Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2017, 2017:15
ISSN: 2052-2541

PERMANENCE OF A FOOD-CHAIN SYSTEM WITH STAGE STRUCTURE AND TIME DELAY

ZHIHUI MA[1,2,*], SHUFAN WANG[3]

[1]School of Mathematics and Statistics, Lanzhou University, Lanzhou, 730000, China

[2]Key Laboratory of Applied Mathematics and Complex Systems, Gansu Province, Lanzhou, 730000, China

[3]School of Mathematics and Computer Science, Northwest University for Nationalities, Lanzhou, 730030, China

Communicated by Y. Tian

Copyright © 2017 Ma and Wang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. A non-autonomous food-chain system incorporating discrete time delay and stage-structure for each species has been presented in this paper. The sufficient conditions are derived for permanence and non-permanence of the considered system by applying the standard comparison theorem. Finally, the ecological meaning of the conclusions are discussed.

Keywords: a food-chain system; stage structure; discrete time delay; permanence; non-permanence.

2010 AMS Subject Classification: 92B05.

*Corresponding author

E-mail address: mazhh@lzu.edu.cn

Received May 31 , 2017

1. Introduction

The permanence of predation systems have been received much attention in recent years (Seeing in [1-18]). Many ecological effects are incorporated in the predation systems when scientists investigate permanence of the considered system with stage-structure and time delay [6-15]. The pioneering work on a single species growth model with stage structure and time delay, which represented a mathematically more careful and biologically meaningful formulation approach, was formulated and discussed by Aiello and Freedman [1]. Motivated by their work, many authors focused on the permanence of the predation system with stage structure and time delay [Seeing in 2-18]. Chen et al. [2] proposed the following system

$$
\begin{aligned}
&\dot{x}_1(t) = r_1(t)x_2(t) - d_{11}x_1(t) - r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1)\\
&\dot{x}_2(t) = r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1) - d_{12}x_2(t) - b_1(t)x_2^2(t) - c_1(t)x_2(t)y_2(t)\\
&\dot{y}_1(t) = r_2(t)y_2(t) - d_{22}y_1(t) - r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2)\\
&\dot{y}_2(t)r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2) - d_{22}y_2(t) - b_2(t)y_2^2(t) + c_2(t)x_2(t)y_2(t)
\end{aligned} \tag{1}
$$

and obtained a set of novel criteria which ensure the permanence of the considered system by introducing a new lemma and applying the standard comparison theorem. Their method and conclusions were novel and interesting. As one type of the predation system, the food-chain system are more realistic and complex than the prey-predator system. Thus, considering the food-chain system is more interesting and challenging. Liu et al. [3] proposed an impulsive reaction-diffusion periodic food-chain system with ratio-dependent functional response, they obtained the sufficient conditions for the ultimate boundedness and permanence of the food-chain system by the standard comparison theory and upper and lower solution method. Liao et al. [4] considered is a delayed discrete time Lotka-Volterra type food-chain model and obtained the sufficient conditions of the permanence. A food-chain predator-prey model with Holling IV type functional response is proposed by Shen [5] and the sufficient conditions for the permanence and the global attractivity of the system were obtained by applying the comparison theorem of the differential equation and constructing the suitable Lyapunov function.

However, the food-chain system with stage-structure and time delay are mainly confined to one or two species having stage-structure in the existing literature. Permanence of the system in which three species have stage-structure are received less attention. Motivated by these, our paper represents the following stage-structured food-chain system with time delay

$$
\begin{aligned}
&\dot{I}_1(t) = \alpha_1 M_1(t) - \gamma_1 I_1(t) - \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) \\
&\dot{M}_1(t) = \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) - a_{11}(t) M_1^2(t) - c_1(t) M_1(t) M_2(t) \\
&\dot{I}_2(t) = \alpha_2 M_1(t) M_2(t) - \gamma_2 I_2(t) - \alpha_2 e^{-\gamma_2\tau_2} M_1(t-\tau_2) M_2(t-\tau_2) \\
&\dot{M}_2(t) = \alpha_2 e^{-\gamma_2\tau_2} M_1(t-\tau_2) M_2(t-\tau_2) - d_2(t) M_2(t) - a_{22}(t) M_2^2 - c_2(t) M_2(t) M_3(t) \\
&\dot{I}_3(t) = \alpha_3 M_2(t) M_3(t) - \gamma_3 I_3(t) - \alpha_3 e^{-\gamma_3\tau_3} M_2(t-\tau_3) M_3(t-\tau_3) \\
&\dot{M}_3(t) = \alpha_3 e^{-\gamma_3\tau_3} M_2(t-\tau_3) M_3(t-\tau_3) - d_3(t) M_3(t) - a_{33}(t) M_3^2(t)
\end{aligned}
\tag{2}
$$

Where $I_1(t)$ and $M_1(t)$ denote the immature and mature population densities of the prey at time t, respectively. $I_2(t)$ and $M_2(t)$ represent the immature and mature population densities of the predator at time t, respectively. $I_3(t)$ and $M_3(t)$ are the immature and mature population densities of the top predator at time t, respectively. $a_{ii}(t)$ $(1 = 1,2,3)$, $d_j(t)$ $(j = 2,3)$, $c_k(t)$ $(k = 1,2)$ are positive and continuous functions for all $t \geq 0$. The above system assumes that mature predators feed only on the mature prey population, and the mature top predators feed only on the mature predators. The birth rate of the prey population is proportional to the existing mature population with a proportionality $\alpha_1 > 0$. $c_1(t)$ and $c_2(t)$ are the capturing rate of mature predators and top predators, respectively. $\alpha_i(c_i(t))^{-1}$ $(i = 2,3)$ is the conversion rate of nutrients into the reproduction of mature predators and top predators, respectively. $a_{ii}(t) > 0$ $(i = 1,2,3)$ denotes the intra-species competition rate of mature prey, mature predators and top predators respectively. γ_i $(i = 1,2,3)$ is the death rate of immature population of prey, predator and top predator respectively. $d_2(t)$ and $d_3(t)$ denote the death rate of mature predators and top predators, respectively. $\tau_i > 0$ $(i = 1,2,3)$ is the length of time from birth to maturity of ith species.

The initial conditions of system (2) are given by

$$
\begin{aligned}
&I_i(\theta) = \varphi_i(\theta) > 0,\ M_i(\theta) = \psi_i(\theta) > 0, \\
&\varphi_i(0) > 0,\ \psi_i(0) > 0,\ \theta \in [-\tau, 0],\ \tau = \max(\tau_1, \tau_2, \tau_3)
\end{aligned}
\tag{3}
$$

For the continuity of initial conditions, we require further that

$$I_1(0)=\int_{-\tau_1}^{0}\alpha_1 e^{\gamma_1 s}\psi_1(s)ds, I_2(0)=\int_{-\tau_2}^{0}\alpha_2 e^{\gamma_2 s}\psi_2(s)ds, I_3(0)=\int_{-\tau_3}^{0}\alpha_3 e^{\gamma_3 s}\psi_3(s)ds. \tag{4}$$

In this paper, we will mainly consider the stage-structured three species food-chain system with discrete time delay, and obtain the sufficient conditions for permanence and non-permanence of the system (2).

2. **Permanence and Non-Permanence**

Throughout this paper, we define that

$$\begin{aligned}
&a_{kk}^s=\sup_{t\in R_+^1} a_{kk}(t)>0,\ c_l^s=\sup_{t\in R_+^1} c_l(t)>0,\\
&d_j^s=\sup_{t\in R_+^1} d_j(t)>0,\ a_{kk}^i=\inf_{t\in R_+^1} a_{kk}^i>0,\\
&c_l^i=\inf_{t\in R_+^1} c_l(t)>0,\ d_j^i=\inf_{t\in R_+^1} d_j(t)(j=2,3;k=1,2,3;l=1,2).
\end{aligned} \tag{5}$$

Definition 3.1 [1]. *If there exist positive constants M^i and M^s, such that each solution $(I_1(t),M_1(t),I_2(t),M_2(t),I_3(t),M_3(t))$ of the system (2) satisfies*

$$\begin{aligned}
&0<M^i\le \liminf_{t\to+\infty} I_i(t)\le \limsup_{t\to+\infty} I_i(t)\le M^s(i=1,2,3),\\
&0<M^i\le \liminf_{t\to+\infty} M_i(t)\le \limsup_{t\to+\infty} M_i(t)\le M^s(i=1,2,3).
\end{aligned} \tag{6}$$

Then the system (2) is permanent. Otherwise, it is called non-permanent.

Lemma 3.2 [2] *Considering the following equation*

$$\dot v(t)=av(t-\tau)-bv(t)-cv^2(t) \tag{7}$$

where a,b,c and τ are positive constants, $v(t)>0$ for all $t\in[-\tau,0]$, we have

(1) If $a>b$, then $\lim_{t\to+\infty} v(t)=\frac{a-b}{c}$,

(2) If $a<b$, then $\lim_{t\to+\infty} v(t)=0$.

Theorem 3.3. *If the assumptions (H_1) and (H_2) hold,*

$$(H_1).\ \alpha_3(\alpha_1\alpha_2 e^{-\gamma_1\tau_1-\gamma_2\tau_2}-a_{11}^i d_2^i)-a_{11}^i a_{22}^i d_3^i e^{\gamma_3\tau_3}>0,$$

$$\begin{aligned}
(H_2).\ &\alpha_1\alpha_2\alpha_3 a_{11}^i a_{22}^i d_3^i e^{-\gamma_1\tau_1-\gamma_2\tau_2}-\alpha_3(c_1^s a_{33}^i\alpha_2 e^{-\gamma_2\tau_2}+c_2^s a_{11}^i\alpha_3 e^{-\gamma_3\tau_3})\\
&\times(\alpha_1\alpha_2 e^{-\gamma_1\tau_1-\gamma_2\tau_2}-a_{11}^i d_2^i)-a_{11}^s a_{11}^i a_{22}^i d_3^i(\alpha_3 a_{22}^s d_2^s+d_3^s e^{\gamma_3\tau_3})>0.
\end{aligned}$$

Then the system (2) is permanent.

Proof. Let $(I_1(t), M_1(t), I_2(t), M_2(t), I_3(t), M_3(t))$ is a solution of the system (2) with initial conditions (3) and (4).

From the second equation of the system (2), we have

$$\dot{M}_1(t) \le \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) - a_{11}(t) M_1^2(t)$$

$$\le \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) - a_{11}^i M_1^2(t).$$

By Lemma 3.2 and standard comparison theorem, we get

$$\limsup_{t\to+\infty} M_1(t) \le \alpha_1 e^{-\gamma_1\tau_1} (a_{11}^i)^{-1} \doteq M_1^s > 0.$$

Then for any $\varepsilon > 0$, there exists a $T_1 > 0$, for any $t > T_1 > 0$, we have $M_1(t) < M_1^s + \varepsilon$.

Therefore, for any $t > T_1 + \tau$, we obtain from the fourth equation of the system (2)

$$\dot{M}_2(t) \le \alpha_2 e^{-\gamma_2\tau_2}(M_1^s + \varepsilon) M_2(t-\tau_2) - d_2(t) M_2(t) - a_{22}(t) M_2^2(t)$$

$$\le \alpha_2 e^{-\gamma_2\tau_2}(M_1^s + \varepsilon) M_2(t-\tau_2) - d_2^i M_2(t) - a_{22}^i M_2^2(t).$$

Noticing that $\alpha_2 e^{-\gamma_2\tau_2} M_1^s - d_2^i > 0$, by Lemma 3.2 and standard comparison theorem, we have

$$\limsup_{t\to+\infty} M_2(t) \le [\alpha_2 e^{-\gamma_2\tau_2}(M_1^s + \varepsilon) - d_2^i](a_{22}^i)^{-1}$$

Since ε is sufficiently small, we conclude that

$$\limsup_{t\to+\infty} M_2(t) \le [\alpha_1\alpha_2 e^{-\gamma_1\tau_1 - \gamma_2\tau_2} - a_{11}^i d_2^i](a_{11}^i a_{22}^i)^{-1} \doteq M_2^s > 0.$$

Then for this $\varepsilon > 0$, there exists a $T_2 > T_1 + \tau > 0$, for any $t > T_2 > 0$, such that $M_2(t) < M_2^s + \varepsilon$.

For any $t > T_2 + \tau$, we drive from the sixth equation of the system (2)

$$\dot{M}_3(t) \le \alpha_3 e^{-\gamma_3\tau_3}(M_2^s + \varepsilon) M_3(t-\tau_3) - d_3(t) M_3(t) - a_{33}(t) M_3^2$$

$$\le \alpha_3 e^{-\gamma_3\tau_3}(M_2^s + \varepsilon) M_3(t-\tau_3) - d_3^i M_3(t) - a_{33}^i M_3^2(t).$$

Noticing that $\alpha_3 e^{-\gamma_3\tau_2} M_2^s - d_3^i > 0$, by Lemma 3.2 and standard comparison theorem, we get

$$\limsup_{t\to+\infty} M_3(t) \le [\alpha_3 e^{-\gamma_3\tau_2}(M_2^s + \varepsilon) - d_3^i](a_{33}^i)^{-1}$$

Since ε is sufficiently small, we obtain that

$$\limsup_{t\to+\infty} M_3(t) \le [\alpha_3 e^{-\gamma_3\tau_3}(\alpha_1\alpha_2 e^{-\gamma_1\tau_1-\gamma_2\tau_2} - a_{11}^i d_2^i) - a_{11}^i a_{22}^i d_3^i][a_{11}^i a_{22}^i a_{33}^i]^{-1} \doteq M_3^s > 0.$$

Then for this $\varepsilon > 0$, there exists a $T_3 > T_2 + \tau > 0$, for any $t > T_3 > 0$, such that $M_3(t) < M_3^s + \varepsilon$.

Set $T_4 = T_3 + \tau > 0$, for any $t > T_4 > 0$. Similarly we have

$$I_1(t) \le \alpha_1 M_1^s (1 - e^{-\gamma_1\tau_1})\gamma_1^{-1} \doteq I_1^s > 0.$$

$$I_2(t) \le \alpha_2 M_1^s M_2^s (1 - e^{-\gamma_2\tau_2})\gamma_2^{-1} \doteq I_2^s > 0.$$

$$I_3(t) \le \alpha_3 M_2^s M_3^s (1 - e^{-\gamma_3\tau_3})\gamma_3^{-1} \doteq I_3^s > 0.$$

Again, for $t > T_4 + \tau$, we derive from the second equation of the system (2)

$$\begin{aligned}\dot{M}_1(t) &\ge \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) - a_{11}(t) M_1^2(t) - c_1(t)(M_2^s + \varepsilon) M_1(t)\\ &\ge \alpha_1 e^{-\gamma_1\tau_1} M_1(t-\tau_1) - a_{11}^s M_1^2(t) - c_1^s (M_2^s + \varepsilon) M_1(t).\end{aligned}$$

According to the assumptions (H_1) and (H_2), it is obtained

$$\alpha_1 e^{-\gamma_1\tau_1} > c_1^s M_2^s.$$

By Lemma 3.2 and standard comparison theorem, we get

$$\liminf_{t\to+\infty} M_1(t) \ge [\alpha_1 e^{-\gamma_1\tau_1} - c_1^s(M_2^s + \varepsilon)](a_{11}^s)^{-1}.$$

Since ε is sufficiently small, we have

$$\liminf_{t\to+\infty} M_1(t) \ge [\alpha_1 e^{-\gamma_1\tau_1} a_{11}^i a_{22}^i - c_1^s(\alpha_1\alpha_2 e^{-\gamma_1\tau_1-\gamma_2\tau_2} - a_{11}^i d_2^i)](a_{11}^i a_{11}^s a_{22}^i)^{-1} \doteq M_1^i > 0.$$

Therefore, for this $\varepsilon > 0$, there exists a $T_5 > T_4 + \tau > 0$, for any $t > T_5 > 0$, such that $M_1(t) > M_1^i - \varepsilon$.

Similarly, we have

$$M_2(t) > M_2^i - \varepsilon, M_3(t) > M_3^i - \varepsilon, I_1(t) \ge \alpha_1 M_1^i (1 - e^{-\gamma_1\tau_1})\gamma_1^{-1} > 0,$$

$$I_2(t) \ge \alpha_2 M_1^i M_2^i (1 - e^{-\gamma_2\tau_2})\gamma_2^{-1} > 0, I_3(t) \ge \alpha_1 M_2^i M_3^i (1 - e^{-\gamma_3\tau_3})\gamma_3^{-1} > 0.$$

By the Definition 3.1, the system (2) is permanent. This completes the Proof.

As can be reached from the Definition 3.1, we also obtain that

Theorem 3.4. *If the assumption* (H_1) *and* (H_2) *do not hold, Then the system (2) is non-permanent.*

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

This work was supported by the National Natural Science Foundation of China(No.11301238) the Fundamental Research Funds for the Central Universities (No.lzujbky-2017-xxx).

REFERENCES

[1] W.G. Aiello, H.I. Freedman, A time delay model of single-species growth with stage structure, Math. Biosci. 101(1990), 139-153.

[2] F.D. Chen, W.L. Chen, Y.M. Wu, Z.Z. Ma Permanence of a stage-structured predator-prey system, Appl. Math. Comput. 219(2013), 8856-8862.

[3] Z. Liu, S. Zhong, X. Liu, Permanence and periodic solutions for an impulsive reaction-diffusion food-chain system with ratio-dependent functional response, Commun. Nonlinear Sci. Numer.Simul. 19(1)(2014), 173-188.

[4] X. Liao, S. Zhou, Y. Chen, Permanence for a discrete time Lotka-Volterra type food-chain model with delays, Appl. Math. Comput. 186(2007), 279-285.

[5] C. Shen, Permanence and global attractivity of the food-chain system with Holling IV type functional response, Appl. Math. Comput. 194(2007), 179-185.

[6] F. Wei, K. Wang, Permanence of variable coefficients predator-prey system with stage structure, Appl. Math. Comput. 180(2006), 594-598.

[7] X. Song, L. Chen, A predator-prey system with stage-structured and havesting for prey. Acta. Math. Appl. Son. 18(3)(2002), 423-430.

[8] H. Hu, Permanence for nonautonomous predator-prey Kolmogorov systems with impulses and its applications, Appl. Math. Comput. 223(2013), 54-75.

[9] X. Ding, C. Wang, P. Chen , Permanence for a two-species Gause-type ratio-dependent predator-prey system with time delay in a two-patch environment, Appl. Math. Comput. 219(2013), 9099-9105.

[10] L.M. Cai, X.Y. Song, Permanence and stability of a predator-prey system with stage structure for predator, J. Compul.Appl.Math. 201(2007), 356-366.

[11] Y. Muroya, Persistence and global stability in discrete models of Lotka-Volterra type, J. Math. Anal. Appl. 330(2007), 24-33.

[12] F.D. Chen, Permanence and global attractivity of a discrete multispecies Lotka-Volterra competition predator-prey systems, Appl. Math. Comput. 182(2006), 3-12.

[13] L.P. Wu, F.D. Chen, Z. Li, Permanence and global attractivity of a discrete Schoener's competition model with delays, Math. Comput. Modelling. 49(2009), 1607-1617.

[14] F.D. Chen, Permanence of periodic Holling type predator-prey system with stage structure for prey, Appl. Math. Comput. 182(2006), 1849-1860.

[15] F.D. Chen, M.S. You, Permanence, extinction and periodic solution of the predator-prey system with Beddington-DeAngelis functional response and stage structure for prey, Nonlinear Anal., Real World Appl. 9(2008), 207-221.

[16] X. Ding, C. Wang, P. Chen, Permanence for a two-species Gause-type ratio-dependent predator-prey system with time delay in a two-patch environment, Appl. Math. Comput. 219(2013), 9099-9105.

[17] J.A. Cui, X.Y. Song, Permanence of a predator-prey system with stage structure, Discrete Contin. Dyn. Syst. Ser. B, 4(3)(2004), 547-554.

[18] Z. Liu, S. Zhong, Z. Teng, L. Zhang, Permanence and global attractivity of an impulsive ratio-dependent predator-prey system in a patchy environment, Appl. Math. Comput. 219(2013), 9791-9804.

[19] J. Hou, Z. Teng, S. Gao, Permanence and global stability for nonautonomous N-species Lotka-Volterra competitive system with impulses, Nonlinear Anal., Real World Appl. 11(2010), 1882-1896.