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ON THE EXISTENCE AND STABILITY OF POSITIVE PERIODIC SOLUTION OF A NONAUTONOMOUS COMMENSAL SYMBIOSIS MODEL WITH MICHAELIS-MENTEN TYPE HARVESTING

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Abstract. A non-autonomous commensal symbiosis model of two populations with Michaelis-Menten type harvesting is proposed and studied in this paper. By using a continuation theorem based on Gaines and Mawhin's coincidence degree, we study the global existence of positive periodic solutions of the system. By constructing a suitable Lyapuonov function, sufficient conditions which ensure the global attractivity of the positive periodic solution are obtained. Numeric simulations are carried out to show the feasibility of the main results.

Keywords: commensal symbiosis model; Michaelis-Menten type harvesting; positive periodic solution; Lyapunov function; global attractivity.

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1. Introduction

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During the last decade, many scholars investigated the dynamic behaviors of the mutualism model ([1]-[15], [42]) or commensalism model ([16]-[26],[41],[43]).

Sun et al.[24] proposed the following two species commensalism system

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big),\\ \frac{dy}{dt} &= r_2 y\Big(1-\frac{y}{K_2}\Big), \end{aligned} \tag{1.1}$$

where r_1, r_2, K_1, K_2, α are all positive constants. By linearizing the system at equilibrium, the authors investigated the local stability property of the equilibria of the system.

Recently, Xue, Han, Yang et al[27] argued that the non-autonomous model is more suitable, since the coefficients of the system are varying with the time, for example, the coefficients could be changed with the seasonal factors. They proposed the following two species non-autonomous commensalism model:

$$\begin{aligned} \frac{dN_1}{dt} &= N_1\Big(a(t)-b(t)N_1+c(t)N_2\Big),\\ \frac{dN_2}{dt} &= N_2\Big(d(t)-e(t)N_2\Big). \end{aligned} \tag{1.2}$$

The authors gave a set of sufficient conditions which ensure the existence of a unique globally attractive positive periodic solution of the system. For the autonomous case of system (1.2), recently, Lin[43] further incorporated the Allee effect to the first species, he found that the final density of the species is increasing if the Allee effect is increased. Such a finding is very different to the property of the predator-prey system incorporating Allee effect.

Xie et al. [21] proposed the following discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\big\{a_1(k)-b_1(k)x_1(k)+c_1(k)x_2(k)\big\},\\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.3}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i>0, i=1,2$. By applying the coincidence degree theory, they showed that the system (1.3) admits at least one positive ω-periodic solution.

Recently, by further incorporating the Holling II functional response to system (1.3), Li et al[22] proposed the following two species discrete commensal symbiosis model

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Big\{a_1(k)-b_1(k)x_1(k)+\frac{c_1(k)x_2(k)}{e_1(k)+f_1(k)x_2(k)}\Big\}, \\ x_2(k+1) &= x_2(k)\exp\big\{a_2(k)-b_2(k)x_2(k)\big\}, \end{aligned} \tag{1.4}$$

where $\{b_i(k)\}, i=1,2, \{c_1(k)\}\{e_1(k)\}, \{f_1(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i>0, i=1,2$. They showed that the system admits at least one positive ω-periodic solution.

Wu et al[19] proposed a two species commensal symbiosis model with Holling type functional response, which takes the form

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1-b_1x+\frac{c_1y^p}{1+y^p}\Big), \\ \frac{dy}{dt} &= y(a_2-b_2y), \end{aligned} \tag{1.5}$$

where $a_i, b_i, i=1,2$ p and c_1 are all positive constants, $p\geq 1$. They showed that the unique positive equilibrium is globally stable and the system always permanent. Recently, Wu, Lin and Li[41] further incorporated the Allee effect to the second species in system (1.5), and they showed that the Allee effect has no influence on the final density of the species, however, the system needs to take much time to approach its' positive steady-state.

Chen and Wu[20] proposed a two species commensal symbiosis model with non-monotonic functional response, which takes the form

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1-b_1x+\frac{c_1y}{d_1+y^2}\Big), \\ \frac{dy}{dt} &= y(a_2-b_2y), \end{aligned} \tag{1.6}$$

where $a_i, b_i, i=1,2$ p and c_1, d_1 are all positive constants. They showed show that the system admits a unique globally asymptotically stable positive equilibrium. Recently, Lin[17] further proposed a commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure. He showed that the system may be collapse, or partial survival, or the two species could be coexist in a stable state. He also showed that if the system admits a unique positive equilibrium, then it is globally asymptotically stable.

Han and Chen[23] considered a commensal symbiosis model with feedback control variables,

i. e.:

$$\begin{aligned}
\dot{x} &= x\big(b_1 - a_{11}x + a_{12}y - \alpha_1 u_1\big),\\
\dot{y} &= y\big(b_2 - a_{22}y - \alpha_2 u_2\big),\\
\dot{u}_1 &= -\eta_1 u_1 + a_1 x,\\
\dot{u}_2 &= -\eta_2 u_2 + a_2 y.
\end{aligned} \tag{1.7}$$

They showed that system (1.7) admits a unique globally stable positive equilibrium.

Miao et al[18] proposed the following periodic Lotka-Volterra commensal symbiosis model with impulsive.

$$\begin{aligned}
\frac{dx_1}{dt} &= x_1\Big(a_1(t) - b_1(t)x_1 + c_1(t)x_2\Big),\\
\frac{dx_2}{dt} &= x_2\Big(a_2(t) - b_2(t)x_2\Big), \quad t \neq \tau_k,\\
x_i(\tau_k^+) &= (1 + h_{ik})x_i(\tau_k), \quad t = \tau_k,\ k = 1, 2, \cdots
\end{aligned} \tag{1.8}$$

Their results indicates that impulsive is one of the important factors that can change the long time behaviors of species.

On the other hand, many scholars investigated the influence of the harvesting to predator-prey or competition system, see [28]-[38] and the references cited therein. Some of them ([36]-[38]) argued that from the biological and economic points of view, nonlinear (Michaelis-Menten type) harvesting is more feasible. To the best of the authors knowledge, to this day, still no scholars consider the influence of harvesting to the commensalism model.

In this paper, we propose the following non-autonomous Lotka-Volterra commensalism model with Michaelis-Menten type harvesting for second species:

$$\begin{aligned}
\frac{dN_1(t)}{dt} &= N_1(t)\Big(a(t) - b(t)N_1(t) + c(t)N_2(t)\Big),\\
\frac{dN_2(t)}{dt} &= N_2(t)\Big(d(t) - e(t)N_2(t)\Big) - \frac{q(t)E(t)N_2(t)}{m_1(t)E(t) + m_2(t)N_2(t)},\\
N_1(0) &> 0, N_2(0) > 0,
\end{aligned} \tag{1.9}$$

where $a(t), b(t), c(t), d(t), e(t)$ have the same meaning as that of the system (1.2), $E(t)$ is the fishing effort used to harvest and $q(t)$ is the catchablity coefficient, $m_1(t)$ and $m_2(t)$ are suitable continuous positive periodic functions.

From now on, we assume that

(H_1) $a(t), b(t), c(t), d(t), e(t)$, $q(t)$, $m_1(t)$, $E(t)$ and $m_2(t)$ are all positive continuous ω-period function.

Here we assume that the coefficients of the system (1.9) are all periodic sequences which having a common integer period. Such an assumption seems reasonable in view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc.

The aim of this paper is to obtain a set of sufficient conditions which ensure the existence of a unique positive periodic solution of system (1.9), which is globally attractive. To the best of our knowledge, this is the first time that the commensalism model with nonlinear harvesting term is considered.

2. Existence of the positive periodic solution

Let $R_+^2 := \{(N_1, N_2) \in R^2, N_i \geq 0, i = 1, 2\}$. For a bounded continuous function $g(t)$ on R, we use the following notations: Define

$$g^l = \inf_{t \in R} g(t), \quad g^u = \sup_{t \in R} g(t).$$

Specially, if $g(t)$, is a continuous ω-periodic function, then define

$$\overline{g} = \frac{1}{\omega} \int_0^\omega g(t) dt.$$

Lemma 2.1. *Assume that*

$$\bar{d}\bar{m}_1 > \bar{q},$$

and

$$\bar{q}\bar{m}_2 < \bar{E}\bar{e}\bar{m}_1^2$$

hold, then

$$\bar{d} - \bar{e}x_2 - \frac{\bar{q}\bar{E}}{\bar{m}_1\bar{E} + \bar{m}_2 x_2} = 0$$

admits a unique positive solution

$$x_2^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A},$$

where

$$B=\bar{E}\bar{e}\bar{m}_1-\bar{d}\bar{m}_2,\ A=\bar{e}\bar{m}_2,\ C=\bar{E}\bar{q}-\bar{E}\bar{d}\bar{m}_1.$$

Proof. Set

$$F=\bar{d}-\bar{e}x_2-\frac{\bar{q}\bar{E}}{\bar{m}_1\bar{E}+\bar{m}_2x_2}.$$

Then

$$F(0)=\bar{d}-\frac{\bar{q}}{\bar{m}_1}>0, F(+\infty)=-\infty.$$

Also, from

$$\frac{dF}{dx_2}=-\bar{e}+\frac{\bar{q}\bar{E}\bar{m}_2}{(\bar{E}\bar{m}_1+\bar{m}_2x)^2}<0,$$

it follows that F is monotonic decreasing on the interval $[0,+\infty)$, thus, $F=0$ has a unique positive solution. Since equation

$$\bar{d}-\bar{e}x_2-\frac{\bar{q}\bar{E}}{\bar{m}_1\bar{E}+\bar{m}_2x_2}=0$$

is equivalent to the equation

$$Ax_2^2+Bx_2+C=0.$$

It immediately follows that the equation has the unique positive solution x_2^*. This ends the proof of Lemma 2.1.

Let X,Z be normed vector spaces, $L: DomL\subset X\to Z$ be a linear mapping, $N: X\to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dimKerL=CodimImL<+\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero there exist continuous projectors $P: X\to X$ and $Q: Z\to Z$ such that $ImP=KerL, ImL=KerQ=Im(I-Q)$. It follows that $L|DomL\cap KerP: (I-P)X\to ImL$ is invertible. We denote the inverse of that map by K_P. If Ω be an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N:\bar{\Omega}\to X$ is compact. Since ImQ is isomorphic to $KerL$, there exists an isomorphisms $J: ImQ\to KerL$.

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and Mawhin([34]).

Lemma 2.2 (Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose* *(a).For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;*

(b).$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and

$$deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

Theorem 2.1 *Assume (H_1) holds. Moreover, if*

(H_2) $\bar{d}\bar{m}_1 > \bar{q}$

and

(H_3) $\bar{q}\bar{m}_2 < \bar{E}\bar{e}\bar{m}_1^2$

hold, then system (1.9) has at least one positive ω-periodic solution, say $(N_1^*(t), N_2^*(t))^T$, and there exist positive constants $\alpha_i^*, \beta_i^*, i = 1,2$ such that $\alpha_j^* \leq N_j^*(t) \leq \beta_j^*, j = 1,2$.

Proof. Making the change of variables

$$N_i(t) = e^{x_i(t)} (i = 1,2),$$

then system (1.9) is reformulated as

$$\begin{aligned} \frac{dx_1(t)}{dt} &= a(t) - b(t)\exp\{x_1(t)\} + c(t)\exp\{x_2(t)\}, \\ \frac{dx_2(t)}{dt} &= d(t) - e(t)\exp\{x_2(t)\} - \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}}. \end{aligned} \tag{2.1}$$

Let $X = Z = \{x(t) = (x_1(t), x_2(t))^T \in C(R, R^2) : x(t+\omega) = x(t)\}$, Set $||x|| = ||(x_1(t), x_2(t))|| = \max_{t\in[0,\omega]} |x_1(t)| + \max_{t\in[0,\omega]} |x_2(t)|$, Then X, Z are both Banach spaces when they are endowed with the above norm $||\cdot||$.

Let

$$Nx = \begin{pmatrix} a(t) - b(t)\exp\{x_1(t)\} + c(t)\exp\{x_2(t)\} \\ d(t) - e(t)\exp\{x_2(t)\} - \dfrac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}} \end{pmatrix}, \quad x \in X,$$

$$Lx = \frac{dx(t)}{dt}, \ Px = \frac{1}{\omega}\int_0^\omega x(t)dt, \ x \in X; \ Qz = \frac{1}{\omega}\int_0^\omega z(t)dt, \ z \in Z,$$

$$Px = \frac{1}{\omega}\int_0^\omega x(t)dt, \ x \in X; \ Qz = \frac{1}{\omega}\int_0^\omega z(t)dt, \ z \in Z.$$

Then it follows

$$KerL = \{x \in X : x = h \in R^2\},\ ImL = \{z \in Z : \int_0^{\omega} z(t)dt = 0\} \text{ is closed in } Z,$$

$$dimKerL = 2 = codimImL < +\infty,$$

and P, Q are continuous projectors such that

$$ImP = KerL,\ KerQ = ImL = Im(I-Q).$$

Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P : ImL \to KerP \cap DomL$ reads $K_P(z) = \int_0^t z(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t z(s)dsdt$. Thus

$$QNx = \Big(\frac{1}{\omega}\int_0^{\omega}\Lambda_1(s)ds, \frac{1}{\omega}\int_0^{\omega}\Lambda_2(s)ds\Big)^T,$$

where

$$\begin{aligned}
\Lambda_1(s) &= a(s) - b(s)\exp\{x_1(s)\} + c(s)\exp\{x_2(s)\},\\
\Lambda_2(s) &= d(s) - e(s)\exp\{x_2(s)\} - \frac{q(s)E(s)}{m_1(s)E(s) + m_2(s)\exp\{x_2(s)\}},
\end{aligned}$$

and

$$K_P(I-Q)Nx = \Big(\Phi_1, \Phi_2\Big),$$

where $\Phi_i = \int_0^t \Lambda_i(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t \Lambda_i(s)dsdt - (\frac{t}{\omega} - \frac{1}{2})\int_0^{\omega}\Lambda_i(s)ds$, Obviously, QN and $K_P(I-Q)N$ are continuous. It is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$ by using Arzela-Ascoli theorem. Moreover, $QN(\bar{\Omega})$ is clearly bounded. Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$. The isomorphism J of ImQ onto $KerL$ can be the identity mapping, since $ImQ = KerL$.

Now we reach the position to search for an appropriate open bounded subset Ω for the application of the continuation theorem (Lemma 2.1). Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, we have

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= \lambda\Big[a(t) - b(t)\exp\{x_1(t)\} + c(t)\exp\{x_2(t)\}\Big],\\
\frac{dx_2(t)}{dt} &= \lambda\Big[d(t) - e(t)\exp\{x_2(t)\} - \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}}\Big].
\end{aligned} \tag{2.2}$$

Suppose that $x = (x_1(t), x_2(t))^T \in X$ is an arbitrary solution of system (2.2) for a certain $\lambda \in (0,1)$. Summing on both sides of (2.2) from 0 to ω, we reach

$$\begin{aligned} &\int_0^\omega \Big[a(t) - b(t)\exp\{x_1(t)\} + c(t)\exp\{x_2(t)\}\Big]dt = 0, \\ &\int_0^\omega \Big[d(t) - e(t)\exp\{x_2(t)\} - \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}}\Big]dt = 0. \end{aligned}$$

That is,

$$\int_0^\omega b(t)\exp\{x_1(t)\}dt = \bar{a}\omega + \int_0^\omega c(t)\exp\{x_2(t)\}dt, \tag{2.3}$$

$$\int_0^\omega \Big(e(t)\exp\{x_2(t)\} + \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}}\Big)dt = \bar{d}\omega. \tag{2.4}$$

$$\begin{aligned} & \int_0^\omega |\dot{x}_1(t)|dt \\ = & \lambda \int_0^\omega |a(t) - b(t)\exp\{x_1(t)\} + c(t)\exp\{x_2(t)\}|dt \\ \le & \int_0^\omega (a(t) + c(t)\exp\{x_2(t)\})dt + \int_0^\omega b(t)\exp\{x_1(t)\}dt \\ \le & \int_0^\omega a(t)dt + 2\int_0^\omega c(t)\exp\{x_2(t)\}dt \\ & -\int_0^\omega c(t)\exp\{x_2(t)\}dt + \int_0^\omega b(t)\exp\{x_1(t)\}dt \\ = & 2\int_0^\omega c(t)\exp\{x_2(t)\}dt, \end{aligned} \tag{2.5}$$

$$\begin{aligned} & \int_0^\omega |\dot{x}_2(t)|dt \\ = & \lambda \int_0^\omega |d(t) - e(t)\exp\{x_2(t)\} - \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}}|dt \\ \le & \lambda \int_0^\omega d(t)dt + \int_0^\omega (e(t)\exp\{x_2(t)\} + \frac{q(t)E(t)}{m_1(t)E(t) + m_2(t)\exp\{x_2(t)\}})dt \\ < & 2\bar{d}\omega. \end{aligned}$$

Since $\{x(t)\} = \{(x_1(t), x_2(t))^T\} \in X$, there exist $\eta_i, \delta_i, i = 1,2$ such that

$$x_i(\eta_i) = \min_{t\in[0,\omega]} x_i(t),\ x_i(\delta_i) = \max_{t\in[0,\omega]} x_i(t). \tag{2.6}$$

By (2.4), one could easily obtain

$$\int_0^\omega \Big(e(t)\exp\{x_2(\eta_2)\}\Big)dt \le \bar{d}\omega. \tag{2.7}$$

Thus,

$$x_2(\eta_2) \le \ln \frac{\bar{d}}{\bar{e}}. \tag{2.8}$$

By (2.4), one could also obtain

$$\int_0^\omega \Big(e(t)\exp\{x_2(\delta_2)\} + \frac{q(t)}{m_1(t)}\Big)dt \ge \bar{d}\omega. \tag{2.9}$$

Thus,

$$x_2(\delta_2) \ge \ln \frac{\bar{d} - \overline{\frac{q}{m_1}}}{\bar{e}}. \tag{2.10}$$

Therefore,

$$x_2(t) \le x_2(\eta_2) + \int_0^\omega |\dot{x}_2(t)|dt \le \ln\frac{\bar{d}}{\bar{e}} + 2\bar{d}\omega. \tag{2.11}$$

$$x_2(t) \ge x_2(\delta_2) - \int_0^\omega |\dot{x}_2(t)|dt \ge \ln \frac{\bar{d} - \overline{\frac{q}{m_1}}}{\bar{e}} - 2\bar{d}\omega. \tag{2.12}$$

So,

$$|x_2(t)| \le \max\Big\{|\ln\frac{\bar{d}}{\bar{e}} + 2\bar{d}\omega|, |\ln\frac{\bar{d} - (\frac{\bar{q}}{m_1})}{\bar{e}} - 2\bar{d}\omega|\Big\} \stackrel{\text{def}}{=} H_2. \tag{2.13}$$

It follows from (2.3) that

$$\int_0^\omega b(t)\exp\{x_1(\eta_1)\}dt \le \bar{a}\omega + \int_0^\omega c(t)\exp\{x_2(t)\}dt \quad \le \quad \bar{a}\omega + \bar{c}\omega\exp\{H_2\},$$

and so,

$$x_1(\eta_1) \le \ln\frac{\Delta_1}{\bar{b}}, \tag{2.14}$$

where

$$\Delta_1 = \bar{a} + \bar{c}\exp\{H_2\}.$$

It follows from (2.5) and (2.14) that

$$\begin{aligned} x_1(t) \quad &\le \quad x_1(\eta_1) + \int_0^\omega |\dot{x}_1(t)|dt \\ &\le \quad \ln\frac{\Delta_1}{\bar{b}} + 2\bar{c}\exp\{H_2\}\omega \stackrel{\text{def}}{=} M_1. \end{aligned} \tag{2.15}$$

It follows from (2.3) that

$$\int_0^\omega b(t)\exp\{x_1(\delta_1)\}dt \ge \bar{a}\omega + \int_0^\omega c(t)\exp\{x_2(t)\}dt \quad \ge \quad \bar{a}\omega + \bar{c}\omega\exp\{-H_2\},$$

and so,

$$x_1(\delta_1) \geq \ln \frac{\Delta_2}{\bar{b}}, \tag{2.16}$$

where

$$\Delta_2 = \bar{a} + \bar{c}\exp\{-H_2\}.$$

It follows from (2.5) and (2.16) that

$$\begin{aligned} x_1(t) &\geq x_1(\delta_1) - \textstyle\int_0^\omega |\dot{x}_1(t)|dt \\ &\geq \ln \tfrac{\Delta_2}{\bar{b}} - 2\bar{c}\exp\{-H_2\}\omega \stackrel{\text{def}}{=} M_2. \end{aligned} \tag{2.17}$$

It follows from (2.15) and (2.17) that

$$|x_1(t)| \leq \max\Big\{|M_1|, |M_2|\Big\} \stackrel{\text{def}}{=} H_1. \tag{2.18}$$

Clearly, H_1 and H_2 are independent on the choice of λ. Obviously, the system of algebraic equations

$$\bar{a} - \bar{b}x_1 + \bar{c}x_2 = 0, \;\; \bar{d} - \bar{e}x_2 - \frac{\bar{q}\bar{E}}{\bar{m}_1\bar{E} + \bar{m}_2 x_2} = 0 \tag{2.19}$$

has a unique positive solution $(x_1^*, x_2^*) \in R_2^+$, where

$$x_1^* = \frac{\bar{a} + \bar{c}x_2^*}{\bar{b}},$$

and x_2^* is defined by Lemma 2.1.

Let $H = H_1 + H_2 + H_3$, where $H_3 > 0$ is taken enough large such that $||(\ln\{x_1^*\},\, \ln\{x_2^*\})^T|| = |\ln\{x_1^*\}| + |\ln\{x_2^*\}| < H_3$.

Let $H = H_1 + H_2 + H_3$, and define

$$\Omega = \Big\{x(t) = (x_1(t), x_2(t))^T \in X : \|x\| < H\Big\}.$$

It is clear that Ω verifies requirement (a) in Lemma 2.2. When $x \in \partial\Omega \cap KerL = \partial\Omega \cap R^2$, x is constant vector in R^2 with $||x|| = H$. Then

$$QNx = \begin{pmatrix} \bar{a} - \bar{b}\exp\{x_1\} + \bar{c}\exp\{x_2\} \\ \bar{d} - \bar{e}\exp\{x_2\} - \dfrac{\bar{q}\bar{E}}{\bar{m}_1\bar{E} + \bar{m}_2\exp\{x_2\}} \end{pmatrix} \neq 0.$$

Moreover, direct calculation shows that

$$deg\{JQN, \Omega \cap KerL, 0\} = \operatorname{sgn}\Big(\bar{b}K\exp\{x_1^*\}\exp\{x_2^*\}\Big) = 1 \neq 0.$$

where

$$K = \bar{e} - \frac{\bar{q}\bar{E}\bar{m}_2}{(\bar{m}_1\bar{E} + \bar{m}_2\exp\{x_2^*\})^2} > \bar{e} - \frac{\bar{q}\bar{m}_2}{(\bar{m}_1)^2\bar{E}} > 0,$$

$deg(.)$ is the Brouwer degree and the J is the identity mapping since $ImQ = KerL$.

By now we have proved that Ω verifies all the requirements in Lemma 2.2. Hence (2.1) has at least one solution $(x_1^*(t), x_2^*(t))^T$ in $DomL \cap \bar{\Omega}$. And so, system (1.9) admits a positive periodic solution $(x_1^*(t), x_2^*(t))^T$, where $N_i^*(t) = \exp\{x_i^*(t)\}, i = 1,2$. This completes the proof of the claim.□

3. Permanence and global attractivity

Lemma 3.1.[40] If $a > 0, b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Theorem 3.1. *Let $(N_1(t), N_2(t))^T$ be any solution of system (1.9), assume that*

$$d^l m_1^l > q^u, \tag{3.1}$$

then the system is permanent, i. e., there exists positive constants $\beta_i, \Gamma_i, i = 1,2$, which independent of the solutions of (1.9), such that

$$\begin{aligned} \beta_1 &\leq \liminf_{t\to+\infty} N_1(t) \leq \limsup_{t\to+\infty} N_1(t) \leq \Gamma_1, \\ \beta_2 &\leq \liminf_{t\to+\infty} N_2(t) \leq \limsup_{t\to+\infty} N_2(t) \leq \Gamma_2. \end{aligned} \tag{3.2}$$

where

$$\beta_1 = \frac{a^l}{b^u};\ \Gamma_1 = \frac{a^u + c^u\dfrac{d^u}{e^l}}{b^u};\ \beta_2 = \frac{d^l - \dfrac{q^u}{m_1^l}}{e^u};\ \Gamma_2 = \frac{d^u}{e^l}.$$

Proof Let $(N_1(t),N_2(t))^T$ be any solution of system (1.9). From the second equation of system (1.9), it immediately follows that

$$\frac{dN_2(t)}{dt} \le N_2(t)\Big(d^u - e^l N_2(t)\Big), \tag{3.3}$$

Applying Lemma 3.1 to above inequality leads to

$$\limsup_{t\to+\infty} N_2(t) \le \frac{d^u}{e^l}. \tag{3.4}$$

For any small positive constants $\varepsilon > 0$, there exists $T_1 > 0$ such that

$$N_2(t) < \frac{d^u}{e^l} + \varepsilon \text{ for all } t \ge T_1. \tag{3.5}$$

Again, from the second equation of system (1.9), we also have

$$\frac{dN_2(t)}{dt} \ge N_2(t)\Big(d^l - \frac{q^u}{m_1^l} - e^u N_2(t)\Big). \tag{3.6}$$

Applying Lemma 3.1 to above inequality leads to

$$\liminf_{t\to+\infty} N_2(t) \ge \frac{d^l - \dfrac{q^u}{m_1^l}}{e^u}. \tag{3.7}$$

From the first equation of system (1.9), it immediately follows that

$$\frac{dN_1(t)}{dt} \ge N_1(t)\Big(a^l - b^u N_1(t)\Big),$$

Applying Lemma 3.1 to above inequality leads to

$$\liminf_{t\to+\infty} N_1(t) \ge \frac{a^l}{b^u}. \tag{3.8}$$

(3.5) together with the first equation of system (1.9) leads to

$$\frac{dN_1(t)}{dt} \le N_1(t)\Big(a^u + c^u(\frac{d^u}{e^l} + \varepsilon) - b^l N_1(t)\Big),$$

Applying Lemma 3.1 to above inequality leads to

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^u + c^u(\dfrac{d^u}{e^l} + \varepsilon)}{b^u}.$$

Setting $\varepsilon \to 0$ leads to

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^u + c^u\dfrac{d^u}{e^l}}{b^u}. \tag{3.9}$$

(3.4), (3.7), (3.8) and (3.9) show that the conclusion of Theorem 3.1 holds, This ends the proof of Theorem 3.1.

Theorem 3.2 *In addition to (3.1), further assume that*

$$e^l > c^u + \frac{m_2^u q^u}{(m_1^l)^2 E^l} \tag{3.10}$$

holds, then system (1.9) admits a unique positive periodic solution $N^*(t) = (N_1^*(t), N_2^*(t))^T$ *which is globally attractive, i. e., for any positive solution* $N(t) = (N_1(t), N_2(t))^T$ *of system (1.9), one has*

$$\lim_{t\to+\infty}\Big(|N_1(t)-N_1^*(t)|+|N_2(t)-N_2^*(t)|\Big)=0.$$

Proof. Let $N(t) = (N_1(t), N_2(t))^T$ be any positive solutions of system (1.9), and $N^*(t) = (N_1^*(t), N_2^*(t))^T$ be the positive periodic solution of the system (1.9). For any enough small positive constants $\varepsilon > 0$, it then follows from Theorem 3.1 that there exists a enough large T_2, such that for all $t \geq T_2$

$$\begin{aligned} &N_1(t), N_1^*(t) < \Gamma_1+\varepsilon,\ N_2(t), N_2^*(t) < \Gamma_2+\varepsilon,\\ &N_1(t), N_1^*(t) > \beta_1-\varepsilon,\ N_2(t), N_2^*(t) > \beta_2-\varepsilon. \end{aligned} \tag{3.11}$$

Now we let

$$V(t) = |\ln N_1(t) - \ln N_1^*(t)| + |\ln N_2(t) - \ln N_2^*(t)|.$$

Then for $t > T_2$, we have

$$\begin{aligned} &D^+V(t)\\ \leq\ & -b(t)|N_1(t)-N_1^*(t))| + c(t)|N_2(t)-N_2^*(t)|\\ & -\Big(e(t) - \frac{m_2(t)q(t)E(t)}{(m_1(t)E(t)+m_2(t)N_2(t))(m_1(t)E(t)+m_2(t)N_2^*(t))}\Big)|N_2(t)-N_2^*(t)|\\ \leq\ & -b^l|N_1(t)-N_1^*(t))| - \Big(e^l - c^u - \frac{m_2^u q^u}{(m_1^l)^2E^l}\Big)|N_2(t)-N_2^*(t)|. \end{aligned} \tag{3.12}$$

Integrating both sides of (3.12) on interval $[T_2, t)$, then, for $t \geq T_2$,

$$V(t) - V(T_2) \leq \int_{T_2}^t \Big[-b^l|N_1(s)-N_1^*(s)| - \Big(e^l - c^u - \frac{m_2^u q^u}{(m_1^l)^2E^l}\Big)|N_2(s)-N_2^*(s)|\Big]ds. \tag{3.13}$$

It follows from (3.13) that for $t \geq T_2$,

$$V(t)+\min\left\{b^l, e^l - c^u - \frac{m_2^u q^u}{(m_1^l)^2 E^l}\right\}\int_{T_2}^{t}\Big[|N_1(s)-N_1^*(s)|+|N_2(s)-N_2(s)|\Big]ds \leq V(T_2). \quad (3.14)$$

Therefore, $V(t)$ is bounded on $[T_2,+\infty)$ and also

$$\int_{T_2}^{t}\Big[|N_1(s)-N_1^*(s)|+|N_2(s)-N_2(s)|\Big]ds < +\infty. \quad (3.15)$$

By (3.11), $|N_1(t)-N_1^*(t)|, |N_2(t)-N_2^*(t)|$ are bounded on $[T_2,+\infty)$. On the other hand, it is easy to see that $\dot{N}_1(t)$, $\dot{N}_2(t)$, $\dot{N}_1^*(t)$ and $\dot{N}_2^*(t)$ are bounded for $t \geq T_2$. Therefore, $|N_1(t)-N_1^*(t)|, |N_2(t)-N_2^*(t)|$ are uniformly continuous on $[T_2,+\infty)$. By Barbălat Lemma, one can conclude that

$$\lim_{t\to+\infty}\Big[|N_1(t)-N_1^*(t)|+|N_2(t)-N_2^*(t)|\Big]=0.$$

This ends the proof of the Theorem 3.2.

4. Numerical simulations

Example 4.1. Consider the following system

$$\begin{aligned}
\frac{dN_1(t)}{dt} &= N_1(t)\Big(2+\tfrac{1}{4}\cos(t)-N_1(t)+(\tfrac{1}{2}+\tfrac{1}{4}\sin(t))N_2(t)\Big),\\
\frac{dN_2(t)}{dt} &= N_2(t)\Big(2+\tfrac{1}{4}\sin(t)-3N_2(t)\Big)-\tfrac{N_2(t)}{1+N_2(t)}, \qquad (4.1)\\
N_1(0) &> 0, N_2(0)>0.
\end{aligned}$$

Corresponding to system (1.9), here we take

$$a(t)=2+\frac{1}{4}\cos(t),\ d(t)=2+\frac{1}{4}\sin(t), \quad (4.2)$$

$$q(t)=E(t)=m_1(t)=m_2(t)=1,\ c(t)=\frac{1}{2}+\frac{1}{4}\sin(t),\ e(t)=3,$$

One could easily verify that conditions (H_1)-(H_3), (3.1) and (3.10) hold, and it follows from Theorem 2.1, 3.1 and 3.2 that the system admits a unique positive equilibrium which is globally attractive. Numeric simulations (Fig. 1, Fig.2) also support this assertion.

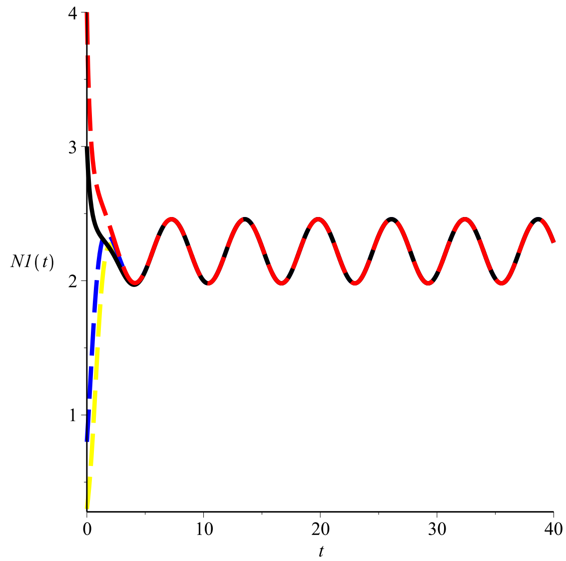

FIGURE 1. Numeric simulations of the first component system (4.1), the initial conditions $(x(0), y(0)) = (3, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(4, 0.9)$, respectively.

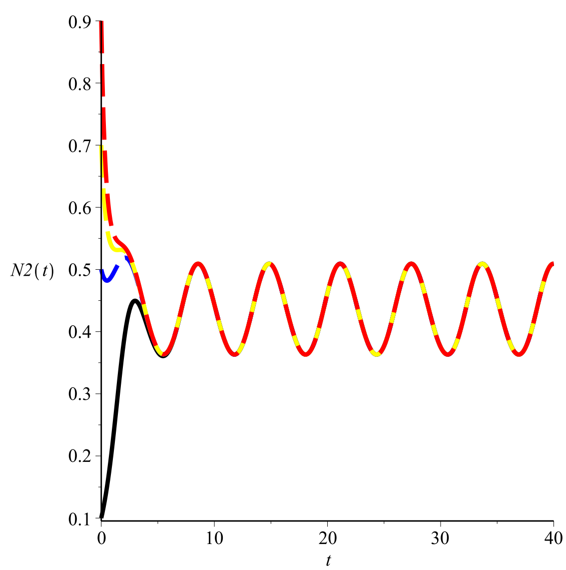

FIGURE 2. Numeric simulations of the second component of system (4.1), the initial conditions $(x(0), y(0)) = (3, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(4, 0.9)$, respectively.

5. Discussion

Recently, many scholars [15]-[27] studied the dynamic behaviors of the commensalism model, however, none of them consider the influence of harvesting. In this paper, we incorporate the Michaelis-Menten type harvesting term to the system (1.2), and this leads to the system (1.9).

Though there are many scholars study the predator-prey system with the Michaelis-Menten type harvesting [35]-[37], all of them are focus on the autonomous case and none of them consider the non-autonomous case. We consider the non-autonomous case of system (1.9), which implies that the coefficients of the system are time-dependent. By using a continuation theorem, a set of sufficient conditions which ensure the global existence of positive periodic solutions of the system are obtained. After that, we also investigated the permanence and global stability of the system. Our study shows that the intrinsic growth rate ($d(t)$) and the intrinsic competition rate ($e(t)$) of the second species are the most important factors to determine the dynamic behaviors of the system.

It seems interesting to incorporating the time delay to system (1.9) and study the influence of the time delay, we will leave this for the future study.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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