LOGARITHMIC AND IDENTRIC MEAN LABELINGS OF GRAPHS

S. ALAGU*, R. KALA

Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli - 627 012, Tamilnadu, India

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Abstract. Graph labeling was first introduced by Rosa in 1966. Labeling of graphs is an assignment of non-negative integers to vertices, edges or both according to some specified conditions. Mean labeling of graphs was introduced by Somasundaram and Ponraj in 2003. Subsequently, labelings of graphs were done with geometric mean, harmonic mean etc. In this paper we introduce two concepts called 'Logarithmic mean labeling’, 'Identric mean labeling’ and acquire their mean labelings for some standard graphs.

Keywords: logarithmic mean; identric mean; labeling; mean graphs; comb; crown; path; cycle.

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1. INTRODUCTION

Labeled graphs provide mathematical models for a broad range of applications. Labelings of graph elements have been used in diverse fields such as Radar location codes, Coding theory, Circuit design, Communication network etc...

In Molecular Chemistry (or Cell Biology), different molecules (or cells) bear different numbers for chemical (or biological) characteristics. The byproducts of a chemical reaction (or a biological process) are supposed to inherit some type of average (equivalently mean) as its characteristic number. What type of mean depends on the particular characteristic and it is determined by

*Corresponding author
E-mail address: alagu391@gmail.com
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observations. Also different graphs represent different molecular structure. This motivates the study of various mean labelings for graphs. Here we come up with new types of mean labeling called Logarithmic and Identric mean labeling.

First we give the definitions of Logarithmic mean labeling and Identric mean labeling and make some observations on the ranges of both the means for any two positive integers upto a distance of 7 which will be helpful while labeling graphs. We collect all the relevant definitions needed in the sequel.

**Definition 1.1.** A graph is an ordered pair $G = (V, E)$ where $V$ is a set of vertices and $E$ is a set of edges which are unordered pair of vertices from $V$. This type of graph is called undirected and simple. In addition, if the number of vertices is finite, then the graph is called finite too.

**Definition 1.2.** Let $V = \{x_1, x_2, \ldots, x_n\}$ and $E = \{(x_i, x_{i+1}) / 0 \leq i \leq n - 1\}$. The graph $(V, E)$ is called a path on $n$ vertices and is denoted by $P_n$.

**Definition 1.3.** Let $V = \{x_1, x_2, \ldots, x_n\}$ and $E = \{(x_i, x_{i+1}) / 0 \leq i \leq n - 1\} \cup \{(x_1, x_n)\}$. The graph $(V, E)$ is called a cycle on $n$ vertices and is denoted by $C_n$.

**Definition 1.4.** Any cycle with a pendent edge attached to each vertex is called a crown.

**Definition 1.5.** The graph obtained by joining a single pendent edge to each vertex of a path is called a comb.

**Definition 1.6.** A graph on $n$ vertices in which any two vertices are adjacent is called a complete graph and is denoted by $K_n$.

**Definition 1.7.** A graph is bipartite if the set of vertices $V(G)$ can be partitioned into two nonempty subsets $X$ with $m$ vertices and $Y$ with $n$ vertices such that each edge has one end in $X$ and the other end in $Y$. It is denoted by $K_{m,n}$.

**Definition 1.8.** The complete bipartite graph $K_{1,n}$ is called a star.

**Definition 1.9.** The bistar $B_{m,n}$ is the graph obtained from $K_2$ by joining $m$ pendent edges to one end of $K_2$ and $n$ pendent edges to the other end of $K_2$.

**Definition 1.10.** A Triangular snake denoted by $T_n$ is obtained from a path $v_1v_2\ldots v_n$ by joining $v_i$ and $v_{i+1}$ to a new vertex $w_i$ for $1 \leq i \leq n - 1$. It is denoted by $T_n$.
Definition 1.11. A **Quadrilateral snake** is obtained from a path $u_1u_2...u_n$ by joining $u_i,u_{i+1}$ to a new pair of vertices $v_i,w_i$ respectively and joining $v_i$ and $w_i$. It is denoted by $Q_n$.

Definition 1.12. A **Ladder** graph, denoted by $L_n$ is a graph obtained from the cartesian product of $P_n$ and $P_1$.

Definition 1.13. A tree which yields a path on removing its pendent vertices is called **caterpillar**.

Definition 1.14. A **dragon** is formed by joining the end of a path to a cycle.

Definition 1.15. Let $a$ and $b$ be two positive integers. The Logarithmic mean of $a$ and $b$, denoted as $L(a,b)$, is defined by

$$L(a,b) = \frac{b-a}{\log b - \log a}.$$ 

Definition 1.16. Let $a$ and $b$ be two positive integers. The Identric mean of $a$ and $b$, denoted as $I(a,b)$, is defined by

$$I(a,b) = \frac{e^{\left(\frac{a^a}{b^b}\right)^{\frac{1}{a-b}}}}{e}.$$ 

Fact 1. [3]. For distinct positive numbers $a$ and $b$, $G(a,b) < L(a,b) < I(a,b) < A(a,b)$, where $G(a,b)$ and $A(a,b)$ denote respectively, the geometric and arithmetic means of $a,b$.

2. **Main Results**

Definition 2.1. Let $G = (V,E)$ be a graph with $p$ vertices and $q$ edges. $G$ is said to be a Logarithmic mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,...q+1$ in such a way that on labeling each edge $e = xy$ with 

$$\left\lfloor \frac{y-x}{\log y - \log x} \right\rfloor \text{ or } \left\lceil \frac{y-x}{\log y - \log x} \right\rceil,$$

the resulting edge labels are distinct and are from $1,2,...q$. In this case $f$ is called a Logarithmic mean labeling.

Definition 2.2. Let $G = (V,E)$ be a graph with $p$ vertices and $q$ edges. $G$ is said to be an Identric mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,...q+1$ in such a way that on labeling each edge $e = xy$ with 

$$\left\lfloor \frac{1}{e} \left(\frac{x^a}{y^b}\right)^{\frac{1}{a-b}} \right\rfloor \text{ or } \left\lceil \frac{1}{e} \left(\frac{x^a}{y^b}\right)^{\frac{1}{a-b}} \right\rceil,$$

the resulting edge labels are distinct and are from $1,2,...q$. In this case $f$ is called an Identric mean labeling.

Remark 2.3. For $G$ to be a Logarithmic/Identric mean graph, $(1,2)$ or $(1,3)$ must be the labels for some pair of vertices, since with respect to Logarithmic/Identric mean labeling, only the labels $(1,2)$ and $(1,3)$ for vertices allow us to give the label 1 for an edge.
**Remark 2.4.** Clearly a \((p, q)\) graph cannot admit Logarithmic and Identric mean labeling if \(p > q + 1\).

**Remark 2.5.** For \(G\) to be a Logarithmic/Identric mean graph, \(q\) must be a label for one of the edges. To obtain \(q\) as one of the edge labels, some pair of vertices must receive one of the following set of labels: \((q-1, q), (q-1, q+1), (q, q+1), (q-2, q+1)\). This claim can be substantiated using the following two propositions.

**Proposition 2.6.** Let \(a, b\) be positive integers with \(a < b\). Then
1. \(a < L(a, b) < b\).
2. \(a < I(a, b) < b\).

**Proof.**
1. To prove: \(a < L(a, b) < b\)

   That is to prove: \(a < \frac{b-a}{\log b - \log a} < b\)

   To prove: (i) \(a < \frac{b-a}{\log b - \log a}\) (ii) \(b > \frac{b-a}{\log b - \log a}\)

   Proof of (i): Enough to prove (henceforth abbreviated as ETP) \(a < \frac{b-a}{\log b}\)

   ETP: \(a \log \frac{b}{a} < b - a\)

   ETP: \(a \log \left(1 + \frac{h}{a}\right) < h\) where \(b = a + h\; ; \; h > 0\)

   ETP: \(\log (1 + x) < x\) for \(x > 0\)

   ETP: \(f(x) = x - \log (1 + x) > 0\), for \(x > 0\)

   Now \(f'(x) = 1 - \frac{1}{1+x} > 0\) for \(x > 0\).

   Therefore, \(f\) is strictly increasing in \(\infty\). Also \(f(0) = 0\).

   Therefore, \(f(x) > f(0) = 0\; \forall x > 0\).

   Hence (i) - the left side of the inequality of (1) is proved.

   Proof of (ii): ETP: \(\frac{b-a}{\log b} < b\)

   ETP: \(b - a < b \log \frac{b}{a}\)

   ETP: \(h < b \log \left(1 + \frac{h}{a}\right)\) where \(b = a + h\; ; \; h > 0\)

   Note that \(\frac{h}{b} = \frac{h}{a+h}\)

   \[= 1 - \frac{a}{a+h}\]

   \[= 1 - \frac{1}{1 + \frac{h}{a}}\]
ETP : \( 1 - \frac{1}{1 + \frac{h}{a}} < \log \left( 1 + \frac{h}{a} \right) \)

ETP : \( 1 - \frac{1}{1 + x} < \log (1 + x) \) for \( x > 0 \).

ETP : \( g(x) = \log (1 + x) - 1 + \frac{1}{1 + x} > 0 \) for \( x > 0 \).

Now \( g'(x) = \frac{1}{1 + x} - \frac{1}{(1 + x)^2} > 0 \) for \( x > 0 \).

Therefore, \( g \) is strictly increasing in \( \infty \). Also \( g(0) = 0 \).

Therefore, \( g(x) > g(0) = 0 \) \( \forall x > 0 \).

Hence (ii) - the right side of the inequality of (1) is proved.

2. To prove : \( a < I(a, b) < b \)

That is to prove : \( a < \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{a-a}} < b \)

It is the same as proving \( a < \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} < b \)

Now to prove : (i) \( a < \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} \) (ii) \( \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} < b \)

Proof of (i):

\[
\begin{align*}
\text{Now } a < \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} & \iff a e < \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} \\
& \iff \log a + 1 < \frac{1}{b-a} \log \left( \frac{b^b}{a^a} \right) = \frac{1}{b-a} (b \log b - a \log a) \\
& \iff (b-a) \log a + b - a < b \log b - a \log a \\
& \iff b - a < b \log b - b \log a = b \log \frac{b}{a} \\
& \iff 1 - \frac{a}{b} < \log \frac{b}{a} \\
& \iff 1 - \frac{a}{a+h} < \log \frac{a+h}{a}, \text{ where } b = a+h; \ h > 0 \\
& \iff 1 - \frac{1}{1+x} < \log (1+x), \text{ where } x = \frac{h}{a} \\
& \iff \frac{x}{1+x} < \log (1+x) \\
& \iff x < (1+x) \log (1+x)
\end{align*}
\]

To prove that \( x < (1+x) \log (1+x) \), it is enough to prove that

\( f(x) = (1+x) \log (1+x) - x > 0 \) for \( x \geq 0 \).

\( f'(x) = \log (1+x) \)

\( f'(x) > 0 \) for \( x > 0 \).

\( f \) is strictly increasing in \( [0, \infty) \). Also \( f(0) = 0 \).
Therefore \( f(x) > f(0) = 0 \)

Therefore \( f(x) > 0 \) for \( x \geq 0 \).

Proof of (ii) : To prove \( \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} < b \)

Now \( \frac{1}{e} \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} < b \iff \left( \frac{b^b}{a^a} \right)^{\frac{1}{b-a}} < be \)

\[ \iff \frac{1}{b-a} \log \frac{b}{a} < \log b + 1 \]

\[ \iff b \log b - a \log a < (b - a)(\log b + 1) \]

\[ \iff a \log \frac{b}{a} < b - a \]

\[ \iff \log \frac{b}{a} < \frac{b-a}{a} = \frac{b}{a} - 1 \]

\[ \iff \log \left( \frac{a+h}{a} \right) < \frac{a+h}{a} - 1 \text{, where } b = a+h; \, h > 0 \]

\[ \iff \log (1+x) < x \text{ [where } x = \frac{h}{a} \text{].} \]

\[ \iff 1+x < e^x \]

For \( x > 0 \), \( 1 + x < e^x \) is always true. Hence proved.

\[ \square \]

**Proposition 2.7.** For \( k > n(n+2) \), \( i+n < L(i,i+k) < I(i,i+k) < i + \frac{k}{2} \).

*Proof.* \( L(i,i+k) < I(i,i+k) \) is always true, by Fact 1. Again by Fact 1, \( L(i,i+k) < I(i,i+k) < A(i,i+k) = i + \frac{k}{2} \). Now to prove that \( i+n < L(i,i+k) \), it is enough if we prove that \( i+n < \)
\(G(i, i + k)\) for \(k > n (n + 2)\).

\[
i + n < G(i, i + k) \iff (i + n)^2 < i(i + k) \\
\iff i^2 + 2ni + n^2 < i^2 + ki \\
\iff n^2 < (k - 2n)i \\
\iff 1 < \left(\frac{k}{n^2} - \frac{2}{n}\right)i \\
\iff \frac{k}{n^2} - \frac{2}{n} > 1 \\
\iff \frac{k - 2n}{n^2} > 1 \\
\iff k - 2n > n^2 \\
\iff k > n^2 + 2n \\
\iff k > n(n + 2)
\]

Hence the proposition. \(\square\)

**Observation 1.** From the above Proposition, we note that \(i + 1 < L(i, i + k) < I(i, i + k) < i + \frac{k}{2}\) for \(k > 3\).

**Observation 2.** If the vertices \(u\) and \(v\) have labels \(i\) and \(i + 1\) respectively, then by (1) and (2) of Proposition 2.6, \(i < L(i, i + 1) < I(i, i + 1) < i + 1\) and hence the edge \(uv\) can be given the label \(i\) or \(i + 1\).

**Observation 3.** If the vertices \(u\) and \(v\) are labeled with \(i\) and \(i + 2\) respectively, then \(i < L(i, i + 2) < I(i, i + 2) < A(i, i + 2) = i + 1\). So the edge \(uv\) can be labeled with \(i\) or \(i + 1\).

**Observation 4.** We have \((i + 1)^2 = i^2 + 2i + 1 \leq i^2 + 3i = i(i + 3)\). Hence \(i + 1 \leq G(i, i + 3)\) and so \(i + 1 < L(i, i + 3) < I(i, i + 3) < A(i, i + 3) = i + \frac{3}{2}\). If the vertices \(u\) and \(v\) have \(i\) and \(i + 3\) respectively as labels, the edge \(uv\) can be given the label \(i + 1\) or \(i + 2\).

**Observation 5.** If the vertices \(u\) and \(v\) are labeled with \(i\) and \(i + 4\) respectively, then by Observation 1, taking \(k = 4\), \(i + 1 < L(i, i + 4) < I(i, i + 4) < i + 2\) and hence the edge \(uv\) can be labeled with \(i + 1\) or \(i + 2\).
Observation 6. By Observation 1, we have
\[ i + 1 < L(i, i+5) < I(i, i+5) < i + \frac{5}{2}. \] Hence on labeling the vertices \( u \) and \( v \) with \( i \) and \( i + 5 \) respectively, the edge \( uv \) can be guaranteed the label \( i + 2 \).

Observation 7. By Observation 1, we have
\[ i + 1 < L(i, i+6) < I(i, i+6) < i + 3. \] Hence on labeling the vertices \( u \) and \( v \) with \( i \) and \( i + 6 \) respectively, the edge \( uv \) can be given the label \( i + 2 \).

Considering all the above observations, we now give the following figures which depict the range of Logarithmic/Identric mean.

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Theorem 2.8. Any path is both a Logarithmic and an Identric mean graph.

Proof. Let \( P_n \) be a path on \( n \) vertices given by \( u_1, u_2, u_3, \ldots, u_n \) with \( n - 1 \) edges.
Let \( f : V(P_n) \to \{1, 2, \ldots, q + 1\} \) be defined by \( f(u_i) = i, \ 1 \leq i \leq n \). Now each edge \( u_iu_{i+1} \) can be labeled as \( i, \ 1 \leq i \leq n \). Thus \( f \) becomes both Logarithmic and Identric mean labeling and hence paths are Logarithmic and Identric mean graphs. \( \square \)

Example 1. A Logarithmic/Identric mean labeling of \( P_5 \) is shown below.

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Theorem 2.9. Any cycle is both a Logarithmic and an Identric mean graph.

Proof. Let \( C_n \) be a cycle on \( n \) vertices \( u_1, u_2, \ldots, u_n \). Rename the vertices
\( u_1, u_{\left\lceil \frac{n}{2} \right\rceil + 1}, u_{\left\lceil \frac{n}{2} \right\rceil + 2}, \ldots, u_n \) as \( v_m, v_{m-1}, \ldots, v_1 \) respectively, where \( m + \left\lceil \frac{n}{2} \right\rceil = n \).
Define \( f : V(C_n) \to \{1, 2, \ldots, q + 1\} \) by \( f(u_i) = 2i - 1 \) and \( f(v_i) = 2i \). The edges \( u_iu_{i+1} \) get labels \( \left\lceil \frac{2i+1}{2} \right\rceil \) and the edges \( v_iv_{i+1} \) get labels \( 2i + 1 \). \( u_1v_1 \) gets the label 1 and \( u_{\left\lceil \frac{n}{2} \right\rceil}v_m \) gets the label \( n \). \( \square \)
Example 2. A Logarithmic/Identric mean labeling of \( C_5 \) and \( C_6 \) are shown below.

![Graphs C5 and C6](image)

Theorem 2.10. Triangular snake \( T_n \) is both a Logarithmic and an Identric mean graph.

Proof. Let \( T_n \) be a triangular snake with \( v_i, 1 \leq i \leq n \) being the vertices of the path and \( w_i, 1 \leq i \leq n \) being the vertices that are joined to \( v_i v_{i+1} \). Let \( f : V(T_n) \to \{1, 2, ..., q+1\} \) be defined by \( f(v_1) = 1 \), \( f(v_i) = 3i - 3 \), \( 2 \leq i \leq n \) and \( f(w_i) = 3i - 1 \), \( 1 \leq i \leq n - 1 \). With respect to this type of labeling for the vertices, for \( 1 \leq i \leq n - 1 \) the edges \( v_i v_{i+1} \) and \( v_i w_i \) get the labels \( 3i - 1 \), \( 3i - 2 \) and \( 3i \) respectively. Thus \( f \) becomes both Logarithmic and Identric mean labeling and hence \( T_n \) is both Logarithmic and Identric mean graphs. \( \square \)

Example 3. A Logarithmic/Identric mean labeling of \( T_5 \) is shown in the figure below.

![Graph T5](image)

Theorem 2.11. Quadrilateral snake \( Q_n \) admits Logarithmic/Identric mean labeling of graphs.

Proof. Let \( Q_n \) be a quadrilateral snake as given in definition.

Define \( f : V(Q_n) \to \{1, 2, 3, ..., q+1\} \) as follows. \( f(u_1) = 1 \), \( f(u_i) = 4i - 4 \) for \( 2 \leq i \leq n \), \( f(v_i) = 4i - 2 \) for \( 1 \leq i \leq n \), \( f(w_i) = 4i - 1 \) for \( 1 \leq i \leq n \). For \( 1 \leq i \leq n - 1 \) Edges \( u_i v_i \), \( v_i w_i \), \( u_i u_{i+1} \), \( u_{i+1} w_i \) get the labels \( 4i - 3, 4i - 1, 4i - 2, 4i \) respectively. This results in distinct edge labels. \( \square \)

Example 4. A Logarithmic/Identric mean labeling of \( Q_3 \).
Theorem 2.12. The Star graph $K_{1,n}, n > 5$ is not a Logarithmic/Identric mean graph.

Proof. Let $K_{1,n}, n > 5$ be a star graph where the vertex $u$ is attached to $v_1, v_2, \ldots, v_n$. By Remark 2.3, to get the label 1, 1 and 2 must be adjacent or 1 and 3 must be adjacent. The central vertex $u$ should receive one of the labels: 1, 2, 3. Consider the case when the central vertex is given 1 or 2. None of $L(1,n)$, $I(1,n)$, $L(2,n)$, $I(2,n)$ is $n$ for $n > 4$, since $A(1,n)$ and $A(2,n)$ are less than $n - 1$ for $n > 4$. Hence the central vertex is labeled with 3. In this case too, none of $L(3,n)$, $I(3,n)$ is $n$ for $n > 5$, since $A(3,n)$ is less than $n - 1$ for $n > 5$. \[\square\]

$K_{1,n}$ is both Logarithmic and Identric mean graph for $n \leq 5$. Logarithmic/Identric mean labeling of $K_{1,5}$ is as follows. For $n < 5$, it is still simpler.

Theorem 2.13. The following standard graphs admit both Logarithmic and Identric mean labeling:
1. Crown
2. Ladder
3. Comb
4. Caterpillar
5. Dragon

Proof. 1. Let $G$ be a crown graph, where $u_1, u_2, \ldots, u_n$ are the vertices of the cycle $C_n$ and $v_1, v_2, \ldots, v_n$ are the pendant vertices such that each $v_i$ is attached to $u_i$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by $f(u_i) = 2i - 1$ for $i = 1, 2$, $f(u_i) = 2i + 1$ for $3 \leq i \leq n$ and $f(v_i) = 2i$ for $1 \leq i \leq n$. \[\square\]
2. Let $L_n$ be a ladder graph with $u_1, u_2, \ldots, u_n$ as vertices of one side of the ladder and $v_1, v_2, \ldots, v_n$ as vertices on the other side of the ladder such that $u_i$ is adjacent to $v_i$. Define $f: V(L_n) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by $f(u_i) = 1$, $f(v_i) = 3i - 3$ for $2 \leq i \leq n$ and $f(v_i)$ and $f(v_i) = f(u_i) + 1$ for $1 \leq i \leq n$. Edges $u_i u_{i+1}, u_i v_i, v_i v_{i+1}$ receive the labels $3i - 1, 3i - 2, 3i$ respectively.

3. Let $G$ be a comb graph such that for $1 \leq i \leq n$, $v_i$ are the vertices of the path and $u_i$ are the pendant vertices with every $u_i$ attached to $v_i$. Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by $f(v_i) = 2i - 1$ and $f(u_i) = 2i$ for $1 \leq i \leq n$. Edges $v_i u_i$ and $v_i v_{i+1}$ get $2i - 1, 2i$ as labels respectively.

4. Let $G$ be a caterpillar graph. For $1 \leq i \leq n$, let $v_i$ be the vertices of the path $P_n$, $u_i, w_i$ be the vertices attached to $v_i$. Thus $G$ has $3n$ vertices and $3n - 1$ edges. $f: V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ is defined by $f(v_i) = 3i - 2$, $f(u_i) = 3i - 1$ and $f(w_i) = 3i$, $1 \leq i \leq n$. Edges $u_i v_i, v_i w_i, v_i v_{i+1}$ are labeled with $3i - 2, 3i - 1, 3i$ respectively.

5. Let $G$ be a dragon graph with $u_i, 1 \leq i \leq n$ as the vertices of the cycle $C_n$ and $v_i, 1 \leq i \leq m$ as the vertices of the path $P_m$. Note that $u_n = v_1$. Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by $f(u_i) = i$ for $1 \leq i \leq n$, $f(v_j) = n + j - 1$ for $2 \leq j \leq m$. Edges $u_i u_{i+1}$ and $v_j v_{j+1}$ receive $i$ and $n + j$ as labels respectively.

With respect to the labelings given above all the graphs $(1) - (5)$ become both Logarithmic and Identric mean graphs.

Throughout we note that the same labeling works for both the means.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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