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NOTE ON THE STABILITY PROPERTY OF A RATIO-DEPENDENT HOLLING-TANNER MODEL

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Abstract. A Holling-Tanner system with ratio-dependence functional response is revisited in this paper. By developing the new analysis technique, two set of new conditions which ensure the global attractivity of the positive equilibrium of the system are obtained. Our results essential improving the main results of Liang and Pan [Qualitative analysis of a ratio-dependent Holling-Tanner model, J. Math. Anal. Appl. 334 (2007) 954-964].

Keywords: ratio-dependence; Holling-Tanner; global attractivity.

2000 Mathematics Subject Classification : 34D23, 92D25, 34D20, 34D40.

1. Introduction

Leslie-Gower type predator-prey system has been extensively investigated during the last decades, see [1]-[20]. Liang and Pan [1] proposed the following ratio-dependent Holling-Tanner

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model

$$\begin{aligned}\frac{dx}{dt} &= rx\big(1-\frac{x}{k}\big)-\frac{mx}{Ay+x}y,\\ \frac{dy}{dt} &= y\big[s(1-h\frac{y}{x})\big],\\ x(0) &> 0,\ y(0)>0,\end{aligned} \tag{1.1}$$

where r,k,m,A,s,h are all positive constants. The system is equivalent to the following system

$$\begin{aligned}\frac{dx}{dt} &= x\big(1-x\big)-\frac{xy}{ay+x},\\ \frac{dy}{dt} &= \delta y\big(\beta-\frac{y}{x}\big),\\ x(0) &> 0,\ y(0)>0,\end{aligned} \tag{1.2}$$

where $a=\dfrac{rA}{m},\delta=\dfrac{sh}{m},\beta=\dfrac{m}{hr}$.

Concerned with the persistent and stability property of the system (1.2), obtained the following results.

Theorem A. *If the condition $a>1$ holds, then system (1.2) is permanent.*

Theorem B.*Assume that the following condition holds:*

$$a\beta+1>\max\big\{\beta,\frac{1}{\delta}\big\}, \tag{1.3}$$

then the positive equilibrium $E_*(x_*,y_*)$ is globally asymptotically stable in the interior of the first quadrant.

Now let's consider the following example.

Example 1.1.

$$\begin{aligned}\frac{dx}{dt} &= x\big(1-x\big)-\frac{xy}{2y+x},\\ \frac{dy}{dt} &= \tfrac{1}{10}y\big(\tfrac{1}{10}-\frac{y}{x}\big),\\ x(0) &> 0,\ y(0)>0.\end{aligned} \tag{1.4}$$

Here, we take $a=2,\delta=\frac{1}{10},\beta=\frac{1}{10}$, hence, we have $a\beta+1=\frac{1}{5}+1,\ \beta=\frac{1}{10},\ \dfrac{1}{\delta}=10$. therefore,

$$a\beta+1<\max\big\{\beta,\frac{1}{\delta}\big\}. \tag{1.5}$$

That is, condition (1.3) in Theorem B did not holds, however, numeric simulations (Fig. 1) shows that system (1.4) admits a unique globally attractive positive equilibrium $E_*(\frac{11}{12}, \frac{11}{120})$.

Above example shows that it is necessary to revisit the stability property of the positive

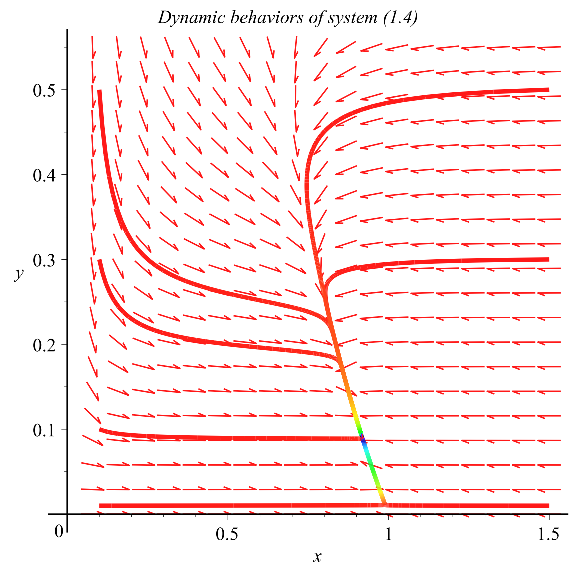

FIGURE 1. Dynamic behavior of system (1.4) with the initial condition $(x(0), y(0)) = (0.1, 0.1)$, $(0.1, 0.3)$, $(0.1, 0.5)$, $(0.1, 0.01)$, $(1.5, 0.01)$, $(1.5, 0.3)$ and $(1.5, 0.5)$, respectively.

equilibrium of system (1.2). Indeed, by using a new method which is very different to that of [1], we could establish the following results:

Theorem 1.1. *Assume that $a \geq 2$ holds, then the positive equilibrium $E_*(x_*, y_*)$ of (1.2) is globally asymptotically stable in the interior of the first quadrant.*

Theorem 1.2. *Assume that $1 < a \leq 2$ holds, assume further that*

$$a^2\beta - a\beta + a - 2 > 0 \tag{1.6}$$

and

$$a\beta - 1 - \beta > 0 \tag{1.7}$$

hold, then the positive equilibrium $E_*(x_*, y_*)$ of (1.2) is globally asymptotically stable in the interior of the first quadrant.

Remark 1.1. Note that conditions (1.6) and (1.7) are independent of δ, that is, δ has no influence to the stability property of the system, hence, Theorem 1.1 and 1.2 are thoroughly new results, and can be seen as the essential improving the main results of [1], since our results reflect some more essential property of the system (1.2).

Remark 1.2. Theorem 1.1 shows that for almost all of the parameters (only require $a \geq 2$) of the system (1.2), two species could be coexist in a stable state, this seems very interesting, a can be seen as the most important parameters in the system. Theorem 1.2 shows that for the case $1 < a < 2$, if β is enough large, i. e., the intrinsic growth rate of the predator species is enough large, then two species could also possible coexist in a stable state.

The paper is arranged as follows: In Section 2, some useful Lemmas are established and then we prove the main results in Section 3. In Section 4, two examples together with their numeric simulations are presented to illustrate the feasibility of the main results. We end this paper by a briefly discussion. For more works on Leslie-Gower predator-prey model, one could refer to [1-16] and the references cited therein.

2. Lemmas

Now we state and prove several useful Lemmas.

Lemma 2.1. *Assume that $a > 1$, then system*

$$\frac{dx}{dt} = x\Big(1 - x - \frac{B}{x + aB}\Big) \tag{2.1}$$

admits a unique positive equilibrium $x^*(B)$ which is globally attractive, where B is some positive constant.

Proof. The positive equilibrium of system (2.1) satisfies the equation

$$1 - x - \frac{B}{x + aB} = 0. \tag{2.2}$$

which is equivalent to

$$x^2 + (Ba - 1)x + B - Ba = 0. \tag{2.3}$$

Obviously, under the assumption $a>1$, system (2.3) has a unique positive solution

$$x^*(B) \quad = \quad \frac{-(Ba-1)+\sqrt{(Ba-1)^2-4B(1-a)}}{2}. \tag{2.4}$$

Set $F(x)=1-x-\dfrac{B}{x+aB}$, since $F(0)=r-\dfrac{c}{d}>0$ and $F(x^*)=0$, from the continuity of the function $F(x)$, it follows that

$$F(x)>0 \text{ for all } x\in(0,x^*)$$

and

$$F(x)<0 \text{ for all } x\in(x^*,+\infty),$$

and so applying Theorem 2.1 in [2] to system (2.1), one could see that x^* is globally stable, i. e., $\lim\limits_{t\to+\infty}x(t)=x^*$. This ends the proof of Lemma 2.1.

Lemma 2.2. *Let $x^*(B)$ be defined by (2.4), assume that $a>1$ and (1.6) holds, then $x^*(B)$, $B\in[\beta(1-\frac{1}{a}),\beta]$ is a strictly decreasing function of B.*

Proof. Since $x^*(B)$ is the positive solution of (2.3). Let's consider the function

$$F(x^*,B)=(x^*)^2+(Ba-1)x^*+B-Ba,\ x^*\in(1-\frac{1}{a},1],\ B\in[\beta(1-\frac{1}{a}),\beta]. \tag{2.5}$$

Noting that

$$\frac{\partial F}{\partial x^*}=Ba+2x^*-1\geq\beta(1-\frac{1}{a})a+2(1-\frac{1}{a})-1=\frac{a^2\beta-a\beta+a-2}{a}>0, \tag{2.6}$$

and

$$\frac{\partial F}{\partial B}=ax^*-a+1>a(1-\frac{1}{a})-a-1=0. \tag{2.7}$$

Then, it follows from implicit function theorem that

$$\frac{dx^*}{dB}=-\frac{\frac{\partial F}{\partial x^*}}{\frac{\partial F}{\partial B}}<0. \tag{2.8}$$

Hence, $x^*(B)$ is the strict decreasing function of B. This ends the proof of Lemma 2.2.

Remark 2.1. (1.6) can be rewrite as follows

$$a\beta(a-1)+a-2>0.$$

Obviously, if $a\geq 2$ holds, then above inequality holds, i. e., under the assumption of Theorem 1.1, the conclusion of Lemma 2.2 holds.

Lemma 2.3. Let $(x(t),y(t))$ be any positive solution of the system (1.2), then

$$\limsup_{t\to+\infty} x(t) \le 1, \limsup_{t\to+\infty} y(t) \le \beta.$$

Proof. From the first equation of (1.2) we have

$$\frac{dx}{dt} \le x\big(1-x\big), \tag{2.9}$$

and so,

$$\limsup_{t\to+\infty} x(t) \le 1. \tag{2.10}$$

For any $\varepsilon > 0$ enough small, it follows from (2.10) that there exists a $T > 0$ such that $x(t) < 1+\varepsilon$. and so, from the second equation of system (1.2), we have

$$\frac{dy}{dt} \le \delta y\big(\beta - \frac{y}{1+\varepsilon}\big), \tag{2.11}$$

thus

$$\limsup_{t\to+\infty} y(t) \le \beta(1+\varepsilon). \tag{2.12}$$

Setting $\varepsilon \to 0$ leads to

$$\limsup_{t\to+\infty} y(t) \le \beta. \tag{2.13}$$

This ends the proof of Lemma 2.3.

Lemma 2.4. Let $(x(t),y(t))$ be any positive solution of the system (1.2), Assume that $a > 1$ holds, then

$$\liminf_{t\to+\infty} x(t) \ge 1-\frac{1}{a},\ \liminf_{t\to+\infty} y(t) \ge \beta(1-\frac{1}{a}).$$

Proof. From the first equation of (1.2) we have

$$\frac{dx}{dt} \ge x\big(1-x-\frac{1}{a}\big), \tag{2.14}$$

thus

$$\liminf_{t\to+\infty} x(t) \ge 1-\frac{1}{a}. \tag{2.15}$$

For any $\varepsilon > 0$ enough small ($\varepsilon < \frac{1}{2}(1-\frac{1}{a})$), it follows from (2.15) that there exists a $T_1 > T$ such that $x(t) > 1-\frac{1}{a}-\varepsilon$. and so, from the second equation of system (1.2), we have

$$\frac{dy}{dt} \ge \delta y\big(\beta - \frac{y}{1-\frac{1}{a}-\varepsilon}\big), \tag{2.16}$$

and so,

$$\liminf_{t\to+\infty} y(t) \geq \beta(1-\frac{1}{a}-\varepsilon). \tag{2.17}$$

Setting $\varepsilon \to 0$ leads to

$$\liminf_{t\to+\infty} y(t) \geq \beta(1-\frac{1}{a}). \tag{2.18}$$

This ends the proof of Lemma 2.4.

3. Proof of the main results

Proof of Theorem 1.1. Let $(x(t),y(t))$ be any positive solution of system (1.2), let $\varepsilon > 0$ be any positive constant enough small which satisfies

$$\beta\frac{-\big(a(\beta+\varepsilon)-1\big)+\sqrt{\big(a(\beta+\varepsilon)-1\big)^2-4(1-a)(\beta+\varepsilon)}}{2}-(\beta+1)\varepsilon>0.$$

It follows from Lemma 2.3 that there exists a $T>0$ such that for all $t \geq T$,

$$x(t) < 1+\varepsilon \stackrel{\text{def}}{=} M_1^{(1)}. \tag{3.1}$$

$$y(t) < \beta+\varepsilon \stackrel{\text{def}}{=} M_2^{(1)}. \tag{3.2}$$

(3.2) together with the first equation of (1.2) leads to

$$\dot{x}(t) \geq x\Big(1-x-\frac{M_2^{(1)}}{x+aM_2^{(1)}}\Big) \text{ for all } t \geq T. \tag{3.3}$$

Consider the auxiliary equation

$$\dot{v}(t) = v\Big(1-v-\frac{M_2^{(1)}}{v+aM_2^{(1)}}\Big). \tag{3.4}$$

Since

$$1-\frac{M_2^{(1)}}{aM_2^{(1)}} = 1-\frac{1}{a} > 0, \tag{3.5}$$

According to Lemma 2.1, (3.4) admits a unique positive equilibrium

$$v_{11} = \frac{-(aM_2^{(1)}-1)+\sqrt{(aM_2^{(1)}-1)^2-4(1-a)M_2^{(1)}}}{2}, \tag{3.6}$$

which is globally attractive. Hence, by using differential inequality theory, there exists a $T_{11} > T$ such that

$$x(t) > v_{11} - \varepsilon \overset{\text{def}}{=} m_1^{(1)} > 0 \text{ for all } t \geq T_{11}. \tag{3.7}$$

(3.7) together with the second equation of (1.2) leads to

$$\frac{dy}{dt} \geq \delta y\big(\beta - \frac{y}{m_1^{(1)}}\big), \tag{3.8}$$

and so

$$\liminf_{t\to+\infty} y(t) \geq \beta m_1^{(1)}. \tag{3.9}$$

That is, for above $\varepsilon > 0$, there exists a $T_{12} > T_{11}$ such that

$$y(t) > \beta m_1^{(1)} - \varepsilon \overset{\text{def}}{=} m_2^{(1)} > 0 \text{ for all } t \geq T_{12}. \tag{3.10}$$

It follows from (3.1),(3.2), (3.7) and (3.10) that for all $t \geq T_{12}$,

$$0 < m_1^{(1)} < x(t) < M_1^{(1)},\ 0 < m_2^{(1)} < y(t) < M_2^{(1)}. \tag{3.11}$$

(3.11) together with the first equation of (1.2) leads to

$$\dot{x}(t) \leq x\Big(1 - x - \frac{m_2^{(1)}}{x + am_2^{(1)}}\Big) \text{ for all } t \geq T_{12}. \tag{3.12}$$

Consider the auxiliary equation

$$\dot{v} = v\Big(1 - v - \frac{m_2^{(1)}}{v + am_2^{(1)}}\Big). \tag{3.13}$$

Since

$$1 - \frac{m_2^{(1)}}{am_2^{(1)}} = 1 - \frac{1}{a} > 0, \tag{3.14}$$

According to Lemma 2.1, equation (3.13) admits a unique positive equilibrium

$$v_{21} = \frac{-(am_2^{(1)} - 1) + \sqrt{(am_2^{(1)} - 1)^2 - 4(1-a)m_2^{(1)}}}{2}, \tag{3.15}$$

which is globally attractive. Hence, by using differential inequality theory, there exists a $T_{21} > T_{12}$ such that

$$x(t) < v_{21} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)} \text{ for all } t \geq T_{21}. \tag{3.16}$$

Since

$$\begin{aligned} v_{21} &= \frac{-(am_2^{(1)}-1)+\sqrt{(am_2^{(1)}-1)^2-4(1-a)m_2^{(1)}}}{2} \\ &= \frac{-(am_2^{(1)}-1)+\sqrt{(am_2^{(1)}+1)^2-4m_2^{(1)}}}{2} \\ &< \frac{-(am_2^{(1)}-1)+\sqrt{(am_2^{(1)}+1)^2}}{2}=1, \end{aligned} \tag{3.17}$$

it follows from (3.1) and (3.17) that

$$M_1^{(2)} < M_1^{(1)}. \tag{3.18}$$

From (3.18) and the second equation of (1.2), we know that for $t \geq T_{21}$,

$$\frac{dy}{dt} \leq \delta y\Big(\beta - \frac{y}{M_1^{(2)}}\Big), \tag{3.19}$$

and so

$$\limsup_{t\to+\infty} y(t) \leq \beta M_1^{(2)}. \tag{3.20}$$

That is, for above $\varepsilon > 0$, there exists a $T_{22} > T_{21}$ such that

$$y(t) < \beta M_1^{(2)} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{(2)} \text{ for all } t \geq T_{22}. \tag{3.21}$$

It follows from (3.2), (3.18) and (3.21) that

$$M_2^{(2)} < M_2^{(1)}. \tag{3.22}$$

Substituting (3.22) into the first equation of system (1.2), we obtain

$$\dot{x}(t) \geq x\Big(1-x-\frac{M_2^{(2)}}{x+aM_2^{(2)}}\Big) \text{ for all } t \geq T_{22}.$$

Similarly to the analysis of (3.3)-(3.7), there exists a $T_{23} > T_{22}$ such that

$$x(t) > v_{22} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_1^{(2)} > 0 \text{ for all } t \geq T_{23}. \tag{3.23}$$

where

$$v_{22} = \frac{-(aM_2^{(2)}-1)+\sqrt{(aM_2^{(2)}-1)^2-4(1-a)M_2^{(2)}}}{2}. \tag{3.24}$$

From (3.22) and Lemma 2.2, we have

$$m_1^{(2)} > m_1^{(1)}. \tag{3.25}$$

From (3.23) and the second equation of (1.2), we know that for $t \geq T_{23}$,

$$\dot{y}(t) \geq \delta y\Big(\beta - \frac{y}{m_1^{(2)}}\Big), \tag{3.26}$$

thus,

$$\liminf_{t\to+\infty} y(t) \geq \beta m_1^{(2)}. \tag{3.27}$$

That is, for above $\varepsilon > 0$, there exists a $T_{24} > T_{23}$ such that

$$y(t) > \beta m_1^{(2)} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)} \text{ for all } t \geq T_{24}. \tag{3.28}$$

From (3.10), (3.25) and (3.28) we have

$$m_2^{(2)} > m_2^{(1)}. \tag{3.29}$$

It follows from (3.18),(3.22), (3.25) and (3.29) that for all $t \geq T_{24}$,

$$\begin{aligned} 0 < m_1^{(1)} < m_1^{(2)} < x(t) < M_1^{(2)} < M_1^{(1)}, \\ 0 < m_2^{(1)} < m_2^{(2)} < y(t) < M_2^{(2)} < M_2^{(1)}. \end{aligned} \tag{3.30}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}, i = 1,2, n = 1,2,....$ such that

$$M_1^{(n)} = v_{n1} + \frac{\varepsilon}{n},\ m_1^{(n)} = v_{n2} - \frac{\varepsilon}{n}, \tag{3.31}$$

$$v_{n1} = \frac{-(am_2^{(n-1)} - 1) + \sqrt{(am_2^{(n-1)} - 1)^2 - 4(1-a)m_2^{(n-1)}}}{2}. \tag{3.32}$$

$$v_{n2} = \frac{-(aM_2^{(n)} - 1) + \sqrt{(aM_2^{(n)} - 1)^2 - 4(1-a)M_2^{(n)}}}{2}. \tag{3.33}$$

$$\beta m_1^{(n)} - \frac{\varepsilon}{n} = m_2^{(n)},\ \beta M_1^{(n)} + \frac{\varepsilon}{n} = M_2^{(n)}. \tag{3.34}$$

Now, we go to show that the sequences $M_i^{(n)}$ is strictly decreasing, and the sequences $m_i^{(n)}$ is strictly increasing for $i = 1,2$ by induction. Firstly, from (3.30), we have

$$m_i^{(1)} < m_i^{(2)},\ M_i^{(2)} < M_i^{(1)},\ i = 1,2. \tag{3.35}$$

Let us suppose that

$$m_i^{(n-1)} < m_i^{(n)},\ M_i^{(n)} < M_i^{(n-1)},\ i = 1,2. \tag{3.36}$$

It then follows from Lemma 2.2, (3.32) and (3.33) that

$$v_{n1} > v_{(n+1)1}. \tag{3.37}$$

From (3.31) we have

$$M_1^{(n)} > M_1^{(n+1)}. \tag{3.38}$$

By using (3.38), it follows from (3.34) that

$$M_2^{(n)} > M_2^{(n+1)}. \tag{3.39}$$

It then follows from Lemma 2.2, (3.32) and (3.39) that

$$v_{(n+1)2} > v_{n2}. \tag{3.40}$$

(3.40) and (3.31) show that

$$m_1^{(n+1)} > m_1^{(n)}.$$

From the relationship of $m_1^{(n)}$ and $m_2^{(n)}$, we have

$$m_2^{(n+1)} > m_2^{(n)}. \tag{3.41}$$

Therefore, we have

$$\begin{aligned} &0 < m_1^{(1)} < m_1^{(2)} < \cdots < m_1^{(n)} < x(t) < M_1^{(n)} < \cdots < M_1^{(2)} < M_1^{(1)}, \\ &0 < m_2^{(1)} < m_2^{(2)} < \cdots < m_2^{(n)} < y(t) < M_2^{(n)} < \cdots < M_2^{(2)} < M_2^{(1)}. \end{aligned} \tag{3.42}$$

Hence, the limits of $M_i^{(n)}$ and $m_i^{(n)}$, $i = 1,2, n = 1,2,...$ exist. Denote that

$$\lim_{n\to+\infty} M_1^{(n)} = \overline{x}, \ \lim_{n\to+\infty} m_1^{(n)} = \underline{x}, \lim_{n\to+\infty} M_2^{(n)} = \overline{y}, \ \lim_{n\to+\infty} m_2^{(n)} = \underline{y}. \tag{3.43}$$

Then $\overline{x} \geq \underline{x}, \overline{y} \geq \underline{y}$. Letting $n \to +\infty$ in (3.31)-(3.34), we obtain

$$\begin{aligned} \overline{x} &= \frac{-(a\underline{y}-1)+\sqrt{(a\underline{y}-1)^2-4(1-a)\underline{y}}}{2}. \\ \underline{x} &= \frac{-(a\overline{y}-1)+\sqrt{(a\overline{y}-1)^2-4(1-a)\overline{y}}}{2}. \\ \beta\underline{x} &= \underline{y}, \ \beta\overline{x} = \overline{y}. \end{aligned} \tag{3.44}$$

(3.44) is equivalent to

$$\begin{aligned} &\overline{x}^2 + (a\underline{y}-1)\overline{x} + \underline{y}(1-a) = 0, \\ &\underline{x}^2 + (a\overline{y}-1)\underline{x} + \overline{y}(1-a) = 0, \\ &\beta\underline{x} = \underline{y}, \ \beta\overline{x} = \overline{y}. \end{aligned} \tag{3.45}$$

Consequently

$$\begin{aligned}\bar{x}^2+(a\beta\underline{x}-1)\bar{x}+\beta\underline{x}(1-a)=0,\\ \underline{x}^2+(a\beta\bar{x}-1)\underline{x}+\beta\bar{x}(1-a)=0.\end{aligned} \tag{3.46}$$

And so,

$$(\bar{x}-\underline{x})\big(\bar{x}+\underline{x}-1-\beta(1-a)\big)=0. \tag{3.47}$$

From Lemma 2.4 and $a\geq 2$ we have

$$\bar{x}+\underline{x}-1-\beta(1-a)>2(1-\frac{1}{a})-1-\beta(1-a)\geq\beta(a-1)>0.$$

Hence, it follows from (3.47) that

$$\bar{x}=\underline{x}.$$

Also, from (3.45) we have

$$\bar{y}=\underline{y}.$$

Under the assumption of Theorem 1.1, system (1.2) admits a unique positive solution (x^*,y^*), hence $\bar{x}=\underline{x}=x^*,\bar{y}=\underline{y}=y^*$. That is to say,

$$\lim_{t\to+\infty}x(t)=x^*,\ \lim_{t\to+\infty}y(t)=y^*. \tag{3.48}$$

This ends the proof of the Theorem 1.1.

Proof of Theorem 1.2. Similarly to the proof of Theorem 1.1, we can finally obtain (3.45)-(3.47). Assume that $\bar{x}\neq\underline{x}$, then from (3.47) we have

$$\bar{x}=-\underline{x}+1+\beta(1-a), \tag{3.49}$$

and

$$\underline{x}=1+\beta(1-a)-\bar{x}. \tag{3.50}$$

Substituting (3.49) and (3.50) to (3.46) leads to

$$\begin{aligned}A_1\bar{x}^2+A_2\bar{x}+A_3=0,\\ A_1\underline{x}^2+A_2\underline{x}+A_3=0,\end{aligned} \tag{3.51}$$

where

$$\begin{aligned} A_1 &= a\beta - 1, \\ A_2 &= (a\beta - 1)(a\beta - \beta - 1), \\ A_3 &= -\beta(a-1)(a\beta - \beta - 1). \end{aligned}$$

And so, $\overline{x}$ and $\underline{x}$ are the positive solution of the equation

$$A_1 x^2 + A_2 x + A_3 = 0. \tag{3.52}$$

From (1.7) one could see that $A_1 > 0$ and $A_3 < 0$. Hence, (3.52) had unique positive solution, this shows that $\overline{x} = \underline{x}$, the rest of the proof is similar to that of the proof of Theorem 1.1, and we omit the detail here. This ends the proof of Theorem 1.2.

4. Numeric example

Now let's consider the following example.

Example 4.1

$$\begin{aligned} \dot{x}(t) &= x\Big(1 - x - \frac{y}{x + \frac{3}{2}y}\Big), \\ \dot{y}(t) &= \frac{1}{10}y\Big(5 - \frac{y}{x}\Big). \end{aligned} \tag{4.1}$$

Corresponding to system (1.2), one has

$$a = \frac{3}{2},\ \delta = \frac{1}{10},\ \beta = 5,$$

Again, in this case

$$a\beta + 1 = \frac{17}{2} < \frac{1}{\delta} = 10.$$

Hence, Theorem A in [1] could not be applied to this system. However, one could easily verify

$$a\beta - \beta - 1 = \frac{3}{2} > 0,$$

and

$$a^2\beta - a\beta + a - 2 = \frac{13}{4} > 0.$$

Hence, from Theorem 1.2, system (4.1) admits a unique globally attractive positive equilibrium $(\frac{7}{17}, \frac{35}{17})$. numeric simulations (Fig.2) also support this findings.

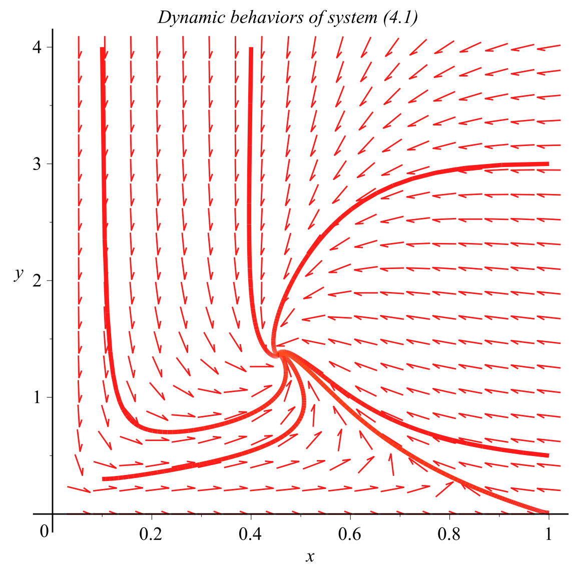

FIGURE 2. Dynamic behavior of the solution $(x(t), y(t))$ of system (4.1) with the initial condition $(x(0), y(0)) = (0.1, 0.3)$, $(0.4, 4)$, $(0.1, 4)$, $(1, 3)$, $(1, 0.5)$ and $(1, 0.01)$, respectively.

5. Discussion

In this paper, we revisit the Holling-Tanner system with ratio-dependence, which was proposed by Liang and Pan[1]. By developing some new analysis technique and using the new method, we obtain the sufficient conditions which ensure the global attractivity of the positive equilibrium.

Theorem 1.1 seems very interesting since it shows that for system (1.2), for $a \in [2, +\infty)$, the system always admits a unique positive equilibrium, which is globally attractive. Theorem 1.2 shows that for the case $1 < a < 2$, one could still obtain a the conditions which is independent of δ, to ensure the global attractivity of the positive equilibrium.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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