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GLOBAL STABILITY OF A STAGE-STRUCTURED PREDATOR-PREY MODEL

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Abstract. In this paper, a two species autonomous Leslie-Gower predator-prey model with stage structure of distributed-delay type for prey species is studied. Using the iterative technique, we investigated the global stability of the positive equilibrium of the system. Our result extend the main result in [Global stability of a stage-structured predator-prey model with modified Leslie-Gower and Holling-Type II schemes, Int. J. Journal of Biomath. 6 (2012), Article ID 1250057].

Keywords: Global stability; Stage-structure; Leslie-Gower; Holling-Type II; Predator-Prey model.

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1. Introduction

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Recently, Huo, Wang and Chavez [1] proposed a stage-structured Leslie-Gower predator-prey model as follows

$$
\begin{aligned}
x_1'(t) &= r_1 x_2(t) - d_{11} x_1(t) - r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1),\\
x_2'(t) &= r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1) - b x_2^2(t) - \frac{a_1 y(t) x_2(t)}{x_2(t)+k_1},\\
y'(t) &= y(t)\Big(r_2 - \frac{a_2 y(t)}{x_2(t)+k_2}\Big),
\end{aligned}
\tag{1}
$$

where x_1, x_2 and y represent the population densities of immature prey, mature prey and predator, respectively; r_1 is the birth rate of immature prey x_1; d_{11} denotes the death rate of immature prey x_1; r_2 is the intrinsic growth rate of predator y; b represents the strength of intra-specific competition in mature prey; a_1 represents the maximum value that mature x_2 can be captured by predator y, and the meaning of a_2 is similar to a_1; k_1 and k_2 measure the protection degree that environment afford to prey x_2 and predator y, respectively; τ_1 is the time to maturity for prey; $r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1)$ represents the prey who were born at time $t-\tau_1$ and survive and become to maturity at time t. The authors of [1] aims at the study of the boundedness of solutions and the persistent property of the system. Sufficient conditions for the local stability of the nonnegative equilibria of the model are also derived, and sufficient conditions for the global attractivity of positive equilibrium are obtained. In [2], Li, Han and Chen noticed that one of the main results in [1] still has room to improve, by using the iterative technique, they obtained a set of weaker condition for the global attractivity of the positive equilibrium of system (1), more precisely, they established the following theorem:

Theorem A [2] *Suppose that*

$$
\lambda_0 = r_1 e^{-d_{11}\tau_1} a_2 k_1 b - a_1 k_2 r_2 b - a_1 r_2 r_1 e^{-d_{11}\tau_1} > 0 \tag{H}
$$

holds, then the system (1) *has a unique globally attractive positive equilibria E.*

An important assumption behind the work of Huo, Wang and Chavez [1] and Li, Han and Chen[2] is that all individuals take the identical amount of time to become mature, which seems biologically unreasonable since individuals in a population do not necessarily always mature at the same age [17]. To solve this problem, stage-structure models of distributed delay type were then proposed([17]-[20]). It bring to our attention that all the works of [17]-[20] are concerned with the competitive system, to this day, there are still seldom scholars investigate the dynamic behaviors of the predator-prey system with stage structure of distributed-delay type.

Mainly motivated by [1, 2, 17], in this paper, we consider a non-autonomous predator-prey model with stage structure of distributed-delay type:

$$
\begin{aligned}
x_1'(t) &= r_1 x_2(t) - d_{11}x_1(t) - r_1\int_0^\infty f(s)e^{-d_{11}s}x_2(t-s)ds,\\
x_2'(t) &= r_1\int_0^\infty f(s)e^{-d_{11}s}x_2(t-s)ds - bx_2^2(t) - \frac{a_1 y(t)x_2(t)}{x_2(t)+k_1},\\
y'(t) &= y(t)\Big(r_2 - \frac{a_2 y(t)}{x_2(t)+k_2}\Big),
\end{aligned}
\tag{2}
$$

where all the parameters have the same meaning as that of system (1.1), $\int_0^{+\infty} f(s)ds = 1$ and $f \geq 0$, because f is a probability density function. System (1.1) is a particular case of system (1.2). It arises when we take $f(s) = \delta(s-\tau)$, where δ is the Dirac delta function.

Noting that the first equation in system (1) is equivalent to integral equations, thus, to investigate the dynamic behaviors of system (1), it is enough to investigate the dynamic of the subsystem which contain by the second and third equations of the system (1). However, in system (2), with the influence of probability density function, the first equation could not be expressed in integral form, hence, the analysis technique of [1, 2] could not be applied to system (2) directly. To overcome this difficulty, we develop some new analysis technique in this paper.

The initial conditions for system (2) take the form of

$$
\begin{aligned}
&x_i(\theta) = \phi_i(\theta), y(\theta) = \psi(\theta) > 0,\\
&\phi_i(0) > 0, \psi(0) > 0, i = 1,2, \theta \in (-\infty, 0],
\end{aligned}
\tag{3}
$$

where $\phi(t) = (\phi_1(t), \phi_2(t), \psi_1(t)) \in UC_g$, which is referred to as the fading memory space [21, p. 46].

From now on, we denote

$$
\mathbf{A} = r_1\int_0^\infty f(s)e^{-d_{11}s}ds. \tag{4}
$$

One could easily see that $\mathbf{A} < r_1$. In this paper, we aim to extend the main results in [2] to system (1.2), Following is the main result of this paper:

Theorem 1.1 *Let* $col(x_1(t), x_2(t), y(t))$ *be a solution of (2) and (3). Assume that the coefficients of system (2) satisfy*

$$\lambda_1 = \mathbf{A}a_2k_1b - a_1k_2r_2b - a_1r_2\mathbf{A} > 0. \tag{H_1}$$

Then the unique interior equilibrium $E^*(x_1^*, x_2^*, y^*)$ *of system* (1.2) *is globally attractive, that is,*

$$\lim_{t\to+\infty} x_i(t) = x_i^*, \;\; i = 1,2, \;\; \lim_{t\to+\infty} y(t) = y^*.$$

For more works on Leslie-Gower predator-prey system and stage structured system, one could refer to [1-20] and the references cited therein.

2. Proof of the main results

The interior positive equilibrium $E^*(x_1^*, x_2^*, y^*)$ of system (2) satisfies the following equations

$$\begin{cases} r_1x_2 - d_{11}x_1 - \mathbf{A}x_2 = 0, \\ \mathbf{A}x_2 - bx_2^2 - \frac{a_1yx_2}{x_2+k_1} = 0, \\ r_2 - \frac{a_2y}{x_2+k_2} = 0. \end{cases} \tag{5}$$

Under the assumption (H_1), system (5) admits a unique positive equilibrium $E^*(x_1^*, x_2^*, y^*)$, where

$$\begin{aligned} x_1^* &= \frac{(r_1-\mathbf{A})x_2^*}{d_{11}}, \\ x_2^* &= \frac{\mathbf{A}a_2 - bk_1a_2 - a_1r_2 + \sqrt{(\mathbf{A}a_2 - bk_1a_2 - a_1r_2)^2 - 4a_2b(a_1r_2k_2 - \mathbf{A}a_2k_1)}}{2ba_2}, \\ y^* &= \frac{r_2(k_2+x_2^*)}{a_2}. \end{aligned} \tag{6}$$

Similarly to the analysis of Theorem 1 in [19], we have

Lemma 2.1. *Solutions of system (2) with the initial condition (3) are positive for all* $t > 0$.

Lemma 2.2 ([19]) *Consider the following system:*

$$\begin{aligned} x'(t) &= b\int_0^{+\infty} f(s)x(t-s)e^{-ds}ds - cx(t) - ax^2(t), \\ x(t) &= \phi(t) \geq 0, \; t \leq 0, \phi(0) > 0 \end{aligned}$$

and assume that $b, a, d > 0, c \geq 0$ are positive constants, $B = b\int_0^{+\infty} f(s)e^{-ds}ds > 0$, then:
(i) $\lim_{t\to+\infty} x(t) = \dfrac{B-c}{a}$ *if* $B - c > 0$.
(ii) $\lim_{t\to+\infty} x(t) = 0$ *if* $B - c \leq 0$.

Lemma 2.3. ([22]) *If $a > 0, b > 0$ and $x' \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $x' \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Proof of Theorem 1.1. We first show that under the assumption of Theorem 1.1,

$$\lim_{t\to+\infty} x_2(t) = x_2^*, \quad \lim_{t\to+\infty} y(t) = y^*$$

hold. For any $\varepsilon > 0$,

$$\varepsilon < \frac{1}{2}\min\Big\{\Big(\frac{a_1}{k_1}\big(\frac{r_2}{a_2}+1\big)+b\Big)^{-1}\frac{\mathbf{A}a_2k_1b - a_1k_2r_2b - a_1r_2\mathbf{A}}{a_2k_1b}, \frac{r_2k_2}{a_2}\Big\},$$

it follows that

$$m_1^{(1)} \stackrel{\text{def}}{=} \frac{\dfrac{\mathbf{A}a_2k_1b - a_1k_2r_2b - a_1r_2\mathbf{A}}{a_2k_1b} - \dfrac{a_1}{k_1}\left(\dfrac{r_2}{a_2}+1\right)\varepsilon}{b} - \varepsilon > 0; \tag{7}$$

$$m_2^{(1)} \stackrel{\text{def}}{=} \frac{r_2(k_2 + m_1^{(1)})}{a_2} - \varepsilon > 0. \tag{8}$$

From the second equation of system (2), we have

$$x_2'(t) < r_1\int_0^{\infty} f(s)e^{-d_{11}s}x_2(t-s)ds - bx_2^2(t).$$

By applying Lemma 2.2(i) and standard comparison theorem, we have

$$\limsup_{t\to+\infty} x_2(t) \leq \frac{\mathbf{A}}{b}.$$

So, for any small constant $\varepsilon > 0$, which satisfies (7) and (8), there exists a $T_1 > 0$ such that

$$x_2(t) \leq \frac{\mathbf{A}}{b} + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)},\ t > T_1. \tag{9}$$

Above inequality together with the third equation of system (2) implies that

$$\dot{y}(t) < y(t)\Big(r_2 - \frac{a_2 y(t)}{M_1^{(1)} + k_2}\Big).$$

It follows from Lemma 2.3 that

$$\limsup_{t\to+\infty} y(t) \le \frac{r_2\big(M_1^{(1)} + k_2\big)}{a_2}.$$

Then for above ε, there exists a $T_2 > T_1$, such that

$$y(t) < \frac{r_2\big(M_1^{(1)} + k_2\big)}{a_2} + \varepsilon \overset{\text{def}}{=} M_2^{(1)},\ t > T_2. \tag{10}$$

Substituting (10) into the second equation of system (2), we have

$$x_2'(t) \ > \ r_1\int_0^{\infty} f(s)e^{-d_{11}s}x_2(t-s)ds - bx_2^2(t) - \frac{a_1 M_2^{(1)} x_2(t)}{k_1}.$$

Noting that from (7) one has

$$\begin{aligned}
r_1\int_0^{\infty} f(s)e^{-d_{11}s}ds - \frac{a_1 M_2^{(1)}}{k_1} &= \mathbf{A} - \frac{a_1 M_2^{(1)}}{k_1}\\
&= \mathbf{A} - \frac{a_1\big(\frac{r_2\left(M_1^{(1)}+k_2\right)}{a_2} + \varepsilon\big)}{k_1}\\
&= \frac{\mathbf{A}a_2k_1b - a_1k_2r_2b - a_1r_2\mathbf{A}}{a_2k_1b} - \frac{a_1}{k_1}\left(\frac{r_2}{a_2}+1\right)\varepsilon > 0.
\end{aligned} \tag{11}$$

By applying Lemma 2.2(i), and standard comparison theorem, we have

$$\liminf_{t\to+\infty} x_2(t) \ge \frac{\mathbf{A} - \frac{a_1 M_2^{(1)}}{k_1}}{b}.$$

Then for above $\varepsilon > 0$, there exists a $T_3 > T_2$, such that

$$x_2(t) > \frac{\mathbf{A} - \frac{a_1 M_2^{(1)}}{k_1}}{b} - \varepsilon \overset{\text{def}}{=} m_1^{(1)},\ t > T_3. \tag{12}$$

For $t > T_3$, substituting (12) into the third equation of system (2), we have

$$y'(t) > y(t)\Big(r_2 - \frac{a_2 y(t)}{m_1^{(1)} + k_2}\Big),\ \ t \ge T_3.$$

By applying Lemma 2.3 and standard comparison theorem, we have

$$\liminf_{t\to+\infty} y(t) \ge \frac{r_2(k_2 + m_1^{(1)})}{a_2}.$$

Then for above $\varepsilon > 0$, there exists a $T_4 > T_3$ such that

$$y(t) > \frac{r_2(k_2 + m_1^{(1)})}{a_2} - \varepsilon \overset{\text{def}}{=} m_2^{(1)},\ t > T_4. \tag{13}$$

According to (9), (10), (12) and (13), we obtain

$$0 < m_1^{(1)} < x_2(t) < M_1^{(1)},\ 0 < m_2^{(1)} < y_2(t) < M_2^{(1)},\ t > T_4. \tag{14}$$

Then for $t > T_4$, it follows from (9) and (13) and the second equation of system (2), we have

$$x_2'(t) \ < \ r_1 \int_0^\infty f(s) e^{-d_{11}s} x_2(t-s) ds - b x_2^2(t) - \frac{a_1 m_2^{(1)} x_2(t)}{k_1 + M_1^{(1)}}.$$

According to the inequalities (11) and (14), we have

$$r_1 \int_0^\infty f(s) e^{-d_{11}s} ds - \frac{a_1 m_2^{(1)}}{k_1 + M_1^{(1)}} = \mathbf{A} - \frac{a_1 m_2^{(1)}}{k_1 + M_1^{(1)}} > \mathbf{A} - \frac{a_1 M_2^{(1)}}{k_1} > 0.$$

By applying Lemma 2.2(i) and standard comparison theorem, we have

$$\limsup_{t \to +\infty} x_2(t) \le \frac{\mathbf{A} - \frac{a_1 m_2^{(1)}}{k_1 + M_1^{(1)}}}{b}.$$

Then for above $\varepsilon > 0$, there exists a $T_5 > T_4$, such that

$$x_2(t) < \frac{\mathbf{A} - \frac{a_1 m_2^{(1)}}{k_1 + M_1^{(1)}}}{b} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)},\ t > T_5. \tag{15}$$

From inequalities (9) and (15), we obtain

$$x_2(t) < M_1^{(2)} < M_1^{(1)},\ t > T_5. \tag{16}$$

For $t > T_5$, it follows from (15) and the third equation of system (2) that

$$\dot{y}(t) < y(t) \Big(r_2 - \frac{a_2 y^2(t)}{M_1^{(2)} + k_2} \Big)\ t \ge T_5.$$

Therefore, by Lemma 2.3, we have

$$\limsup_{t \to +\infty} y(t) \le \frac{r_2 \big(M_1^{(2)} + k_2 \big)}{a_2}.$$

Then for above ε, there exists a $T_6 > T_5$, such that

$$y(t) < \frac{r_2 \big(M_1^{(2)} + k_2 \big)}{a_2} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{(2)},\ t > T_6. \tag{17}$$

From inequalities (10), (16) and (17), we have

$$y(t) < M_2^{(2)} < M_2^{(1)},\ t > T_6. \tag{18}$$

For $t > T_6$, substituting inequalities (12) and (18) into the second equation of system (2), we have

$$x_2'(t) \quad > \quad r_1 \int_0^\infty f(s)e^{-d_{11}s}x_2(t-s)ds - bx_2^2(t) - \frac{a_1 M_2^{(2)} x_2(t)}{k_1 + m_1^{(1)}}.$$

According to inequalities (11), we can obtain

$$r_1 \int_0^\infty f(s)e^{-d_{11}s}ds - \frac{a_1 M_2^{(2)}}{k_1 + m_1^{(1)}} > \mathbf{A} - \frac{a_1 M_2^{(2)}}{k_1} > 0,$$

By applying Lemma 2.2(i) and standard comparison theorem, we have

$$\liminf_{t\to+\infty} x_2(t) \geq \frac{\mathbf{A} - \frac{a_1 M_2^{(2)}}{k_1 + m_1^{(1)}}}{b}.$$

Then for above $\varepsilon > 0$, there exists a $T_7 > T_6$, such that

$$x_2(t) > \frac{\mathbf{A} - \frac{a_1 M_2^{(2)}}{k_1 + m_1^{(1)}}}{b} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)},\ t > T_7. \tag{19}$$

According to the inequalities (12), (18) and (19), we can obtain

$$x_2(t) > m_1^{(2)} > m_1^{(1)},\ t > T_7. \tag{20}$$

Substituting inequality (20) into the third equation of system (2), we have

$$y'(t) > y(t)\Big(r_2 - \frac{a_2 y(t)}{m_1^{(2)} + k_2}\Big),\ t > T_7.$$

By applying Lemma 2.3 and standard comparison theorem, we have

$$\liminf_{t\to+\infty} y(t) \geq \frac{r_2\big(m_1^{(2)} + k_2\big)}{a_2}.$$

Then for above $\varepsilon > 0$, there exists a $T_8 > T_7$, such that

$$y(t) > \frac{r_2\big(m_1^{(2)} + k_2\big)}{a_2} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)},\ t > T_8. \tag{21}$$

According to the inequalities (13), (20), and (21), we can obtain

$$y(t) > m_2^{(2)} > m_2^{(1)},\ t > T_8. \tag{22}$$

For $t > T_8$, according to (16), (18), (20) and (22), we have

$$m_1^{(1)} < m_1^{(2)} < x_2(t) < M_1^{(2)} < M_1^{(1)}, m_2^{(1)} < m_2^{(2)} < y(t) < M_2^{(2)} < M_2^{(1)}. \tag{23}$$

Repeating above process, we get four sequences

$$\begin{aligned} M_1^{(n)} &= \frac{\mathbf{A} - \frac{a_1 m_2^{(n-1)}}{k_1 + M_1^{(n-1)}}}{b} + \frac{\varepsilon}{n}, \quad M_2^{(n)} = \frac{r_2\left(M_1^{(n)} + k_2\right)}{a_2} + \frac{\varepsilon}{n}, \\ m_1^{(n)} &= \frac{\mathbf{A} - \frac{a_1 M_2^{(n)}}{k_1 + m_1^{(n-1)}}}{b} - \frac{\varepsilon}{n}, \quad m_2^{(n)} = \frac{r_2\left(m_1^{(n)} + k_2\right)}{a_2} - \frac{\varepsilon}{n}. \end{aligned} \tag{24}$$

For $i = 1,2$, we assert that $M_i^{(n)}$ are monotonic decreasing sequences, and $m_i^{(n)}$ are monotonic increasing sequences. Following we will prove this assertion by induction. Firstly, according to inequalities (23), we have

$$m_i^{(1)} < m_i^{(2)},\ M_i^{(2)} < M_i^{(1)},\ i = 1,2.$$

Secondly, we suppose that our assertion is true for n, that is,

$$m_i^{(n-1)} < m_i^{(n)},\ M_i^{(n)} < M_i^{(n-1)},\ i = 1,2. \tag{25}$$

Noting that

$$\begin{aligned} M_1^{(n+1)} &= \frac{\mathbf{A} - \frac{a_1 m_2^{(n)}}{k_1 + M_1^{(n)}}}{b} + \frac{\varepsilon}{n+1}, \quad M_2^{(n+1)} = \frac{r_2\left(M_1^{(n+1)} + k_2\right)}{a_2} + \frac{\varepsilon}{n+1}, \\ m_1^{(n+1)} &= \frac{\mathbf{A} - \frac{a_1 M_2^{(n+1)}}{k_1 + m_1^{(n)}}}{b} - \frac{\varepsilon}{n+1}, \quad m_2^{(n+1)} = \frac{r_2\left(m_1^{(n+1)} + k_2\right)}{a_2} - \frac{\varepsilon}{n+1}. \end{aligned} \tag{26}$$

According to inequalities (24), (25) and (26), one could easily verified that

$$M_i^{(n+1)} < M_i^{(n)},\ m_i^{(n)} < m_i^{(n+1)},\ i = 1,2.$$

Then for $t > T_{4n}$, we have

$$0 < m_1^{(1)} < m_1^{(2)} < \cdots < x_2(t) < M_1^{(n)} < \cdots < M_1^{(2)} < M_1^{(1)},$$

$$0 < m_2^{(1)} < m_2^{(2)} < \cdots < y(t) < M_2^{(n)} < \cdots < M_2^{(2)} < M_2^{(1)}.$$

Therefore sequences $M_i^{(n)}, m_i^{(n)}, i = 1,2, n = 1,2,\ldots$ all have limit. Denote that

$$\lim_{t\to+\infty} M_1^{(n)} = \bar{x}_2,\ \lim_{t\to+\infty} m_1^{(n)} = \underline{x}_2,\ \lim_{t\to+\infty} M_2^{(n)} = \bar{y},\ \lim_{t\to+\infty} m_2^{(n)} = \underline{y}.$$

Consequently, $\bar{x}_2 \geq \underline{x}_2, \bar{y} \geq \underline{y}$. Now we show that under the assumption of Theorem 1.1, $\bar{x}_2 = \underline{x}_2, \bar{y} = \underline{y}$. Letting $n \to +\infty$ in (24), we have

$$\begin{aligned} b\bar{x}_2 &= \mathbf{A} - \frac{a_1 \underline{y}}{\bar{x}_2 + k_1}, a_2\bar{y} = r_2(\bar{x}_2 + k_2), \\ b\underline{x}_2 &= \mathbf{A} - \frac{a_1 \bar{y}}{\underline{x}_2 + k_1}, a_2\underline{y} = r_2(\underline{x}_2 + k_2). \end{aligned} \tag{27}$$

By using (27), similarly to the analysis of (2.17)-(2.21) in [2], we can show that under the assumption (H_1) holds,

$$\lim_{t\to+\infty} x_2(t) = x_2^*, \lim_{t\to+\infty} y(t) = y^*. \tag{28}$$

To end the proof of Theorem 1.1, it is enough to show that $\lim_{t\to+\infty} x_1(t) = x_1^*$ holds. Setting $M > \sup\{x_2(t), t \in R\}$, it follows from $\int_0^{+\infty} f(s)ds = 1$ and (28) for any enough small $\varepsilon > 0$ ($\varepsilon < \frac{1}{2}\frac{(r_1-\mathbf{A})x^*}{r_1+(M+1)\mathbf{A}}$), there exists a positive number T^* such that for all $t \geq T^*$,

$$x_2^* - \varepsilon < x_2(t) < x_2^* + \varepsilon,\ r_1 \int_0^{T^*} f(s)e^{-d_{11}s}ds > (1-\varepsilon)\mathbf{A}. \tag{29}$$

Now, for $t \geq 2T^*$, from the first equation of (1.2), we have

$$\begin{aligned} \dot{x}_1(t) &= r_1x_2(t) - d_{11}x_1(t) - r_1 \int_0^{\infty} f(s)e^{-d_{11}s}x_2(t-s)ds \\ &\leq r_1x_2(t) - d_{11}x_1(t) - r_1 \int_0^{T^*} f(s)e^{-d_{11}s}x_2(t-s)ds \\ &\leq r_1(x_2^*+\varepsilon) - d_{11}x_1(t) - (x_2^*-\varepsilon)(1-\varepsilon)\mathbf{A} \end{aligned} \tag{30}$$

Applying Lemma 2.3 to (30) leads to

$$\limsup_{t\to+\infty} x_1(t) \leq \frac{r_1(x_2^*+\varepsilon) - (x_2^*-\varepsilon)(1-\varepsilon)\mathbf{A}}{d_{11}}. \tag{31}$$

Setting $\varepsilon \to 0$ in (31), we obtain

$$\limsup_{t\to+\infty} x_1(t) \leq \frac{(r_1-\mathbf{A})x_2^*}{d_{11}} = x_1^*. \tag{32}$$

Also, for $t \geq 2T^*$, from the first equation of (1.2), we have

(33)

$$\begin{aligned} \dot{x}_1(t) &= r_1x_2(t) - d_{11}x_1(t) - r_1 \int_0^{\infty} f(s)e^{-d_{11}s}x_2(t-s)ds \\ &\geq r_1x_2(t) - d_{11}x_1(t) - r_1 \int_0^{T^*} f(s)e^{-d_{11}s}x_2(t-s)ds - r_1 \int_{T^*}^{\infty} f(s)e^{-d_{11}s}x_2(t-s)ds \\ &\geq r_1(x_2^*-\varepsilon) - d_{11}x_1(t) - (x_2^*+\varepsilon)\mathbf{A} - M\varepsilon\mathbf{A} \end{aligned}$$

From the definition of ε,

$$r_1(x_2^* - \varepsilon) - (x_2^* + \varepsilon)\mathbf{A} - M\varepsilon\mathbf{A} > \frac{1}{2}(r_1 - \mathbf{A})x_2^* > 0. \tag{34}$$

And so, applying Lemma 2.3 to (33) leads to

$$\liminf_{t\to+\infty} x_1(t) \geq \frac{r_1(x_2^* - \varepsilon) - (x_2^* + \varepsilon)\mathbf{A} - M\varepsilon\mathbf{A}}{d_{11}}. \tag{35}$$

Setting $\varepsilon \to 0$ in (35), we obtain

$$\liminf_{t\to+\infty} x_1(t) \geq \frac{(r_1 - \mathbf{A})x_2^*}{d_{11}} = x_1^*. \tag{36}$$

(32) and (36) implies that

$$\lim_{t\to+\infty} x_1(t) = x_1^*. \tag{37}$$

Thus, (36) and (37) show that the unique interior equilibrium $E^*(x_1^*, x_2^*, y^*)$ is globally attractive. This completes the proof of Theorem 1.1.

3. Discussion

In this paper, we study the predator-prey model with modified Leslie-Gower and stage-structure of distributed delay, which is an extension of the discrete delayed stage structured model studied by Huo Wang and Chavez [1] and Li, Han and Chen[2]. By applying iterative technique, we obtain a set of sufficient conditions which guarantee the globally attractivity of the coexistence equilibrium.

Comparing Theorem 1.2 of [2] for system (1.1) with Theorem 1.1 for (1.2), we find out that the term

$$\mathbf{A} = r_1 \int_0^\infty f(s)e^{-d_{11}s}ds$$

in our result is corresponding to the $r_1 e^{-d_{11}\tau}$ in [2]. It's in this sense that we extend the main results of Huo Wang and Chavez [1] and Li, Han and Chen[2].

Conflict of Interests

The authors declare that there is no conflict of interests.

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