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A COMMENSAL SYMBIOSIS MODEL WITH NON-MONOTONIC FUNCTIONAL RESPONSE

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Abstract. A two species commensal symbiosis model with non-monotonic functional response takes the form

$$\begin{aligned}\frac{dx}{dt} &= x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big),\\ \frac{dy}{dt} &= y(a_2 - b_2 y)\end{aligned}$$

is proposed and studied, where $a_i, b_i, i = 1,2$ c_1 and d_1 are all positive constants. We show that the system admits a unique globally asymptotically stable positive equilibrium.

Keywords: commensal symbiosis model; non-monotonic; stability.

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1. Introduction

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The aim of this paper is to investigate the dynamic behaviors of the following two species commensal symbiosis model with non-monotonic functional response

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big), \\ \frac{dy}{dt} &= y(a_2 - b_2 y), \end{aligned} \tag{1.1}$$

where $a_i, b_i, i = 1,2$ and c_1 are all positive constants. Here we make the following assumption:

(1) Two species obey the Logistic type growing;

(2) The commensal of the second species to the first one obey the non-monotonic functional response, i.e., $\frac{y}{1+y^2}$, which is a humped function and declines with the high densities of the second species. That is, with the increasing of the density of second species, the commensal between two species declined.

During the last decade, many scholars investigated the dynamic behaviors of the mutualism model ([1]-[9]), however, only recently did scholars paid attention to the commensal model, a model describes a relationship which is only favorable to the one side and have no influence to the other side([10]-[14]).

Sun and Wei[11] first time proposed a intraspecific commensal model:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(\frac{k_1 - x + ay}{k_1}\Big), \\ \frac{dy}{dt} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big). \end{aligned} \tag{1.2}$$

They investigated the local stability of all equilibrium points.

Han and Chen[12] proposed the following commensal symbiosis model with feedback controls:

$$\begin{aligned} \dot{x} &= x\big(b_1 - a_{11}x + a_{12}y - \alpha_1 u_1\big), \\ \dot{y} &= y\big(b_2 - a_{22}y - \alpha_2 u_2\big), \\ \dot{u}_1 &= -\eta_1 u_1 + a_1 x, \\ \dot{u}_2 &= -\eta_2 u_2 + a_2 y. \end{aligned} \tag{1.3}$$

They showed that system (1.3) admits a unique globally stable positive equilibrium.

Xie et al. [13] proposed the following discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\big\{a_1(k)-b_1(k)x_1(k)+c_1(k)x_2(k)\big\}, \\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.4}$$

They showed that the system (1.4) admits at least one positive ω-periodic solution. Xue et al[13] further incorporate the delay to system (1.4), and they investigated the almost periodic solution of the system.

It brings to our attention that all of the above works are based on the traditional Lotka-Volterra model, which suppose that the influence of the second species to the first one is linearize. This may not be suitable since the commensal between two species become infinity as the density of the species become infinity. Now, if we assume that the the functional response between two species is of non-monotonic type ([15, 16]), it then follows the system (1.1).

The aim of this paper is to investigate the local and global stability property of the possible equilibria of system (1.1). We arrange the paper as follows: In the next section, we will investigate the existence and local stability property of the equilibria of system (1.1). In Section 3, we will investigate the global stability property of the positive equilibrium; In Section 4, an example together with its numeric simulation is presented to show the feasibility of our main results.

2. The existence and local stability of the equilibria

The equilibria of system (1.1) is determined by the system

$$\begin{aligned} & x\Big(a_1-b_1x+\frac{c_1y}{d_1+y^2}\Big)=0, \\ & y(a_2-b_2y)=0. \end{aligned} \tag{2.1}$$

Hence, system (1.1) admits four nonnegative equilibria, $A_0(0,0)$, $A_1\big(\frac{a_1}{b_1},0\big)$, $A_2\big(0,\frac{a_2}{b_2}\big)$ and $A_3\big(x^*,y^*\big)$, where

$$x^*=\frac{a_1b_2^2d_1+a_1a_2^2+a_2b_2c_1}{b_1(b_2^2d_1+a_2^2)},\ y^*=\frac{a_2}{b_2}. \tag{2.2}$$

Concerned with the local stability property of the above four equilibria, we have

Theorem 2.1. *$A_0(0,0)$, $A_1(\frac{a_1}{b_1},0)$ and $A_2\big(0,\frac{a_2}{b_2}\big)$ are unstable; $A_3\big(x^*,y^*\big)$ is locally stable.*

Proof. The Jacobian matrix of the system (1.1) is calculated as

$$J(x,y)=\begin{pmatrix} a_1-2b_1x+\dfrac{c_1y}{1+y^2} & \dfrac{c_1x(d_1-y^2)}{(d_1+y^2)^2} \\ 0 & -2b_2y+a_2 \end{pmatrix}. \tag{2.3}$$

Then the Jacobian matrix of the system (1.1) about the equilibrium $A_0(0,0)$, $A_1(\frac{a_1}{b_1},0)$ and $(0,\frac{a_2}{b_2})$ are given by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}. \tag{2.4}$$

$$\begin{pmatrix} -a_1 & \dfrac{c_1a_1}{b_1d_1} \\ 0 & a_2 \end{pmatrix}. \tag{2.5}$$

and

$$\begin{pmatrix} a_1+\dfrac{c_1a_2}{b_2\Big(d_1+(\frac{a_2}{b_2})^2\Big)} & 0 \\ 0 & -a_2 \end{pmatrix} \tag{2.6}$$

respectively. One could easily see that all of the above three matrix has at least one positive eigenvalues, which means that $A_0(0,0)$, $A_1(\frac{a_1}{b_1},0)$ and $A_2(0,\frac{a_2}{b_2})$ are all unstable.

The Jacobian matrix about the equilibrium A_3 is given by

$$\begin{pmatrix} -\dfrac{a_1b_2^2d_1+a_1a_2^2+a_2b_2c_1}{b_2^2d_1+a_2^2} & A \\ 0 & -a_2 \end{pmatrix}, \tag{2.7}$$

where

$$A=\frac{c_1(a_1b_2^2d_1+a_1a_2^2+a_2b_2c_1)b_2^2(-b_2^2d_1+a_2^2)}{b_1(b_2^2d_1+a_2^2)^3}. \tag{2.8}$$

The eigenvalues of the above matrix are $\lambda_1=-\dfrac{a_1b_2^2d_1+a_1a_2^2+a_2b_2c_1}{b_2^2d_1+a_2^2}<0, \lambda_2=-a_2<0$. Hence, $A_3(x^*,y^*)$ is locally stable.

This ends the proof of Theorem 2.1.

3. Global stability of the positive equilibrium

Theorem 2.1 shows that the system always admits a positive equilibrium, and this equilibrium is locally stable. One interesting issue is to find out the conditions which ensure the global stability of the positive equilibrium.

Lemma 3.1.[17] System

$$\frac{dy}{dt} = y(a - by) \tag{3.1}$$

has a unique globally attractive positive equilibrium $y^* = \frac{a}{b}$.

Theorem 3.1. *$A_3\big(x^*, y^*\big)$ is globally stable.*

Proof. Firstly we proof that every solution of system (1.1) that starts in R_+^2 is uniformly bounded. Noting that the second equation of (1.1) takes the form

$$\frac{dy}{dt} = y(a_2 - b_2 y). \tag{3.2}$$

By applying Lemma 3.1 to system (3.2), we know that system (3.2) has a unique globally attractive positive equilibrium $y^* = \frac{a_2}{b_2}$.

It follows from the first equation of system (1.1) that

$$\frac{dx}{dt} \le x\Big(a_1 - b_1 x + \frac{c_1}{2\sqrt{d_1}}\Big),$$

and so

$$\limsup_{t\to+\infty} x(t) \le \frac{a_1 + \frac{c_1}{2\sqrt{d_1}}}{b_1}. \tag{3.3}$$

Hence, there exists a $\varepsilon > 0$ such that for all $t > T$

$$x(t) < \frac{a_1 + \frac{c_1}{2\sqrt{d_1}}}{b_1} + \varepsilon,\ y(t) < \frac{a_2}{b_2} + \varepsilon. \tag{3.4}$$

Let

$$D = \{(x,y) \in R_+^2 : x < \frac{a_1 + \frac{c_1}{2\sqrt{d_1}}}{b_1} + \varepsilon,\ y < \frac{a_2}{b_2} + \varepsilon\}.$$

Then every solution of system (1.1) starts in R_+^2 is uniformly bounded on D. Also, from Theorem 2.1 there is a unique local stable positive equilibrium $A_3(x^*, y^*)$. To show that $A_3(x^*, y^*)$ is globally stable, it's enough to show that the system admits no limit cycle in the area D, Let's consider the Dulac function $B(x,y) = x^{-1}y^{-1}$, then

$$\frac{\partial(BP)}{\partial x} + \frac{\partial(BQ)}{\partial y} = -b_1 y^{-1} - b_2 x^{-1} < 0,$$

where $P(x,y) = x\Big(a_1 - b_1 x + \dfrac{c_1 y}{d_1 + y^2}\Big), Q(x,y) = y(a_2 - b_2 y)$. By Dulac Theorem[18], there is no closed orbit in area D. Consequently, $A_3(x^*, y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.1.

4. Numeric simulation

Now let us consider the following example.

Example 4.1. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x\Big(3 - 6x + \frac{2y}{10 + y^2}\Big), \\ \frac{dy}{dt} &= y(1 - 2y). \end{aligned} \tag{4.1}$$

In this system, corresponding to system (1.1), we take $a_1 = 3, b_1 = 6, c_1 = 2, d_1 = 10, a_2 = 1, b_2 = 2$. From Theorem 3.1, the unique positive equilibrium $\left(\frac{127}{246}, \frac{1}{2}\right)$ is globally asymptotically stable. Numeric simulation (Fig.1) also support this assertion.

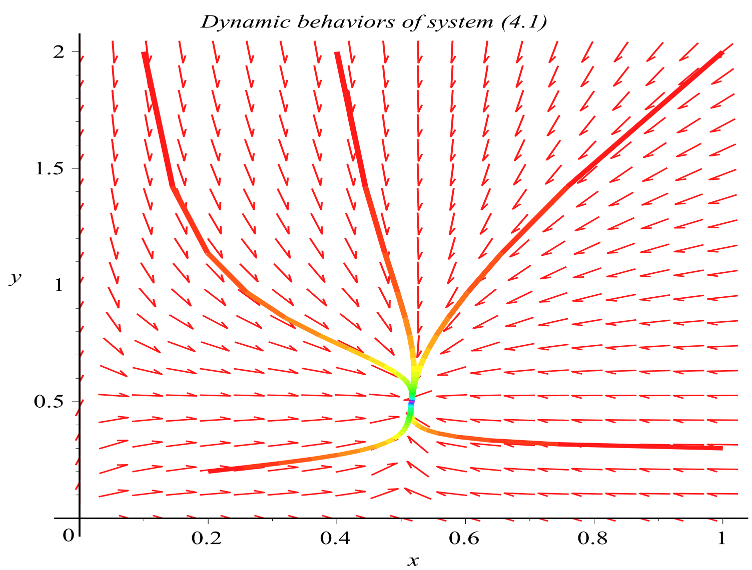

FIGURE 1. Numeric simulations of system (4.1) with the initial conditions $(x(0), y(0)) = (0.4, 2), (1, 0.3), (0.2, 0.2), (1, 2)$ and $(0.1, 2)$, respectively.

5. Conclusion

We propose a two species commensal symbiosis model with non-monotonic functional response, to the best of the authors knowledge, this is the first time such kind of model is proposed. We show that, despite the functional response, the dynamic behaviors of the system is very simple, i.e., the system admits a unique globally stable positive equilibrium.

Conflict of Interests

The authors declare that there is no conflict of interests.

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REFERENCES

[1] K. Yang, Z. S. Miao, F. D. Chen, X. D. Xie, Influence of single feedback control variable on an autonomous Holling-II type cooperative system, Journal of Mathematical Analysis and Applications, 435(1)(2016), 874-888.

[2] F. D. Chen, X. D. Xie, X. F. Chen, Dynamic behaviors of a stage-structured cooperation model, Communications in Mathematical Biology and Neuroscience, 2015 (2015), Article ID 4.

[3] Y. K. Li, T. Zhang, Permanence of a discrete N-species cooperation system with time-varying delays and feedback controls, Math. Comput. Modelling 53(2011) 1320-1330.

[4] X. D. Xie, F. D. Chen, Y. L. Xue, Note on the stability property of a cooperative system incorporating harvesting, Discrete Dynamics in Nature and Society, 2014 (2014), Article ID 327823, 5 pages.

[5] X. D. Xie, F. D. Chen, K. Yang and Y. L. Xue, Global attractivity of an integrodifferential model of mutualism, Abstract and Applied Analysis, 2014 (2014), Article ID 928726, 6 pages.

[6] F. D. Chen, X. D. Xie, Study on the dynamic behaviors of cooperation population modeling. Beijing: Science Press, 2014.

[7] Z. J. Liu, J. H. Wu, R. H. Tan, et al., Modeling and analysis of a periodic delays two-species model of facultative mutualism, Appl. Math. Comput., 217(2010), 893-903.

[8] W. S. Yang, X. P. Li, Permanence of a discrete nonlinear N-species cooperation system with time delays and feedback controls, Appl. Math. Comput., 218(7)(2011), 3581-3586.

[9] L. J. Chen, X. D. Xie, Permanence of an n-species cooperation system with continuous time delays and feedback controls, Nonlinear Anal. Real World Appl. 12(2011), 34-38.

[10] F. D. Chen, L. Q. Pu, L. Y. Yang, Positive periodic solution of a discrete obligate Lotka-Volterra model Communications in Mathematical Biology and Neuroscience, 2015 (2015), Article ID 14.

[11] G. C. Sun, W. L. Wei, The qualitative analysis of commensal symbiosis model of two populations, Mathematical Theory and Application, 23(3)(2003) 64-68.

[12] R. Y. Han, F. D. Chen, Global stability of a commensal symbiosis model with feedback controls, Communications in Mathematical Biology and Neuroscience, 2015 (2015), Article ID 15.

[13] X. D. Xie, Z. S. Miao, Y. L. Xue, Positive periodic solution of a discrete Lotka-Volterra commensal symbiosis model, Communications in Mathematical Biology and Neuroscience, 2015 (2015), Article ID 2.

[14] Y. L. Xue, X. D. Xie, F. D. Chen, R. Y. Han, Almost periodic solution of a discrete commensalism system, Discrete Dynamics in Nature and Society, 2015 (2015), Article ID 295483, 11 pages.

[15] C. X. Shen, Permanence and global attractivity of the food-chain system with Holling IV type functional response, Applied Mathematics and Computation, 194(1)(2007), 179-185.

[16] Y. Chen, Multiple periodic solutions of delayed predator-prey systems with type IV functional responses, Nonlinear Analysis Real World Applications, 5(5)(2004), 45-53.

[17] L. S. Chen, X. Y. Song, Z. Y. Lu, Mathematical models and methods in Ecology, Shichuan Science and Technology Press, 2002.

[18] Y. C. Zhou, Z. Jin, J. L. Qin, Ordinary differential equaiton and its application, Science Press, 2003.