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FIXED POINT THEOREMS OF GENERALIZED T-CYCLIC WEAK $\varphi - \phi$-CONTRACTION OPERATORS

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Abstract. In this paper, we introduce a new cyclic weak contraction type of mapping and prove the existence of a fixed point for such a type. The results presented in this paper mainly generalize the corresponding results in [9] and [11].

Keywords: Fixed point; Contraction; Cyclic weak contraction types of mapping; Metric space.

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1. Introduction

The Banach contraction mapping principle [2] is a very popular tool for solving the existence of problems in many branches of mathematical analysis. Generalizations of this principle have been established in various settings, for more details; see the references therein. Alber and Guerre-Delabriere in [1] introduced the definition of weakly contractive mappings and they proved some fixed point theorems in the context of Hilbert spaces. Rhoades in [13] and [14] extended the results of [1] to complete metric spaces. The following definitions are necessary.

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Definition 1.1. A self mapping T on a metric space X is called weak ϕ- contraction if and only if there exists a continuous nondecreasing function $\phi:[0,+\infty)\to[0,+\infty)$ with $\phi(t)=0$ if and only if $t=0$ such that

$$d(T(x),T(y))\leq d(x,y)-\phi(d(x,y)),\quad \forall x,y\in X.$$

Definition 1.2. Let X be a nonempty set, j a positive integer, T a self operator on X, and $X=\cup_{i=1}^{j}A_i$. Then $X=\cup_{i=1}^{j}A_i$ is said to be cyclic representation of X with respect to T if

(1) A_i, $i=1,2,\ldots,j$ are nonempty subsets of X.

(2) $T(A_1)\subset A_2$, $T(A_2)\subset A_3$, $\ldots$, $T(A_{j-1})\subset A_j$, and $T(A_j)\subset A_1$.

Definition 1.3. Let (X,d) be a metric space, j a positive integer, $\{A_i\}_{i=1}^{j}$ a nonempty closed subsets of X, and $X=\cup_{i=1}^{j}A_i$. A self mapping T on a metric space X is said to be cyclic weak contraction mapping on X if and only if there is a continuous nondecreasing mapping $\phi:[0,+\infty)\to[0,+\infty)$ with $\phi(t)=0$ if and only if $t=0$ such that the following are true:

(1) $X=\cup_{i=1}^{j}A_i$ is a cyclic representation of X with respect to T.

(2)

$$d(T(x),T(y))\leq d(x,y)-\phi(d(x,y))$$
$$\text{for every } x\in A_i, y\in A_{i+1}, i=1,2,\ldots,j, \text{ where} A_{j+1}=A_1.$$

Recent results related to cyclic weak ϕ- contraction mappings have appeared in [9] for mappings with cyclic representations in complete metric spaces, the considered mapping ϕ need not be continuous. Another fixed point had been given for cyclic weak ϕ-contraction mappings in the frame of compact metric spaces [5], the considered mapping ϕ need not be continuous.

They separately formulated the following results.

Theorem 1.1. [9] *Let (X,d) be a complete metric space, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X=\cup_{i=1}^{j}A_i$, and T be cyclic weak ϕ- contraction on X for some mapping ϕ (not necessarily be continuous) and with respect to $\{A_i\}_{i=1}^{j}$. Then T has a unique fixed point x, $x\in\cap_{i=1}^{j}A_i$.*

Theorem 1.2. [5] *Let (X,d) be a compact metric space, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X=\cup_{i=1}^{j}A_i$, and T be a continuous cyclic weak ϕ- contraction on X for some mapping ϕ (not necessarily be continuous) and with respect to $\{A_i\}_{i=1}^{j}$. Then T has a unique fixed point x, $x\in\cap_{i=1}^{j}A_i$.*

Since each compact metric space is complete, the fixed point theorem given in [9] is concerned with a wider class of metric spaces and the second fixed point theorem is a special case of the second. On the other side, Morales and Rojas [11] defined TZ type operators or the T-zamfirescu operators, the operators of any of the following types:

(1) TB-type or T-Banach contraction,
(2) TK type or T-Kannan contraction,
(3) TC type or T-Chatterjea contraction,

which are defined as follows:

Definition 1.4. Let X be metric space and S, T be self operators on X. Then the operator S is said to be T-zamfirescu operator if and only if there are real numbers $0\le a<1$ and $0\le b,c<\frac{1}{2}$ such that at least one of the following is true

(1) TB-type or T-Banach contraction,

$$d(T(S(x)),T(S(y)))\le d(T(x),T(y))\ \forall x,y\in X.$$

(2) TK type or T-Kannan contraction,

$$d(T(S(x)),T(S(y)))\le b[d(T(x),T(S(x)))+d(T(y),T(S(y)))],\quad \forall x,y\in X.$$

(3) TC type or T-Chatterjea contraction,

$$d(T(S(x)),T(S(y)))\le b[d(T(x),T(S(y)))+d(T(y),T(S(x)))],\quad \forall x,y\in X.$$

They proved the exitance of a unique fixed point for such types of mappings for the continuous, one to one, and subsequentially convergent operator T. For Generalized $\{a,b,c\}$-generalized contraction and non-expansive type mappings, see [19] and [20].

In this paper, we consider a self generalized contraction type of mapping S, the mapping that is satisfying some T-cyclic weak $\varphi-\phi$ contraction condition.

Definition 1.5. Let X be a nonempty set, j be a positive integer, T and S be self operators on X, and $X=\cup_{i=1}^{j}A_i$. Then $X=\cup_{i=1}^{j}A_i$ is said to be T-cyclic representation of X with respect to S if and only if

(1) A_i, $i=1,2,\ldots,j$ are nonempty subsets of X.

(2) $T(S(A_1))\subset A_2$, $T(S((A_2))\subset A_3,\ \ldots\ ,T(S(A_{j-1}))\subset A_j$, and $T(S((A_j))\subset A_1$.

Definition 1.6. Let (X,d) be a metric space, j be a positive integer, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X=\cup_{i=1}^{j}A_i$, and T be a self operator on X. Then the self operator S on a metric space X is said to be T-cyclic weak $\varphi-\phi$ contraction operator on X if and only if there are a subadditive nondecreasing continuous mapping, $\varphi:[0,+\infty)\to[0,+\infty)$ with $\varphi(t)=0$ if and only if $t=0$ and a nondecreasing mapping $\phi:[0,+\infty)\to[0,+\infty)$ with $\phi(t)=0$ if and only if $t=0$ such that the following are true:

(1) $X=\cup_{i=1}^{j}A_i$ is a T-cyclic representation of X with respect to S.

(2)

$$\varphi(d(T(S(x)),T(S(y))))\leq\varphi(d(T(x),T(y)))-\phi(d(T(x),T(y)))$$

$$\text{for every } x\in A_i, y\in A_{i+1}, i=1,2,\ldots,j, \text{ where} A_{j+1}=A_1.$$

Remark 1.1. If T and φ in particular are taken to be the identity operator and mapping, then we will have in particular Karapinar, Sadarangani and Morales, Rojas types of operators, hence our defined operator generalizes the definition of cyclic weak ϕ-contraction of Erdal Karapinar, Kishin Sadarangani, and TB contraction operators of Morales, Rojas [9] and [11].

We are in need of the following definition:

Definition 1.7. The self operator on the metric space (X,d) is said to be sequentially convergent if we have, for every sequence $\{x_n\}_{n=1}^{\infty}$, if $\{T(x_n)\}_{n=1}^{\infty}$ is convergent, then $\{x_n\}_{n=1}^{\infty}$ is convergent.

2. Main results

Our main results are depending on the following Propositions:

Proposition 2.1. *Let (X,d) be a metric space, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X = \cup_{i=1}^{j} A_i$, φ and ϕ are two nondecreasing functions such that $\phi(t) = 0$ if and only if $t = 0$. If T and S are self operators on X, $\{A_i\}_{i=1}^{j}$ is T-cyclic with respect to S and S is weak $\varphi - \phi$-contraction on X, then*

$$\inf\{\|T(S(x)) - T(x)\| : x \in X\} = 0.$$

Proof. Choose $x_0 \in X$ and consider the iterated sequence given by

$$T(x_{n+1}) = T(S(x_n)) = T(S^n(x_0)), \quad \forall n = 0, 1, 2, \ldots.$$

If there exists a natural number n_0 such that $S(x_{n+1}) = S(x_n)$, $\forall n \geq n_0$, then the existence of the fixed point of S is proved which insures that such infimum is zero. Suppose that $T(S(x_{n+1})) \neq T(S(x_n))$ for all $n = 0, 1, 2, \ldots$. Then, since $X = \cup_{i=1}^{j} A_i$, for any $n > 0$ there exists $i_n \in \{1, 2, \ldots, j\}$ such that $T(x_{n-1}) \in A_{i_{n-1}}$ and $T(x_n) \in A_{i_n}$ and consequently using the contraction condition we get

$$\begin{aligned}
\varphi(d(T(x_n), T(x_{n+1}))) &= \varphi(d(T(S(x_{n-1})), T(S(x_n)))) \\
&\leq \varphi(d(T(x_{n-1}), T(x_n))) - \phi(d(T(x_{n-1}), T(x_n))) \qquad (1) \\
&\leq \varphi(d(T(x_{n-1}), T(x_n))).
\end{aligned}$$

Since φ is nondecreasing function, we see that

$$d(T(x_n), T(x_{n+1})) \leq d(T(x_{n-1}), T(x_n)), \forall n \in N.$$

This proves that the sequence $\{d(T(x_n), T(x_{n+1}))\}_{n \in N}$ is a nondecreasing sequence of positive real numbers, hence the limit,

$$\lim_{n \to \infty} d(T(x_n), T(x_{n+1}))$$

exits and it is equal to the infimum of the sequence, say r,

$$r = \lim_{n \to \infty} d(T(x_n), T(x_{n+1})) = \inf\{d(T(x_n), T(x_{n+1})) : n \in N\},$$

$$r \leq d(T(x_n), T(x_{n+1})), \forall n \in N.$$

On the other side, we have the same conclusions for the two sequences $\{\varphi(d(T(x_n),T(x_{n+1})))\}_{n\in N}$ and $\{\phi(d(T(x_n),T(x_{n+1})))\}_{n\in N}$, consider the two positive real numbers ϕ_0 and φ_0 given as follows:

$$\begin{aligned}\phi(r) \leq \phi_0 &= \lim_{n\to\infty} \phi(d(T(x_n),T(x_{n+1}))) \\ &= \inf\{\phi(d(T(x_n),T(x_{n+1}))) : n\in N\}\end{aligned}$$

and

$$\begin{aligned}\varphi(r) \leq \varphi_0 &= \lim_{n\to\infty} \varphi(d(T(x_n),T(x_{n+1}))) \\ &= \inf\{\varphi(d(T(x_n),T(x_{n+1}))) : n\in N\}.\end{aligned}$$

Taking the limit of each side of the inequalities (1) as $n\to\infty$ gives

$$\varphi_0 \leq \varphi_0 - \lim_{n\to\infty} \phi(d(T(x_{n-1}),T(x_n))) \leq \varphi_0$$

and, therefore

$$\lim_{n\to\infty} \phi(d(T(x_{n-1}),T(x_n))) = 0.$$

Now, suppose that $r>0$, we have $\phi(r)>0$, thus

$$0<\phi(r) \leq \phi(d(T(x_{n-1}),T(x_n)))\,\forall n\in N$$

Letting $n\to\infty$ in the last inequalities, we get the following contradiction

$$0<\phi(r) \leq \lim_{n\to\infty} \phi(d(T(x_{n-1}),T(x_n))) = 0.$$

That is; the assumption $r>0$ is not true, hence $r=0$, thus

$$\lim_{n\to\infty} d(T(x_{n+1}),T(x_n)) = \inf\{d(T(x_{n+1}),T(x_n)) : n\in N\} = 0$$

Hence, there is a sequence $\{x_n\}_{n=1}^{\infty}$ in X such that

$$\lim_{n\to\infty} d(T(S(x_n)),T(x_n)) = 0.$$

This is sufficiently proved that $\inf\{\|T(S(x))-T(x)\| : x\in X\} = 0$.

Proposition 2.2. Let (X,d) be a metric space, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X=\cup_{i=1}^{j}A_i$, φ and ϕ be two nondecreasing continuous functions such that φ is subadditive and

$\phi(t) = 0$ if and only if $t = 0$. If T and S are self operators on X, $\{A_i\}_{i=1}^{j}$ is T cyclic with respect to S and S is weak $\varphi - \phi$- contraction on X, then the sequence of iterates $\{T(x_n) = T(S^n(x_0))\}_{n\in N}$ is a Cauchy sequence.

Proof. We prove by contradiction that for every $\varepsilon > 0$ there exists $n_0 \in N$ such that if $m, n > n_0$ with x_n and x_m lie in different adjacently labeled sets A_i and A_{i+1} for some $i \in \{1, 2, \ldots, j\}$ $(n - m \equiv 1(j))$ then

$$d(T(x_n), T(x_m)) < \varepsilon.$$

For the purpose suppose that there exists $\varepsilon > 0$ such that for any $n_0 \in N$ we can find $m, n > n_0$ with $T(x_n)$ and $T(x_m)$ lie in different adjacently labeled sets A_i and A_{i+1} for some $i \in \{1, 2, \ldots, j\}$ $(n - m \equiv 1(j))$ satisfying

$$\varepsilon \leq d(T(x_n), T(x_m)).$$

Since φ and ϕ are nondecreasing, we see that

$$0 < \varphi(\varepsilon) \leq \varphi(d(T(x_n), T(x_m))) \,\text{and}\, 0 < \phi(\varepsilon) \leq \phi(d(T(x_n), T(x_m))) \tag{2}$$

and by the triangle inequality, we have

$$\begin{aligned}
\varepsilon &\leq d(T(x_n), T(x_m)) \\
&\leq d(T(x_n), T(x_{n+1})) + d(T(x_{n+1}), T(x_{m+1})) + d(T(x_{m+1}), T(x_m)) \\
&= d(T(x_n), T(x_{n+1})) + d(T(S(x_n)), T(S(x_m))) + d(T(x_{m+1}), T(x_m)).
\end{aligned}$$

Since φ is subadditive and nondecreasing, we get

$$\begin{aligned}
0 < \varphi(\varepsilon) &\leq \varphi(d(T(x_n), T(x_m))) \\
&\leq \varphi(d(T(x_n), T(x_{n+1}))) + \varphi(d(T(S(x_n)), T(S(x_m)))) \\
&+ \varphi(d(T(x_{m+1}), T(x_m))).
\end{aligned}$$

Using the contraction assumption of S gives

$$\begin{aligned}\varphi(d(T(x_n),T(x_m))) \leq\; &\varphi(d(T(x_n),T(x_{n+1})))\\ &+[\varphi(d(T(x_n),T(x_m)))-\phi(d(T(x_n),T(x_m)))]\\ &+\varphi(d(T(x_{m+1}),T(x_m))).\end{aligned}$$

These inequalities prove the following:

$$\phi(d(T(x_n),T(x_m))) \leq \varphi(d(T(x_n),T(x_{n+1})))+\varphi(d(T(x_{m+1}),T(x_m))). \tag{3}$$

Using equations (2) and (3), we see that

$$0<\phi(d(T(x_n),T(x_m))) \leq \varphi(d(T(x_n),T(x_{n+1})))+\varphi(d(T(x_{m+1}),T(x_m))). \tag{4}$$

Using Proposition (2.1) and the continuity of φ give

$$\lim_{n\to\infty}\varphi(d(T(x_n),T(x_{n+1})))=0.$$

Now; letting $n,m\to\infty$ in (4) with $n-m\equiv 1(j)$ proves the following contradiction:

$$0<\phi(\varepsilon)\leq \lim_{n,m\to\infty_{n-m\equiv 1(j)}}\phi(d(T(x_n),T(x_m)))\leq 0.$$

Hence, we have

$$\lim_{n,m\to\infty_{n-m\equiv 1(j)}}\phi(d(T(x_n),T(x_m)))=0.$$

Since ϕ is continuous, we have

$$\phi(\lim_{n,m\to\infty_{n-m\equiv 1(j)}}d(T(x_n),T(x_m)))=\lim_{n,m\to\infty_{n-m\equiv 1(j)}}\phi(d(T(x_n),T(x_m)))=0.$$

This proves that

$$\lim_{n,m\to\infty_{n-m\equiv 1(j)}}d(T(x_n),T(x_m))=0.$$

That is, the sequence of iterates is having a Cauchy subsequence. Hence; the sequence of iterates itself is a Cauchy sequence.

Theorem 2.1. *Let (X,d) be a complete metric space, $\{A_i\}_{i=1}^{j}$ be a nonempty closed subsets of X, $X=\cup_{i=1}^{j}A_i$, φ and ϕ are two nondecreasing continuous functions such that φ is subadditive, $\varphi(t)=0$ if and only if $t=0$ and $\phi(t)=0$ if and only if $t=0$. If T is a one to one continuous self sequentially convergent operator on X, $\{A_i\}_{i=1}^{j}$ is T-cyclic representation of X with respect* *to the self operator S and S is weak $\varphi-\phi$- contraction on X, then T has fixed point z and there is $y\in\cap_{i=1}^{j}A_i$ such that $T(z)=y$. Moreover two consecutive sets of $\{A_i\}_{i=1}^{j}$ can not contain two different fixed points.*

Proof. Using Proposition (2.2), the sequence of iterates $\{T(S^n(x_0))\}_{n\in N}$ is Cauchy. Using the completeness of the metric space X, there exists a point $y\in X$ such that

$$\lim_{n\to\infty}T(S^n(x_0))=y. \tag{5}$$

Such a limit point is lying in the set $\cap_{i=1}^{j}A_i$, in fact; since $X=\cup_{i=1}^{j}A_i$ is a T-cyclic representation of X with respect to S and the sequence $\{T(S^n(x_0))\}_{n\in N}$ is infinite sequence, each A_i, $i=1,2,\ldots,j$ contains infinitely many members of $\{T(S^n(x_0))\}_{n\in N}$, this shows that y is a limit point for A_i for each $i=1,2,\ldots,j$, giving that A_i is closed for each $i=1,2,\ldots,j$ shows that $y\in A_i$ for each $i=1,2,\ldots,j$ (as closed set contains all its limit points), thus

$$y\in\cap_{i=1}^{j}A_i.$$

Since T is sequentially convergent operator, the sequence $\{S^n(x_0)\}_{n=1}^{\infty}$ has a convergent subsequence $\{S^{n_k}(x_0)\}_{n=1}^{\infty}$, hence there is $z\in X$ such that

$$\lim_{k\to\infty}S^{n_k}(x_0)=z.$$

Using the continuity of T gives that

$$\lim_{k\to\infty}T(S^{n_k}(x_0))=T(z). \tag{6}$$

Using (5) and (6), we see that $T(z)=y$. We claim that such a z is fixed point of S. In fact; there is $i\in\{1,2,\ldots,j\}$ such that $z\in A_i$, for $i+1$ there exists $n_i\in N$ such that

$$\{x_{n_i},x_{n_i+1},x_{n_i+2},\ldots\}\subset A_{i+1}.$$

Therefore z and $\{x_{n_i},x_{n_i+1},x_{n_i+2},\ldots\}$ are lying in two consecutive sets, hence we can use the contraction property of S as:

$$\begin{aligned}d(T(S(z)),T(z))\leq{}& d(T(S(z)),T(S^{n_k}(x_0)))+d(T(S^{n_k}(x_0)),T(S^{n_{k+1}}(x_0)))\\&+d(T(S^{n_{k+1}}(x_0)),T(z)).\end{aligned}$$

Thus, we have

$$\varphi(d(T(S(z)),T(z))) \leq \varphi(d(T(S(z)),T(S^{n_k}(x_0)))) + \varphi(d(T(S^{n_k}(x_0)),T(S^{n_{k+1}}(x_0))))$$
$$+ \varphi(d(T(S^{n_{k+1}}(x_0)),T(z))).$$

Hence, we have

$$\varphi(d(T(S(z)),T(z))) \leq [\varphi(d(T(z)),T(S^{n_{k-1}}(x_0))) - \phi(d(T(z)),T(S^{n_{k-1}}(x_0)))]$$
$$+ \varphi(d(T(S^{n_k}(x_0)),T(S^{n_{k+1}}(x_0)))) + \varphi(d(T(S^{n_{k+1}}(x_0)),T(z))).$$

$$\varphi(d(T(S(z)),T(z))) \leq \varphi(d(T(z)),T(S^{n_{k-1}}(x_0)))) + \varphi(d(T(S^{n_k}(x_0)),T(S^{n_{k+1}}(x_0))))$$
$$+ \varphi(d(T(S^{n_{k+1}}(x_0)),T(z))).$$

Taking the limit as $k \to \infty$ proves that $\varphi(d(T(S(z)),T(z))) = 0$, hence $d(T(S(z)),T(z)) = 0$, therefore, $T(S(z)) = T(z)$, since T is one to one, we get $S(z) = z$ and hence proves that z is a fixed point of S.

To show that two consecutive sets of $\{A_i\}_{i=1}^{j}$ can not contain two different fixed points, by contrary assume that there are two different fixed points of S, namely w and z, $S(w) = w$ and $S(z) = z$ those are lying in two consecutive sets, we have the following:

$$\varphi(d(T(w),T(z))) = \varphi(d(T(S(w)),T(S(z))))$$
$$\leq \varphi(d(T(w),T(z))) - \phi(d(T(w),T(z)))$$

hence $\phi(d(T(w),T(z))) = 0$. Since $\phi(t) > 0$ for $t \in (0,\infty)$, we get $d(T(w),T(z)) = 0$, consequently $T(w) = T(z)$, since T is one to one, $w = z$. This completes the proof.

In the case of normed spaces, we have the following reduced results.

Theorem 2.2. *Let $(X, \|.\|)$ be a weakly complete metric space, C be a closed convex subset of X, $\{C_i\}_{i=1}^{j}$ be a nonempty closed subsets of C, $C = \cup_{i=1}^{j} C_i$, φ and ϕ are two nondecreasing continuous functions such that φ is subadditive and nondecreasing mapping, $\varphi(t) = 0$ if and only if $t = 0$ and $\phi(t) = 0$ if and only if $t = 0$. If T and S are self operators on C, $\{C_i\}_{i=1}^{j}$ is T cyclic with respect to S and S is weak $\varphi - \phi$- contraction on C, then S has fixed points.*

Proof. Using Proposition (2.2) the sequence of iterates $\{T(S^n(x_0))\}_{n\in N}$ is Cauchy, using the weak completeness assumption of X there exists $x \in X$ such that

$$w - \lim_{n\to\infty} T(S^n(x_0)) = x.$$

Since C is closed convex subset of X, the sequence $\{T(S^n(x_0))\}_{n\in N}$ converges strongly to x and $x \in C$. The other parts of the proof is similar to Theorem 2.1.

Remark Reduced assumptions have been given in Theorem 2.2 to prove the existence of fixed point of cyclic weak φ-ϕ-contraction operators.

Conclusion This paper suggests new cyclic weak contraction type of operators and proved the existence of unique fixed point for such types of operators.

Conflict of Interests

The authors declare that there is no conflict of interests.

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