PERMANENCE OF A LOTKA-VOLterra COOPERATIVE SYSTEM WITH TIME DELAYS AND FEEDBACK CONTROLS

JUNYAN XU*, FENGDE CHEN

College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian, 350002, China

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Abstract. In this paper, we investigate a Lotka-Volterra cooperative system with time delays and feedback controls. By applying new inequalities, we obtain some new sufficient conditions which ensure the system to be permanent. Our results show that feedback control variables have no influence on the permanence of the system, which enrich the previous corresponding research results.

Keywords: Lotka-Volterra system; Cooperative; Discrete delay; Feedback control; Permanence

2010 AMS Subject Classification: 92B20, 34K20, 45J05.

1. Introduction

In virtue of their significance in theory and practice, the Lotka-Volterra systems with time delays have been extensively studied. As one of the important interactions among species, cooperative behavior is commonly seen in animal society and human society. The study of Lotka-Volterra cooperative models with delays have attracted the interest of many researchers(see for example, [1-7] and references cited therein). In [1], Lu, Lu and Lian studied the permanence of...
the following Lotka-Volterra system with discrete delays

\[ \begin{align*}
\dot{x}_1(t) &= x_1(t) \left[ r_1 - a_{11} x_1(t) - a_{11} x_1(t - \tau_{11}) + a_{12} x_2(t - \tau_{12}) \right], \\
\dot{x}_2(t) &= x_2(t) \left[ r_2 - a_{22} x_2(t) + a_{21} x_1(t) - a_{22} x_2(t - \tau_{22}) \right],
\end{align*} \tag{1.1} \]

with initial conditions

\[ x_i(t) = \phi_i(t) \geq 0, \ t \in [-\tau_0, 0]; \ \phi_i(0) > 0 \ (i = 1, 2) \]

where \( r_i, a_i, a_{ij} \) and \( \tau_{ij} \) are constants with \( a_i > 0, \tau_{ij} \geq 0 (i, j = 1, 2) \) and \( \tau_0 = \max\{\tau_{ij} : i, j = 1, 2\} \), \( \phi_i(t) \) is continuous on \([-\tau_0, 0]\). In the cooperative case \( a_{12} > 0, a_{21} > 0 \), they obtained the main result.

**Theorem A.** If \( \tau_{11} = \tau_{12} = \tau_{22} = \tau, \ a_i = a_{ji} < a_{ii} (i \neq j, i, j = 1, 2) \) and \( (a_1 + a_{11})(a_2 + a_{22}) - a_{12}a_{21} > 0 \), then system (1.1) is permanent.

Moreover, they gave an example (see example 4.1 in [1]) to show that the size of time delays can destroy the permanence of system (1.1). To eliminate the influence of the size of delays on the permanence, Nakata and Muroya [2] considered the following nonautonomous two species Lotka-Volterra cooperative population systems

\[ \begin{align*}
\dot{x}_1(t) &= x_1(t) \left[ r_1(t) - a_{11}^1 x_1(t - \tau) - a_{11}^2 t_1 x_1(t - 2\tau) + a_{12}^1 t_2 x_2(t - \tau) \right], \\
\dot{x}_2(t) &= x_2(t) \left[ r_2(t) + a_{21}^0 x_1(t) + a_{21}^1 t_1 x_1(t - \tau) - a_{22}^0 t_2 x_2(t) - a_{22}^1 t_2 x_2(t - \tau) \right],
\end{align*} \tag{1.2} \]

with initial conditions

\[ x_1(t) = \Phi_1(t) \geq 0, \ t \in [-2\tau, 0]; \ \Phi_1(0) > 0, \]

\[ x_2(t) = \Phi_2(t) \geq 0, \ t \in [-\tau, 0); \ \Phi_2(0) > 0. \]

Through establishing new inequalities (one can see Lemma 2.2 - Lemma 2.4 in [2]), they obtained weaker conditions which is not dependent on the size of time delays to prove that the cooperative system (1.2) is permanent. For more works on the permanence of cooperative system with delays, one could refer to [1-3,6,14] and the references cited therein.

In another aspect, ecosystems in the real world are often distributed by unpredictable forces which can result in changes in biological parameters such as survival rates, so it is necessary to
study models with control variables which are so-called disturbance functions [8-15]. Recently, Fan and Wang [10], Nie, Peng and Teng [11], Chen [12,13] and Chen [14] investigated the persistent property of discrete or continuous feedback control systems with delays. It is interesting that all of their research results show that feedback controls have no influence on the persistent property of these systems.

However, to the best of the authors’ knowledge, there are few scholars who study the Lotka-Volterra cooperative system with feedback control. Whether the feedback control variables play an essential role on the persistent property of Lotka-Volterra cooperative system or not? To find an answer to this question, we investigate the following Lotka-Volterra cooperative system with time delays and feedback controls

\[
\begin{align*}
\dot{x}_1(t) &= x_1(t) \left[ r_1(t) - a_1(t)x_1(t) - a_{11}(t)x_1(t - \tau) + a_{12}(t)x_2(t - \tau) - b_1(t)u_1(t - \sigma_1) \right], \\
\dot{x}_2(t) &= x_2(t) \left[ r_2(t) - a_2(t)x_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t - \tau) - b_2(t)u_2(t - \sigma_2) \right], \\
\dot{u}_1(t) &= -c_1(t)u_1(t) + d_1(t)x_1(t - \eta_1), \\
\dot{u}_2(t) &= -c_2(t)u_2(t) + d_2(t)x_2(t - \eta_2),
\end{align*}
\]

where \( x_i(t) \) denotes the density of \( i \)th cooperative species \( X_i \), \( u_i(t) \) is the control variable, \( i = 1, 2 \). \( r_i(t), a_i(t), b_i(t), c_i(t), d_i(t), a_{ij}(t) (i, j = 1, 2) \) are all continuous, real-valued functions which are bounded above and below by positive constants. \( \tau, \sigma, \eta_i (i = 1, 2) \) are positive constants and \( \delta = \max \{ \tau, \sigma_1, \sigma_2, \eta_1, \eta_2 \} \).

We consider system (1.3) with the following initial conditions

\[
\begin{align*}
x_i(s) &= \varphi_i(s), \quad s \in [-\delta, 0], \quad \varphi_i(0) > 0, \\
u_i(s) &= \psi_i(s), \quad s \in [-\delta, 0], \quad \psi_i(0) > 0,
\end{align*}
\]

where \( \varphi_i(s) \) and \( \psi_i(s) \) are continuous on \([-\delta, 0]\). It is not difficult to see that solutions of (1.3) and (1.4) are well defined for all \( t \geq 0 \) and satisfy

\[
x_i(t) > 0, u_i(t) > 0 \quad \text{for} \quad t \geq 0, \quad i = 1, 2.
\]

For a continuous bounded function \( f(t) \) defined on \([0, +\infty)\), we set

\[
f^u = \sup_{0 \leq t < +\infty} f(t) \quad \text{and} \quad f^l = \inf_{0 \leq t < +\infty} f(t).
\]
More precisely, in this paper, we will prove the following results.

**Theorem 1.1.** Assume that $a_{11} > a_{21}^\mu, a_1^l > a_{21}^\mu$ and $a_{22}^l > a_{12}^\mu, a_2^l > a_{12}^\mu$. Then system (1.3) is permanent.

As a direct consequence, the following conditions for autonomous case to (1.3) are obtained.

Now we set $a_i(t) = a_i, a_{ij}(t) = a_{ij}$ for $i, j = 1, 2$.

**Corollary 1.2.** Assume that $a_{11} > a_{21}, a_1 > a_{21}$ and $a_{22} > a_{12}, a_2 > a_{12}$. Then system (1.3) is permanent.

**Remark 1.3.** Theorem 1.1 and Corollary 1.2 show that feedback control variables have no influence on the persistent property of the system (1.3). Obviously, compared with Theorem A of Lu, Lu and Lian [1], Corollary 1.2 not only is founded in weaker conditions, but also is suitable for more extensive application. Therefore, our results give some more deeply insight on the dynamic behaviors of the Lotka-Volterra cooperative system with feedback controls.

2. Lemmas and proof of theorem

At first, let us introduce some basic and important lemmas.

**Lemma 2.1.** [9] If $a > 0, b > 0$ and $\frac{dx}{dt} \geq b - ax$, when $t \geq 0$ and $x(0) > 0$, we have $\liminf_{t \to +\infty} x(t) \geq \frac{b}{a}$. If $a > 0, b > 0$ and $\frac{dx}{dt} \leq b - ax$, when $t \geq 0$ and $x(0) > 0$, we have $\limsup_{t \to +\infty} x(t) \leq \frac{b}{a}$.

**Lemma 2.2.** [9] If $a > 0, b > 0$ and $\frac{dx}{dt} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have $\liminf_{t \to +\infty} x(t) \geq \frac{b}{a}$. If $a > 0, b > 0$ and $\frac{dx}{dt} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have $\limsup_{t \to +\infty} x(t) \leq \frac{b}{a}$.

**Lemma 2.3.** (see [13, Lemma 2.2]) Assume that $a > 0, b(t) > 0$ is a boundedness continuous function and $x(0) > 0$. Further suppose that

$$\frac{dx(t)}{dt} \leq -ax(t) + b(t),$$

then for all $t \geq s \geq 0$,

$$x(t) \leq x(t - s) \exp\{-as\} + \int_{t-s}^t b(\tau) \exp\{a(\tau - t)\} d\tau. \quad (2.1)$$
Especially, if $b(t)$ is bounded above with respect to $M$, then
\[
\limsup_{t \to +\infty} x(t) \leq \frac{M}{a},
\]  
(2.2)

**Lemma 2.4.** (see [2, Lemma 2.2]) Assume that for $y(t) > 0$, it holds that
\[
\frac{dy(t)}{dt} \leq y(t)\left(\lambda - \sum_{l=0}^{m} \mu^l y(t-l\tau)\right) + D,
\]
with initial conditions $y(t) = \phi(t) \geq 0$ for $t \in [-m\tau, 0)$ and $\phi(0) > 0$, where
\[
\lambda > 0, \quad \mu^l \geq 0 (l = 0, 1, 2, \cdots, m), \quad \mu = \sum_{l=0}^{m} \mu^l > 0\]
and $D \geq 0$ are constants. Then there exists a positive constant $M_y < +\infty$ such that
\[
\limsup_{t \to +\infty} y(t) \leq M_y = -\frac{D}{\lambda} + \left(\frac{D}{\lambda} + y^*\right)\exp(\lambda m\tau) < +\infty,
\]
where $y = y^*$ is the unique solution of $y(\lambda - \mu y) + D = 0$.

From now on, we will prove the boundedness of $x_i(t)$ and $u_i(t)$, for $i = 1, 2$.

**Lemma 2.5.** For system (1.3) with initial conditions (1.4), suppose that $a_{11}^{1} > a_{21}^{\mu}, a_{1}^{l} > a_{21}^{\mu}$ and $a_{22}^{l} > a_{12}^{\mu}, a_{2}^{l} > a_{12}^{\mu}$ hold, then there exist positive constants $P_1$ and $P_2$ such that
\[
\limsup_{t \to +\infty} x_1(t)x_2(t) \leq P_1 = \frac{(r_1^{\mu} + r_2^{\mu})^2}{a_{11}^{l}(a_{22}^{l} - a_{12}^{l})} \exp\left((r_1^{\mu} + r_2^{\mu})\tau\right) < +\infty,
\]
(2.3)
\[
\limsup_{t \to +\infty} x_1(t)x_2(t-\tau) \leq P_2 = \frac{(r_1^{\mu} + r_2^{\mu})^2}{a_{22}^{l}(a_{11}^{l} - a_{21}^{l})} \exp\left((r_1^{\mu} + r_2^{\mu})\tau\right) < +\infty.
\]
(2.4)

**Proof.** Firstly, suppose that $\limsup_{t \to +\infty} x_1(t)x_2(t) = +\infty$. Then there exists a subsequence $\{t_k^{(1)}\}_{k=1}^{+\infty}$ such that
\[
\lim_{k \to +\infty} x_1(t_k^{(1)})x_2(t_k^{(1)}) = +\infty\quad \text{and} \quad \frac{d}{dt}(x_1(t)x_2(t))|_{t=t_k^{(1)}} \geq 0, \quad k = 1, 2, \cdots.
\]
(2.5)

From (1.3), we obtain the following equation
\[
\frac{d}{dt}(x_1(t)x_2(t)) = x_1(t)x_2(t)\left[r_1(t) + r_2(t) - (a_1(t) - a_{21}(t))x_1(t) - a_{11}(t)x_1(t-\tau) - a_{2}(t)x_2(t)
\right.
\]
\[
- (a_{22}(t) - a_{12}(t))x_2(t-\tau) - b_1(t)u_1(t-\sigma_1) - b_2(t)u_2(t-\sigma_2)
\]
\[
\left. \leq x_1(t)x_2(t)\left[r_1^{\mu} + r_2^{\mu} - (a_1^{l} - a_{21}^{\mu})x_1(t) - a_{11}(t)x_1(t-\tau) - (a_{22}^{l} - a_{12}^{\mu})x_2(t-\tau)\right]\right]
\]
(2.6)
From (2.5), (2.6) and the assumption of the lemma, it holds that
\[ r_1^\mu + r_2^\mu \geq (a_1^l - d_2^{12})x_1(t_k^{(1)}) + a_1^l x_1(t_k^{(1)}) - \tau + (a_2^{22} - d_1^{12})x_2(t_k^{(1)}) - \tau. \]

It implies that \( x_1(t_k^{(1)}) - \tau \leq \frac{r_1^\mu + r_2^\mu}{a_1^l} \) and \( x_2(t_k^{(1)}) - \tau \leq \frac{r_1^\mu + r_2^\mu}{a_2^{22} - d_1^{12}} \). Moreover, integrating both sides of (2.6) from \( t_k^{(1)} - \tau \) to \( t_k^{(1)} \), we have
\[ x_1(t_k^{(1)})x_2(t_k^{(1)}) \leq x_1(t_k^{(1)})x_2(t_k^{(1)}) \exp \left( (r_1^\mu + r_2^\mu) \tau \right). \]
\[ \leq \frac{(r_1^\mu + r_2^\mu)^2}{a_1^l(a_2^{22} - d_1^{12})} \exp \left( (r_1^\mu + r_2^\mu) \tau \right) < +\infty. \]
It leads to a contradiction to our consumption. Thus, we obtain that
\[ \limsup_{t \to +\infty} x_1(t)x_2(t) < +\infty. \]

Furthermore, similar to the above discussion, we get that
\[ \limsup_{t \to +\infty} x_1(t)x_2(t) \leq \frac{(r_1^\mu + r_2^\mu)^2}{a_1^l(a_2^{22} - d_1^{12})} \exp \left( (r_1^\mu + r_2^\mu) \tau \right) = P_1 < +\infty. \]

Secondly, suppose that \( \limsup_{t \to +\infty} x_1(t)x_2(t - \tau) = +\infty \). Then there exists a subsequence \( \{t_k^{(2)}\}_{k=1}^{+\infty} \) such that
\[ t_k^{(2)} > \tau, \quad \lim_{k \to +\infty} x_1(t_k^{(2)})x_2(t_k^{(2)} - \tau) = +\infty \quad \text{and} \quad \frac{d}{dt}(x_1(t)x_2(t - \tau)) \big|_{t = t_k^{(2)}} \geq 0, \quad k = 1, 2, \ldots. \]

From (1.3), we obtain the following equation
\[ \frac{d}{dt}(x_1(t)x_2(t - \tau)) = x_1(t)x_2(t - \tau) \left[ r_1(t) + r_2(t - \tau) - a_1(t)x_1(t) - (a_1(t) - a_2(t - \tau))x_1(t - \tau) \right. \]
\[ - (a_2(t - \tau) - a_2(t - \tau))x_2(t - \tau) - a_2(t - 2\tau)x_2(t - 2\tau) - b_1(t)u_1(t - \sigma_1) \]
\[ - b_2(t - \tau)u_2(t - \tau - \sigma_2) \bigg] \]
\[ \leq x_1(t)x_2(t - \tau) \left[ r_1^\mu + r_2^\mu - (a_1^{12} - a_2^{21})x_1(t - \tau) - (a_2^{22} - d_1^{12})x_2(t - \tau) \right. \]
\[ - a_2^{22}x_2(t - 2\tau) \bigg]. \]

From (2.7), (2.8) and our assumption, it holds that
\[ r_1^\mu + r_2^\mu \geq (a_1^{11} - a_2^{21})x_1(t_k^{(2)} - \tau) + (a_2^{22} - d_1^{12})x_2(t_k^{(2)} - \tau) + a_2^{22}x_2(t_k^{(2)} - 2\tau). \]
It implies that $x_1(t_k^{(2)} - \tau) \leq \frac{r_1^\mu + r_2^\mu}{a_{11}^l - a_{21}^l}$ and $x_2(t_k^{(2)} - 2\tau) \leq \frac{r_1^\mu + r_2^\mu}{a_{22}^l}$. Moreover, integrating both sides of (2.8) from $t_k^{(2)} - \tau$ to $t_k^{(2)}$, we have

$$x_1(t_k^{(2)})x_2(t_k^{(2)} - \tau) \leq x_1(t_k^{(2)} - \tau)x_2(t_k^{(2)} - 2\tau) \exp \left( (r_1^\mu + r_2^\mu) \tau \right) \leq \frac{(r_1^\mu + r_2^\mu)^2}{a_{22}^l(a_{11}^l - a_{21}^l)} \exp \left( (r_1^\mu + r_2^\mu) \tau \right) < +\infty.$$ 

It leads to a contradiction to our consumption. Thus, we obtain that $\limsup_{t\to+\infty} x_1(t)x_2(t - \tau) < +\infty$.

Furthermore, similar to the above discussion, we get that

$$\limsup_{t\to+\infty} x_1(t)x_2(t - \tau) \leq \frac{(r_1^\mu + r_2^\mu)^2}{a_{22}^l(a_{11}^l - a_{21}^l)} \exp \left( (r_1^\mu + r_2^\mu) \tau \right) = P_2 < +\infty.$$ 

The proof is complete.

**Lemma 2.6** Assume that $a_{11}^l > a_{21}^l, a_1^l > a_{12}^l$ and $a_{22}^l > a_{12}^l, a_2^l > a_{12}^l$, then for any positive solution $(x_1(t), x_2(t), u_1(t), u_2(t))^T$ of system (1.3), there exists a positive constant $M$ such that

$$\limsup_{t\to+\infty} x_i(t) \leq M, \quad \limsup_{t\to+\infty} u_i(t) \leq M, \quad i = 1, 2.$$ 

**Proof.** Let $(x_1(t), x_2(t), u_1(t), u_2(t))^T$ be a solution of system (1.3) satisfies the initial conditions (1.4). From Lemma 2.5, there exists a large enough $T_1 > 0$ such that $x_1(t)x_2(t) \leq 2P_1$ and $x_1(t)x_2(t - \tau) \leq 2P_2$, for all $t \geq T_1$. And according to the first equation of system (1.3), for $t \geq T_1$,

$$\frac{dx_1(t)}{dt} \leq x_1(t) \left[ r_1^\mu - a_1^l x_1(t) - a_{11}^l x_1(t - \tau) + a_{12}^l x_2(t - \tau) - b_1^l u_1(t - \sigma_1) \right] \leq x_1(t) \left[ r_1^\mu - a_1^l x_1(t) - a_{11}^l x_1(t - \tau) \right] + 2a_{12}^l P_2.$$ 

By applying Lemma 2.4 to above inequality, we have

$$\limsup_{t\to+\infty} x_1(t) \leq -\frac{2a_{12}^l P_2}{r_1^\mu} + \left( \frac{2a_{12}^l P_2}{r_1^\mu} - y^*_1 \right) \exp(r_1^\mu \tau) \triangleq M_{x_1} < +\infty, \quad (2.9)$$

where $y^*_1$ is the unique positive solution of $y[r_1^\mu - (a_1^l + a_{11}^l)y] + 2a_{12}^l P_2 = 0$.

Similarly, for $t \geq T_1$,

$$\frac{dx_2(t)}{dt} \leq x_2(t) \left[ r_2^\mu - a_2^l x_2(t) + a_{21}^l x_1(t) - a_{22}^l x_2(t - \tau) - b_2^l u_2(t - \sigma_2) \right] \leq x_2(t) \left[ r_2^\mu - a_2^l x_2(t) - a_{22}^l x_2(t - \tau) \right] + 2a_{21}^l P_1.$$
Again from Lemma 2.4, we have
\[
\limsup_{t \to +\infty} x_2(t) \leq - \frac{2a_{21}^\mu P_1}{r_2^\mu} + \left( \frac{2a_{21}^\mu P_1}{r_2^\mu} + y_2^* \right) \exp(r_2^\mu \tau) \triangleq M_{x_2} < +\infty, \tag{2.10}
\]
where \(y_2^*\) is the unique positive solution of \(y[r_2^\mu - (d_2^1 + d_2^2)y] + 2a_{21}^\mu P_1 = 0\).

From (2.9) and (2.10), there exists a \(T_2 \geq T_1\), such that \(x_i(t) \leq 2M_{x_i} \) \((i = 1, 2)\) for all \(t \geq T_2\). According to the latter two equations of system (1.3), for all \(t \geq T_2 + \delta\)
\[
\frac{du_i(t)}{dt} \leq -c_i^l u_i(t) + 2d_i^\mu M_{x_i}, \quad i = 1, 2.
\]
By Lemma 2.1, we obtain
\[
\limsup_{t \to +\infty} u_i(t) \leq \frac{2d_i^\mu M_{x_i}}{c_i^l} \triangleq M_{u_i} < +\infty, \quad i = 1, 2.
\]
Set \(M = \max\{M_{x_1}, M_{x_2}, M_{u_1}, M_{u_2}\}\), obviously, for \(i = 1, 2\), we have
\[
\limsup_{t \to +\infty} x_i(t) \leq M, \quad \limsup_{t \to +\infty} u_i(t) \leq M.
\]
This completes the proof.

**Lemma 2.7** Assume that \(a_{11}^l > a_{21}^l, a_1^l > a_2^l\) and \(a_{22}^l > a_{12}^l, a_2^l > a_{12}^l\), then for any positive solution \((x_1(t), x_2(t), u_1(t), u_2(t))^T\) of system (1.3), there exists a positive constant \(m\) such that
\[
\liminf_{t \to +\infty} x_i(t) \geq m, \quad \liminf_{t \to +\infty} u_i(t) \geq m, \quad i = 1, 2.
\]

**Proof.** Let \(i \in \{1, 2\}\) and \((x_1(t), x_2(t), u_1(t), u_2(t))^T\) be a solution of system (1.3) satisfies the initial conditions (1.4). From Lemma 2.6, there exists a \(T_3 \geq T_2 + \delta\) such that \(x_i(t) \leq 2M, u_i(t) \leq 2M\) for all \(t \geq T_3\). And so, for \(t \geq T_3 + \delta\), from the former two equations of system (1.3), it follows that
\[
\frac{dx_i(t)}{dt} \geq x_i(t) \left[ r_i^l - a_i^l x_i(t) - a_i^\mu x_i(t - \tau) - b_i^\mu u_i(t - \sigma_i) \right]
\]
\[
\geq x_i(t) \left[ - (a_i^l + a_i^\mu + b_i^\mu) 2M \right] \triangleq \xi_i x_i(t) \tag{2.11}
\]
where \(\xi_i = -2(a_i^l + a_i^\mu + b_i^\mu)M < 0\). Integrating both sides of (2.11) from \(s(s \leq t)\) to \(t\), it leads to
\[
x_i(s) \leq x_i(t) \exp\{-\xi_i(t - s)\}. \tag{2.12}
\]
Take $s = t - \tau, t - \eta_i$ and $t - \sigma_i$, respectively, one has

\begin{align*}
x_i(t - \tau) & \leq x_i(t) \exp(-\xi_i \delta), \\
x_i(t - \eta_i) & \leq x_i(t) \exp(-\xi_i \delta), \\
x_i(t - \sigma_i) & \leq x_i(t) \exp(-\xi_i \delta).
\end{align*}

(Sub. 2.3 and (2.12), for all $A$

Note that for large $u$ we obtain

Substituting (2.13)-(2.18) into the former two equations of system (1.3), for $t$

Then fix $\Gamma_i$, for all $t \geq T_3 + \delta + \Gamma_i$, (2.16) can be expressed in the following form

\begin{align*}
u_i(t) & \leq u_i(t - \Gamma_i) \exp(-c_i^1 \Gamma_i) + d_i^\mu x_i(t) \frac{1}{\xi_i}(1 - \exp(-\xi_i \Gamma_i)) \exp(-\xi_i \delta) \\
& \leq 2M \exp(-c_i^1 \Gamma_i) + A_i x_i(t),
\end{align*}

where $A_i = d_i^\mu \frac{1}{\xi_i}(1 - \exp(-\xi_i \Gamma_i)) \exp(-\xi_i \delta) > 0$. Then, for all $t \geq T_3 + 2\delta + \Gamma_i$, we have

\begin{align*}
u_i(t - \sigma_i) & \leq 2M \exp(-c_i^1 \Gamma_i) + A_i x_i(t - \sigma_i).
\end{align*}

(Sub. (2.13)-(2.18) into the former two equations of system (1.3), for $t \geq T_3 + 2\delta + \Gamma_i$, we obtain

\begin{align*}
\frac{dx_i(t)}{dt} & \geq x_i(t) \left[r_i^1 - d_i^\mu x_i(t) - d_i^\mu x_i(t) \exp(-\xi_i \delta) - b_i^\mu 2M \exp(-c_i^1 \Gamma_i) - b_i^\mu A_i x_i(t - \sigma_i)\right] \\
& \geq x_i(t) \left[r_i^1 - b_i^\mu 2M \exp(-c_i^1 \Gamma_i) - [d_i^\mu + d_i^\mu \exp(-\xi_i \delta) + b_i^\mu A_i \exp(-\xi_i \delta)]x_i(t)\right]
\end{align*}
\[ \geq x_i(t) \left[ \frac{1}{2} r_i^f - B_i x_i(t) \right], \]

where \( B_i = d_i^u + d_i^\mu \exp(-\xi_i \delta) + b_i^\mu A_i \exp(-\xi_i \delta) > 0. \) Using the first part of Lemma 2.2, we get

\[ \liminf_{t \to +\infty} x_i(t) \geq \frac{r_i^f}{2B_i} \triangleq m_{x_i}. \tag{2.19} \]

From (2.19), there exists a \( T_4 > \max \{ T_3 + 2\delta + \Gamma_i \} \) such that

\[ x_i(t) \geq \frac{1}{2} m_{x_i} \quad \text{for all} \quad t \geq T_4. \]

Together with the latter two equations of system (1.3), it is easy to see that

\[ \frac{d u_i(t)}{d t} \geq -c_i^u u_i(t) + d_i^l \frac{1}{2} m_{x_i} \quad \text{for all} \quad t \geq T_4 + \delta. \]

Thus, by applying Lemma 2.1 to the above differential inequality, we have

\[ \liminf_{t \to +\infty} u_i(t) \geq \frac{d_i^l m_{x_i}}{2c_i^u} \triangleq m_{u_i}. \]

Set \( m = \min \{ m_{x_1}, m_{x_2}, m_{u_1}, m_{u_2} \} \), obviously, for \( i = 1, 2 \), we have

\[ \liminf_{t \to +\infty} x_i(t) \geq m, \quad \liminf_{t \to +\infty} u_i(t) \geq m. \]

This completes the proof.

**Proof of the theorem 1.1.** From Lemmas 2.6 and 2.7 and the definition of permanence, the conclusion is obvious.

### 3. Conclusion

With the help of a series of inequalities, some new sufficient conditions of the permanence of system (1.3) are obtained in this paper. More meaningfully, our results show that feedback control variables have no influence on the persistent property of the system (1.3). As for Lotka-Volterra cooperative systems with more general delays, we will carry out the research works in the future.

**Conflict of Interests**

The authors declare that there is no conflict of interests.
Acknowledgements

The authors would like to express their gratitude to the reviewers and editor for their valuable comments and suggestions. Without the expert comments made by the reviewers, the paper would not be of this quality. This work was supported by the Program for Science and Technology Development Foundation of Fuzhou University (NO. 2011-XY-18).

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