) \\ &\leq (1-\beta_n)\|x_n - p\| + \beta_n \|z_n - p\| \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} \|z_n - p\| &= \|(1-\gamma_n) x_n + \gamma_n v_n - p\| \\ &\leq (1-\gamma_n)\|x_n - p\| + \gamma_n \|v_n - p\| \\ &\leq (1-\gamma_n)\|x_n - p\| + \gamma_n H(P_T(x_n), P_T(p)) \\ &\leq (1-\gamma_n)\|x_n - p\| + \gamma_n \|x_n - p\| \\ &= \|x_n - p\|. \end{aligned} \tag{2.3}$$

From (2.3), thus (2.2) becomes

$$\|y_n - p\| \leq (1-\beta_n)\|x_n - p\| + \|x_n - p\| = \|x_n - p\|. \tag{2.4}$$

From (2.1),

$$\|x_{n+1} - p\| \leq (1-\alpha_n)\|x_n - p\| + \alpha_n \|x_n - p\| = \|x_n - p\|.$$

Thus $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F$. □

Lemma 5. [7] *Let E be a uniformly convex Banach space and K a nonempty closed convex subset of E. Let $T : K \to P(K)$ be a multivalued mapping such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping .Let $\{x_n\}$ be the sequence as defined in (1.3). Then $\lim_{n\to\infty} d\ (x_n, Tx_n) = 0$.*

Proof. From Lemma 4, $\lim_{n\to\infty} \|x_n - p\|$ exists for any $p \in F(T)$. We suppose that $\lim_{n\to\infty} \|x_n - p\| = c$ for some $c \geq 0$. Since

$$\limsup_{n\to\infty} \|v_n - p\| \leq \limsup_{n\to\infty} H(P_T(x_n), P_T(p)) \leq \limsup_{n\to\infty} \|x_n - p\| = c,$$

so

$$\limsup_{n\to\infty} \|v_n - p\| \leq c. \tag{2.5}$$

Similarly, from (2.3) and (2.4)

$$\limsup_{n\to\infty} \|u_n - p\| \leq c, \tag{2.6}$$

and

$$\limsup_{n\to\infty} \|w_n - p\| \leq c. \tag{2.7}$$

Applying Lemma 2 by (2.5) and (2.6), we get

$$\lim_{n\to\infty} \|v_n - u_n\| = 0.$$

Taking lim sup on both sides of (2.4), we obtain

$$\limsup_{n\to\infty} \|y_n - p\| \leq c. \tag{2.8}$$

Also

$$\begin{aligned}\|x_{n+1}-p\| &= \|(1-\alpha_n)\,v_n+\alpha_n u_n-p\| \\ &= \|(v_n-p)+\alpha_n\,(u_n-v_n)\| \\ &\leq \|v_n-p\|+\alpha_n\,\|u_n-v_n\|\end{aligned}$$

Implies that

$$c \leq \liminf_{n\to\infty} \|v_n-p\|\,. \tag{2.9}$$

Combining (2.5) and (2.9), we have

$$\lim_{n\to\infty} \|v_n-p\| = c.$$

Thus

$$\begin{aligned}\|v_n-p\| &\leq \|v_n-u_n\|+\|u_n-p\| \\ &\leq \|v_n-u_n\| + H(P_T\,(y_n)\,,P_T\,(p)) \\ &\leq \|v_n-u_n\|+\|y_n-p\|\end{aligned}$$

gives

$$c \leq \liminf_{n\to\infty} \|y_n-p\|\,. \tag{2.10}$$

From (2.8) and (2.10), we have

$$\lim_{n\to\infty} \|y_n-p\| = c$$

Since from (2.5) and (2.7) appliyng Lemma 2, we get

$$\lim_{n\to\infty} \|v_n-w_n\| = 0.$$

Also taking limsup on both sides of (2.3), we obtain

$$\limsup_{n\to\infty} \|z_n-p\| \leq c. \tag{2.11}$$

Thus

$$\begin{aligned}\|v_n - p\| &\leq \|v_n - w_n\| + \|w_n - p\| \\ &\leq \|v_n - w_n\| + H(P_T(z_n), P_T(p)) \\ &\leq \|v_n - w_n\| + \|z_n - p\|\end{aligned}$$

give

$$c \leq \limsup_{n\to\infty} \|z_n - p\| \tag{2.12}$$

and, from (2.11) and (2.12) we get

$$\lim_{n\to\infty} \|z_n - p\| = c.$$

Applying Lemma 2 once again,

$$\lim_{n\to\infty} \|x_n - v_n\| = 0. \tag{2.13}$$

Since $d(x_n, Tx_n) \leq \|x_n - v_n\|$, we have

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

□

Now we approximate fixed points of the mapping T through weak convergence of the sequence $\{x_n\}$ defined in (1.3).

Theorem 2. *Let E be a uniformly convex Banach space satisfying Opial's condition and K a nonempty closed convex subset of E. Let $T : K \to P(K)$ be a multivalued mapping such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping. Let $\{x_n\}$ be the sequence as defined in (1.3). Let $I - P_T$ be demiclosed with respect to zero, then $\{x_n\}$ converges weakly to fixed point of T.*

Proof. Let $p \in F(T) = F(P_T)$. As in the proof of Lemma 4, $\lim_{n\to\infty} \|x_n - p\|$ exists. Now we prove that $\{x_n\}$ has a unique weak subsequently limit in $F(T)$. To prove this, let z_1 and z_2 be weak ences limits of the subsequences $\{x_{n_i}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. By (2.13), there exists $v_n \in Tx_n$ such that $\lim_{n\to\infty} \|x_n - v_n\| = 0$. Since $I - P_T$ is demiclosed with respect to zero, therefore we obtain $z_1 \in F(P_T) = F(T)$. In the same way, we can prove that $z_2 \in F(T)$. Next, we prove uniqueness. For this, suppose that $z_1 \neq z_2$. Then by Opial's condition, we have

$$\begin{aligned}
\lim{}_{n\to\infty} \|x_n - z_1\| &= \lim_{n_i\to\infty} \|x_{n_i} - z_1\| \\
&< \lim_{n_i\to\infty} \|x_{n_i} - z_2\| \\
&= \lim_{n\to\infty} \|x_n - z_2\| \\
&= \lim_{n_j\to\infty} \left\|x_{n_j} - z_2\right\| \\
&< \lim_{n_i\to\infty} \left\|x_{n_j} - z_1\right\| \\
&= \lim_{n\to\infty} \|x_n - z_1\|.
\end{aligned}$$

This is a contradiction. Hence $\{x_n\}$ converges weakly to a point in F. $\square$

We now give some strong convergence theorems. Our first strong convergence theorem is valid in general real Banach spaces. We then apply this theorem to obtain a result in uniformly convex Banach spaces.

Theorem 3. *Let E be a real Banach space and K a nonempty closed convex subset of E. Let $T : K \to P(K)$ be a a multivalued mapping such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping. Let $\{x_n\}$ be the sequence as defined in (1.3). Then $\{x_n\}$ converges strongly to fixed point of $F(T)$ if and only $\liminf_{n\to\infty} d(\ x_n, F(T)) = 0$.*

Proof. The necessity is obvious. Conversely, suppose that $\liminf_{n\to\infty} d(\ x_n, F(T)) = 0$. By Lemma 4, we have

$$\|x_{n+1} - p\| \leq \|x_n - p\|.$$

This gives

$$d(x_{n+1}, F(T) \leq d(x_n, F(T)).$$

Hence $\lim_{n\to\infty} d(\ x_n, F(T))$ exists. By hypothesis, $\liminf_{n\to\infty} d(\ x_n, F(T)) = 0$ so we must have $\lim_{n\to\infty} d(\ x_n, F(T)) = 0$. Next, we show that $\{x_n\}$ is a Cauchy sequence in

K. Let $\varepsilon > 0$ be arbitrarily chosen. Since $\lim_{n\to\infty} d(\ x_n, F(T) = 0$, there exists n_0 such that for all $n \geq n_0$. We have

$$d\left(x_n, F\left(T\right)\right) < \frac{\varepsilon}{4}.$$

In particular, $\inf\left\{\|x_{n_0} - p\| : p \in F(T)\right\} < \frac{\varepsilon}{4}$; so there must exists a $p^* \in F(T)$ such that

$$\|x_{n_0} - p^*\| < \frac{\varepsilon}{2}.$$

Now for m,n $\geq n_0$, we have

$$\begin{aligned} \|x_{n+m} - x_n\| &\leq \|x_{n+m} - p^*\| + \|x_n - p^*\| \\ &\leq 2\left\|x_{n_0} - p^*\right\| \\ &< 2\left(\frac{\varepsilon}{2}\right) = \varepsilon. \end{aligned}$$

Hence $\{x_n\}$ is a Cauchy sequence in closed subset K of a Banach space E, and therefore it must converge in K. Let $\lim_{n\to\infty} x_n = q$. Now

$$\begin{aligned} d\left(q, P_T(q)\right) &\leq \|x_n - q\| + d\left(x_n, P_T(x_n)\right) + H\left(P_T(x_n), P_T(q)\right) \\ &\leq \|x_n - q\| + \|x_n - v_n\| + \|x_n - q\| \\ &\to 0 \text{ as } n \to \infty \end{aligned}$$

Which gives that implies $d\left(q, P_T\left(q\right)\right) = 0$. But P_T is a nonexpansive mapping so $F\left(P_T\right)$ is closed. Therefore, $q \in F\left(P_T\right) = F\left(T\right)$. □

Theorem 4. *Let E be a uniformly convex Banach space and K a nonempty closed convex subset of E. Let $T : K \to P\left(K\right)$ be a multivalued mapping satisfying condition (I) such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping. Let $\{x_n\}$ be the sequence as defined in (1.3). Then $\{x_n\}$ converges strongly to point of $F(T)$.*

Proof. By lemma 2, $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T)$. Let this limit be c for some $c \geq 0$.If $c = 0$, there is nothing to prove. Suppose $c > 0$. Now $\|x_{n+1} - p\| \leq \|x_n - p\|$ gives

$$\inf_{p\in F(T)} \|x_{n+1} - p\| \leq \inf_{p\in F(T)} \|x_n - p\|,$$

which implies that $d(x_{n+1}, F(T)) \leq d(x_n, F(T))$ and so $\lim_{n\to\infty} d(x_n, F(T))$ exists. By using condition (I) and Lemma 5, we have

$$\lim_{n\to\infty} f(d(x_n, F(T))) \leq \lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

That is,

$$\lim_{n\to\infty} f(d(x_n, F(T))) = 0$$

Since f is a decreasing function and $f(0) = 0$, it follows that $\lim_{n\to\infty} d(x_n, F(T)) = 0$.

□

Remark 1. *Our iteration scheme extends results of Khan and Yildirim* [7] *with Mann type one-step iteration schemes for multivalued nonexpansive mappings.*

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