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DISCRETE MATHEMATICAL MODELING OF THE PROPAGATION OF RELIGIOUS IDEAS: OPTIMAL CONTROL APPROACH

ABDERRAHIM LABZAI^{1,*}, BOUCHAIB KHAJJI¹, OMAR BALATIF², MOSTAFA RACHIK¹,

¹Laboratory of Analysis Modeling and Simulation, Department of Mathematics and Computer Science, Faculty of Sciences Ben M'Sik, Hassan II University, Sidi Othman, Casablanca, Morocco

²Laboratory of Dynamical Systems, Mathematical Engineering Team (INMA), Department of Mathematics, Faculty of Sciences El Jadida, Chouaib Doukkali University, El Jadida, Morocco

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Abstract. The aim of this paper is to study and investigate the optimal control strategy of a discrete mathematical model of the propagation of religious ideas. The population that we are going to study is divided into five compartments: Potential individuals who become religious, Religious individuals who are convinced of the religious ideas and indirectly affect society, Religious individuals who are convinced of the religious ideas and directly affect society, Religious individuals who are satisfied with the religious ideas and practise religious rites and Individuals who renounce religion and do not practice religious rites. Our objective is to find the best strategy to maximize the number of religious individuals who are convinced of the religious ideas and practise religious rites and to minimize the individuals who renounce religion and do not practise it. We use two control strategies that are: firstly, the efforts to raise religious awareness, early education on learning and practising religion, advocating the conformity of behavior with the religious ideas and secondly, the efforts made through intellectual seminars and deep religious debates on the part of great scholars and thinkers. Pontryagin's maximum principle in discrete time is used to characterize the optimal controls. The numerical simulation is carried out using MATLAB. Consequently, the obtained results confirm the effectiveness of the optimization strategy.

Keywords: religious model; discrete mathematical modeling; optimal control.

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*Corresponding author

E-mail address: labzaiabdo1977@gmail.com

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1. INTRODUCTION

The religions existing in the world nowadays have a significant influence on people's lives, either because they are involved in religious practices or because they become affected by their cultural milieu. The relationship between religion and the individual makes us question the reason behind people's concern about the existence and exchange of certain religious beliefs, ideas and opinions, and how these beliefs, ideas, and opinions affect the worldview and behavior of the individuals and decision-making. In fact, one of the most psychological and behavioral functions served by religions is that they provide proper guidance to the individual and society, not only in relation to the general perspectives of life but also with respect to the important choices faced by individuals. The propagation of religious beliefs occurs in different levels such as family, school, community, media etc. Family exercises a significant influence on people's religious choices. It is the smallest unit in a society where generations transmit religious beliefs sequentially and are therefore crucial for the continuation of religious traditions and communities. Schools, especially the religious ones, play an important role in the propagation of religious ideas and practices by teaching the students the principles of religion and the right way to practise these principles. At the level of community, the relations that link people with different background and age result in a potential influence between these people[8]. Actually, in the last two millennia, many religions disseminated around the globe mainly the three vast religions: Buddhism, Christianity and Islam. These three religions have been introduced repeatedly to groups and societies and have subsequently been accepted by large numbers of people as part of their social identities or have been largely rejected and considered primarily as "foreign" religions[6]. The impact of religion on the psychological and social conditions of individuals is apparent as many studies confirm. Such studies have often mentioned that religions can make people healthier, happier and more engaged in their communities. According to a survey conducted by the Pew Research Center, it is proven that religious participation does make a difference in some of these areas. The researchers categorized the survey-takers into three types: the "actively religious," belonging to a religion and attend a house of worship at least monthly; the "inactively religious," belonging to a religion but attend less frequently; and the unaffiliated (or "nones"), who do not identify with any religion [1,9,15]. The findings

expressed in this survey demonstrate that religious affiliation especially with highly active practice could have a positive impact on such individuals. These advantages of religious practice on people drives us to investigate how religious beliefs spread and to think of the best ways to model the spread of religious beliefs mathematically. Many studies and research in social sciences have focused on this topic and other related topics. But the mathematical studies and research on this topic are still limited and most of them have focused on the statistical aspect of the phenomenon. In this work, we propose an epidemiological approach (see [8],[12],[16]) to describe and study the behavior of individuals who renounce religion and do not practise religious rites as well as individuals who are relatively satisfied with the religious ideas and do not practise religious rites. In epidemiology, we generally use compartments model to describe the spread of an infectious disease. In these epidemiological models, the population is divided into different classes according to people's status versus the disease (susceptible to catch the disease, infected or removed) and the infection process depends on the contact with infectious individuals. Similarly, during the process of spreading religious ideas, the population can be divided into several classes (potential individuals who become religious, religious individuals who are convinced of the religious ideas and directly affect society, religious individuals who are satisfied with the religious ideas and practise religious rites, and individuals who renounce religion and do not practise religious rites ...). Furthermore, the interaction between people is a key factor in the religiosity of the individuals and society. It is very similar to the contagion phenomenon since religious individuals can affect the potential individuals who may become religious in their network (family, friends, and coworkers). Therefore, the epidemic approach is more appropriate to model the religiosity behavior in the religious practitioners. We propose a discrete mathematical model that describes the propagation of religious ideas and we work on finding an optimal control strategy for this model in order to reduce the individuals who renounce religion and do not practise religious rites and also to increase the number of religious individuals who are convinced of the religious beliefs and ideas with a minimal cost. The optimal control problem was subject of an optimization criterion represented by the minimization of an objective function. To achieve this objective, we use some theoretical results of optimal

control theory. The paper is organized as follows: In Section 2, we present our discrete mathematical model that describes the propagation of religious ideas within the population dynamic. In Section 3, we present the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and we characterize these optimal controls using Pontryagin's Maximum Principle in discrete time. Numerical simulations are given in Section 4. Finally, the conclusion is given in section 5.

2. MODEL DESCRIPTION

In this section, we consider a discrete mathematical model $P_i R_i^{ic} R_i^{dc} R_i^c R_i^{nc}$ that describes the propagation of religious ideas within the population. The individuals here are divided into five compartments: Potential individuals who may become religious P_i , Religious individuals who are convinced of the religious ideas and indirectly affect society R_i^{ic} , Religious individuals who are convinced of the religious ideas and directly affect society R_i^{dc} , Religious individuals who are convinced of the religious ideas and practise devotional rites R_i^c and Individuals who renounce religion and do not practise religious rites R_i^{nc} respectively.

The compartment P: represents the potential individuals who may become religious. Their age is over adolescence and adulthood as they can distinguish between good behavior and bad behavior. This compartment is increased by the recruitment of individuals at rate Λ and it is decreased with the rates $\beta_1 \frac{P_i R_i^{ic}}{N}$ and $\beta_2 \frac{P_i R_i^{dc}}{N}$. It is assumed that potential religious individuals can acquire the behavior of being convinced of religious ideas and the practice of devotional rituals via effective contact with religious individuals who are convinced of religious ideas and indirectly affect society at a rate of β_1 or official preachers at a rate of β_2 . Some of the people vacate at a constant death rate of μ due to the total natural death rate μP_i . In other words, the acquisition of religious beliefs behavior is analogous to acquiring disease infection.

The compartment R^{ic} : is composed of the religious individuals who are convinced of the religious ideas and practise devotional rites and indirectly influence society (parents, family, friends ...). They are increased by the rate $\alpha_4 R_i^c$ (α_4 is the rate of the religious individuals who are convinced of the religious ideas who become influencing on society and they transform into the religious individuals who are convinced of the religious ideas and practise devotional rites and indirectly influence society). Also, it is decreased when the religious individuals who are

convinced of the religious ideas and indirectly influence society transform into religious individuals convinced of the religious ideas and directly influence society through official preachers with the rate $\alpha_3 R_i^{ic}$ and μR_i^{ic} . Here, α_3 is the rate of the religious individuals who are convinced of the religious ideas and indirectly influence society. They transform into official preachers and directly influence society. μ is the total natural death rate.

The compartment R^{dc} : includes preachers who are convinced of the religious ideas and practise them and directly influence society through official religious institutions (temples, churches, mosques, and the ministry of religious affairs in some countries ...). They are increased by the rate $\alpha_3 R_i^{ic}$ and some others will leave this compartment due to the total natural death rate μR_i^{dc} .

The compartment R^c : contains the individuals who become convinced of the religious ideas and practise devotional rites due to the influence of the individuals in the compartment R^{ic} and R^{dc} . They are decreased by the rate $\alpha_4 R_i^c$. Also they are increased at the rates $\sigma_1 \beta_1 \frac{P_i R_i^{ic}}{N}$, $\beta_2 (1 - \sigma_2) \frac{P_i R_i^{dc}}{N}$ and $\alpha_2 R_i^{ic}$ where, $(1 - \sigma_2)$ is the fraction of the potential religious individuals who become convinced of the religious ideas and practise them in their lives (at a rate β_2) and σ_1 is the remaining fraction of the potential religious individuals who become convinced of the religious ideas and practise them in their lives (at a rate β_1). They are decreased at the rates μR_i^c and $\alpha_1 R_i^c$. Here, α_1 is the rate of the individuals who were convinced of the religious ideas and practise them in their lives and transform into individuals who renounce religion after they changed their conviction.

The compartment R^{nc} : encompasses the individuals who renounce religion and then renounce practising it. They are increased at the rates $\beta_1 (1 - \sigma_1) \frac{P_i R_i^{ic}}{N}$, $\beta_2 \sigma_2 \frac{P_i R_i^{dc}}{N}$ and $\alpha_2 R_i^{ic}$ where, $(1 - \sigma_1)$ is the fraction of the potential religious individuals who are not convinced of the religious ideas and do not practise them in their lives (at a rate β_1) and σ_2 is the remaining fraction of the potential religious individuals who are not convinced of the religious ideas and do not practise them in their lives (at a rate β_2). They are decreased at the rates μR_i^{nc} and $\alpha_2 R_i^{nc}$. Here, α_2 is the rate of the individuals who renounce religion and transform into the individuals convinced of the religious ideas and practising them.

The following diagram will demonstrate the flow directions of individuals among the compartments. These directions are going to be represented by directed arrows; (See FiGURE 1).

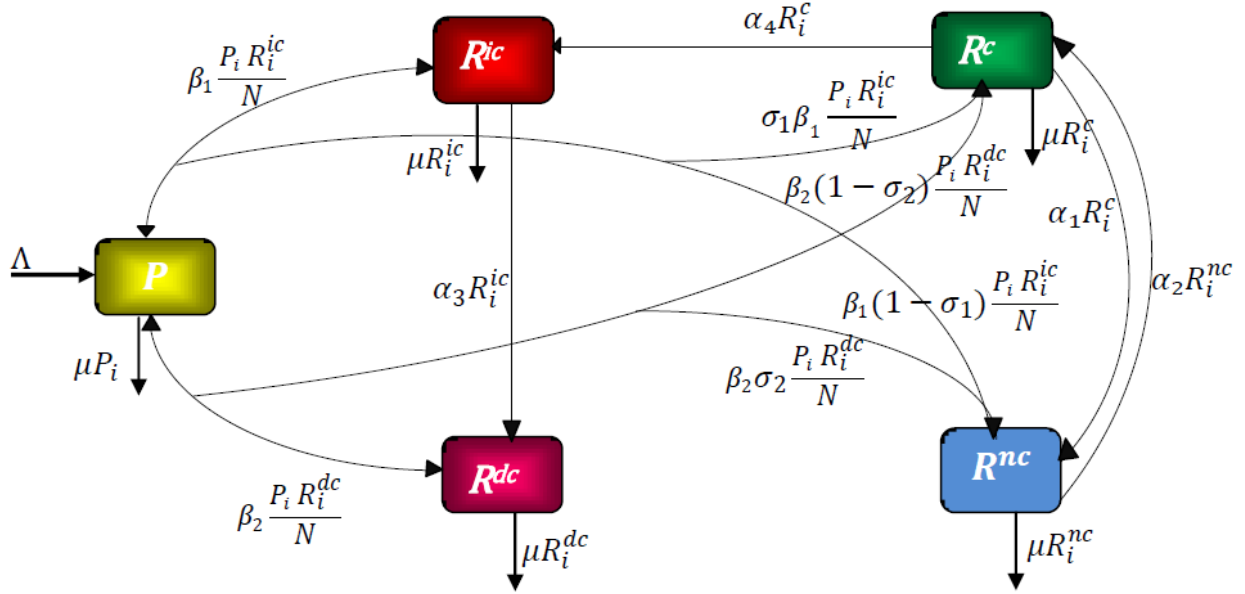


FIGURE 1. The flow between the five compartments $PR^{ic}R^{dc}R^cR^{nc}$

$$(1) \quad \begin{cases} P_{i+1} = \Lambda + (1 - \mu)P_i - \beta_1 \frac{P_i R_i^{ic}}{N} - \beta_2 \frac{P_i R_i^{dc}}{N} \\ R_{i+1}^{ic} = (1 - \mu - \alpha_3)R_i^{ic} + \alpha_4 R_i^c \\ R_{i+1}^{dc} = (1 - \mu)R_i^{dc} + \alpha_3 R_i^{ic} \\ R_{i+1}^c = (1 - \mu - \alpha_1 - \alpha_4)R_i^c + \sigma_1 \beta_1 \frac{P_i R_i^{ic}}{N} + \beta_2 (1 - \sigma_2) \frac{P_i R_i^{dc}}{N} + \alpha_2 R_i^{nc} \\ R_{i+1}^{nc} = (1 - \mu)R_i^{nc} + \beta_1 (1 - \sigma_1) \frac{P_i R_i^{ic}}{N} + \sigma_2 \beta_2 \frac{P_i R_i^{dc}}{N} + \alpha_1 R_i^c - \alpha_2 R_i^{nc} \end{cases}$$

where $P_0, R_0^{ic}, R_0^{dc}, R_0^c$ and R_0^{nc} are none negative.

3. THE OPTIMAL CONTROL PROBLEM

The strategies of control that we adopt consist of the efforts exerted in raising religious awareness through missionary campaigns and advocating the conformity of behavior with the

religious principles. Our main goal in adopting those strategies is to maximize the number of religious individuals who are convinced of the religious ideas and practise devotional rites, minimize the individuals who renounce religion and do not practise it during the time steps $i = 0$ to T and also to minimize the cost spent in applying the two strategies. In this model, we include the two controls u_i and v_i that represent consecutively the efforts to raise religious awareness, early education on religion, advocating the conformity of behavior with the religious ideas and the efforts made on intellectual seminars and deep religious debates on the part of great scholars and thinkers at time i . So, the controlled mathematical system is given by the following system of difference equations:

$$(2) \quad \begin{cases} P_{i+1} = \Lambda + (1 - \mu)P_i - \beta_1 \frac{P_i R_i^{ic}}{N} - \beta_2 \frac{P_i R_i^{dc}}{N} - u_i P_i \\ R_{i+1}^{ic} = (1 - \mu - \alpha_3)R_i^{ic} + \alpha_4 R_i^c \\ R_{i+1}^{dc} = (1 - \mu)R_i^{dc} + \alpha_3 R_i^{ic} \\ R_{i+1}^c = (1 - \mu - \alpha_1 - \alpha_4)R_i^c + \sigma_1 \beta_1 \frac{P_i R_i^{ic}}{N} + \beta_2 (1 - \sigma_2) \frac{P_i R_i^{dc}}{N} \\ \quad + \alpha_2 R_i^{nc} + u_i P_i + v_i R_i^{nc} \\ R_{i+1}^{nc} = (1 - \mu)R_i^{nc} + \beta_1 (1 - \sigma_1) \frac{P_i R_i^{ic}}{N} + \sigma_2 \beta_2 \frac{P_i R_i^{dc}}{N} + \alpha_1 R_i^c \\ \quad - \alpha_2 R_i^{nc} - v_i R_i^{nc} \end{cases}$$

where $P_0, R_0^{ic}, R_0^{dc}, R_0^c$ and R_0^{nc} are none negative.

The problem is to minimize the objective functional

$$(3) \quad J(u, v) = \sum_{i=0}^T (A R_i^{nc} - B R_i^c) + \sum_{i=0}^{T-1} \left(\frac{M_1}{2} u_i^2 + \frac{M_2}{2} v_i^2 \right)$$

Where, A and B are positive constants to keep a balance in the size of R_i^{nc} and R_i^c respectively. In the objective functional, M_1 and M_2 are the positive weight parameters which are associated with the controls u_i and v_i at time i . T is the final time.

In other words, we seek the optimal controls (u^*, v^*) such that

$$(4) \quad J(u^*, v^*) = \min_{(u, v) \in U_{ad}} J(u, v),$$

where U_{ad} is the set of admissible controls defined by

$$(5) \quad U_{ad} = \{(u_i, v_i) : u^{\min} \leq u_i \leq u^{\max}, v^{\min} \leq v_i \leq v^{\max}; i = 0, 1, 2, \dots, T-1\},$$

where $(u^{\min}, u^{\max}, v^{\min}, v^{\max}) \in [0, 1]^4$

The sufficient condition for the existence of the optimal control (u, v) for the problems (2-3) comes from the following theorem.

Theorem 1. *There exists an optimal control (u^*, v^*) such that*

$$(6) \quad J(u^*, v^*) = \min_{(u,v) \in U_{ad}} J(u, v)$$

subject to the control system (2) with initial conditions.

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $P = (P_0, P_1, \dots, P_T)$, $R^{ic} = (R_0^{ic}, R_1^{ic}, \dots, R_T^{ic})$, $R^{dc} = (R_0^{dc}, R_1^{dc}, \dots, R_T^{dc})$, $R^c = (R_0^c, R_1^c, \dots, R_T^c)$ and $R^{nc} = (R_0^{nc}, R_1^{nc}, \dots, R_T^{nc})$ are uniformly bounded for all $(u; v)$ in the control set U_{ad} ; thus $J(u; v)$ is bounded for all $(u; v) \in U_{ad}$. Since $J(u; v)$ is bounded, $\inf_{(u,v) \in U_{ad}} J(u, v)$ is finite, and there exists a sequence $(u^j; v^j) \in U_{ad}$ such that $\lim_{j \rightarrow +\infty} J(u^j, v^j) = \inf_{(u,v) \in U_{ad}} J(u, v)$ and corresponding sequences of states $P^j, R^{icj}, R^{dcj}, R^{cj}$ and R^{ncj} . Since there is a finite number of uniformly bounded sequences, there exist $(u^*, v^*) \in U_{ad}$ and $P^*, R^{ic*}, R^{dc*}, R^{h*}$ and $R^{nc*} \in IR^{T+1}$ such that on a subsequence, $\lim_{j \rightarrow +\infty} (u^j, v^j) = (u^*, v^*)$, $\lim_{j \rightarrow +\infty} P^j = P^*$, $\lim_{j \rightarrow +\infty} R^{icj} = R^{ic*}$, $\lim_{j \rightarrow +\infty} R^{dcj} = R^{dc*}$, $\lim_{j \rightarrow +\infty} R^{cj} = R^{c*}$ and $\lim_{j \rightarrow +\infty} R^{ncj} = R^{nc*}$. Finally, due to the finite dimensional structure of system (2) and the objective function $J(u; v)$; $(u^*; v^*)$ is an optimal control with corresponding states $P^*, R^{ic*}, R^{dc*}, R^{c*}$ and R^{nc*} . Therefore $\inf_{(u,v) \in U_{ad}} J(u, v)$ is achieved. \square

As regards the necessary condition and the characterization of our discrete optimal control, we use a discrete time version of the pontryagin's maximum principle [11],[14], [17], [5], [8],[9]. This principle converts into a problem of minimizing a Hamiltonian H_i at time step i defined by:

$$(7) \quad H_i = AR_i^{nc} - BR_i^c + \frac{M_1}{2}u_i^2 + \frac{M_2}{2}v_i^2 + \sum_{j=1}^5 \lambda_{j,i+1}f_{j,i+1},$$

where $f_{j,i+1}$ is the right side of the system of difference equations (2) of the j^{th} state variable at time step $i + 1$.

Theorem 2. *Given an optimal control $(u_i^*, v_i^*) \in U_{ad}$ and the solutions P_i^* , R_i^{ic*} , R_i^{dc*} , R_i^{c*} and R_i^{nc*} of the corresponding state system (2), there exist adjoint functions $\lambda_{1,i}$, $\lambda_{2,i}$, $\lambda_{3,i}$, $\lambda_{4,i}$ and $\lambda_{5,i}$ satisfying*

$$(8) \quad \begin{aligned} \lambda_{1,i} &= \lambda_{1,i+1} \left[(1 - \mu) - \beta_1 \frac{R_i^{ic}}{N} - \beta_2 \frac{R_i^{dc}}{N} - u_i \right] + \lambda_{4,i+1} \left[\sigma_1 \beta_1 \frac{R_i^{ic}}{N} + (1 - \sigma_2) \beta_2 \frac{R_i^{dc}}{N} + u_i \right] \\ &+ \lambda_{5,i+1} \left[\beta_1 (1 - \sigma_1) \frac{R_i^{ic}}{N} + \sigma_2 \beta_2 \frac{R_i^{dc}}{N} \right] \\ \lambda_{2,i} &= -\lambda_{1,i+1} \beta_1 \frac{P_i}{N} + \lambda_{2,i+1} (1 - \mu - \alpha_3) + \lambda_{3,i+1} \alpha_3 + \lambda_{4,i+1} \sigma_1 \beta_1 \frac{P_i}{N} + \lambda_{5,i+1} \beta_1 (1 - \sigma_1) \frac{P_i}{N} \\ \lambda_{3,i} &= -\lambda_{1,i+1} \beta_2 \frac{P_i}{N} + \lambda_{3,i+1} (1 - \mu) + \lambda_{4,i+1} (1 - \sigma_2) \beta_2 \frac{P_i}{N} + \lambda_{5,i+1} \sigma_2 \beta_2 \frac{P_i}{N} \\ \lambda_{4,i} &= -B + \lambda_{2,i+1} \alpha_4 + \lambda_{4,i+1} (1 - \mu - \alpha_1 - \alpha_4) + \lambda_{5,i+1} \alpha_1 \\ \lambda_{5,i} &= A + \lambda_{4,i+1} (\alpha_2 + v_i) + \lambda_{5,i+1} (1 - \mu - v_i - \alpha_2) \end{aligned}$$

With the transversality conditions at time T . $\lambda_{1,T} = \lambda_{2,T} = \lambda_{3,T} = 0$, $\lambda_{4,T} = -B$ and $\lambda_{5,T} = A$.

Furthermore, for $i = 0, 1, 2, \dots, T - 1$, the optimal controls u_i^* and v_i^* are given by

$$(9) \quad \begin{aligned} u_i^* &= \min \left[u^{\max}; \max \left(u^{\min}, \frac{1}{M_1} [(\lambda_{1,i+1} - \lambda_{4,i+1}) P_i] \right) \right] \\ v_i^* &= \min \left[v^{\max}; \max \left(v^{\min}, \frac{1}{M_2} [(\lambda_{5,i+1} - \lambda_{4,i+1}) R_i^{nc}] \right) \right] \end{aligned}$$

Proof. The Hamiltonian at time step i is given by

$$\begin{aligned} H_i &= AR_i^{nc} - BR_i^c + \frac{M_1}{2}u_i^2 + \frac{M_2}{2}v_i^2 + \lambda_{1,i+1} (\Lambda + (1 - \mu)P_i - \beta_1 \frac{P_i R_i^{ic}}{N} - \beta_2 \frac{P_i R_i^{dc}}{N} - u_i P_i) \\ &+ \lambda_{2,i+1} ((1 - \mu - \alpha_3)R_i^{ic} + \alpha_4 R_i^c) + \lambda_{3,i+1} ((1 - \mu)R_i^{dc} + \alpha_3 R_i^{ic}) \\ &+ \lambda_{4,i+1} ((1 - \mu - \alpha_1 - \alpha_4)R_i^c + \sigma_1 \beta_1 \frac{P_i R_i^{ic}}{N} + \beta_2 (1 - \sigma_2) \frac{P_i R_i^{dc}}{N} + \alpha_2 R_i^{nc} + u_i P_i + v_i R_i^{nc}) \\ &+ \lambda_{5,i+1} ((1 - \mu - \alpha_2 - v_i)R_i^{nc} + \beta_1 (1 - \sigma_1) \frac{P_i R_i^{ic}}{N} + \sigma_2 \beta_2 \frac{P_i R_i^{dc}}{N} + \alpha_1 R_i^c) \end{aligned}$$

Using Pontryagin's maximum principle [11] and setting P_i^* , R_i^{ic*} , R_i^{dc*} , R_i^{c*} , R_i^{nc*} and (u_i^*, v_i^*) , we obtain the following adjoint equations:

$$\begin{aligned}\lambda_{1,i} &= \frac{\partial H_i}{\partial P_i} \\ \lambda_{1,i} &= \lambda_{1,i+1} \left[(1 - \mu) - \beta_1 \frac{R_i^{ic}}{N} - \beta_2 \frac{R_i^{dc}}{N} - u_i \right] + \lambda_{4,i+1} \left[\sigma_1 \beta_1 \frac{R_i^{ic}}{N} + (1 - \sigma_2) \beta_2 \frac{R_i^{dc}}{N} + u_i \right] \\ &\quad + \lambda_{5,i+1} \left[\beta_1 (1 - \sigma_1) \frac{R_i^{ic}}{N} + \sigma_2 \beta_2 \frac{R_i^{dc}}{N} \right] \\ \lambda_{2,i} &= \frac{\partial H_i}{\partial R_i^{ic}} \\ \lambda_{2,i} &= -\lambda_{1,i+1} \beta_1 \frac{P_i}{N} + \lambda_{2,i+1} (1 - \mu - \alpha_3) + \lambda_{3,i+1} \alpha_3 + \lambda_{4,i+1} \sigma_1 \beta_1 \frac{P_i}{N} + \lambda_{5,i+1} \beta_1 (1 - \sigma_1) \frac{P_i}{N} \\ \lambda_{3,i} &= \frac{\partial H_i}{\partial R_i^{dc}} \\ \lambda_{3,i} &= -\lambda_{1,i+1} \beta_2 \frac{P_i}{N} + \lambda_{3,i+1} (1 - \mu) + \lambda_{4,i+1} (1 - \sigma_2) \beta_2 \frac{P_i}{N} + \lambda_{5,i+1} \sigma_2 \beta_2 \frac{P_i}{N} \\ \lambda_{4,i} &= \frac{\partial H_i}{\partial R_i^c} \\ \lambda_{4,i} &= -B + \lambda_{2,i+1} \alpha_4 + \lambda_{4,i+1} (1 - \mu - \alpha_1 - \alpha_4) + \lambda_{5,i+1} \alpha_1 \\ \lambda_{5,i} &= \frac{\partial H_i}{\partial R_i^{nc}} \\ \lambda_{5,i} &= A + \lambda_{4,i+1} (\alpha_2 + v_i) + \lambda_{5,i+1} (1 - \mu - \alpha_2 - v_i)\end{aligned}$$

with transversality conditions

$$\lambda_{1,T} = \lambda_{2,T} = \lambda_{3,T} = 0, \lambda_{4,T} = -B \text{ and } \lambda_{5,T} = A.$$

To obtain the optimality conditions, we take the variation with respect to control (u_i^*, v_i^*) and set it equal to zero

$$\begin{aligned}\frac{\partial H_i}{\partial u_i} &= M_1 u_i - \lambda_{1,i+1} P_i + \lambda_{4,i+1} P_i = 0 \\ \frac{\partial H_i}{\partial v_i} &= M_2 v_i + \lambda_{4,i+1} R_i^{nc} - \lambda_{5,i+1} R_i^{nc} = 0\end{aligned}$$

Then we obtain the optimal controls:

$$(10) \quad \begin{aligned}u_i &= \frac{1}{M_1} (\lambda_{1,i+1} - \lambda_{4,i+1}) P_i \\ v_i &= \frac{1}{M_2} (\lambda_{5,i+1} - \lambda_{4,i+1}) R_i^{nc}\end{aligned}$$

By the bounds in U_{ad} of the controls, it is easy to obtain u_k^* and v_k^* in the form of (9). \square

4. SIMULATION

In this formulation, there were initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with

separated boundary conditions at time steps $k=0$ and $k=T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization. We continue until convergence of successive iterates is achieved.

4.1. Discussion. In this section, we study and analyse numerically the effects of optimal control strategies such as the efforts to raise religious awareness, early education on religion, advocating the conformity of behavior with the religious ideas for the potential individuals who may become religious and the efforts made through intellectual seminars and deep religious debates on the part of great scholars and thinkers for the individuals who renounce religion.

The numerical solution of model (1) is executed using Matlab with the following parameter values and initial values of state variable to illustrate our results. By choosing $\Lambda = 1 \times 10^3$, $P_0 = 5.10^3$, $R_0^{ic} = 2.10^3$, $R_0^{dc} = 1.10^3$, $R_0^c = 2.10^3$, $R_0^{nc} = 5.10^2$, $\alpha_1 = 0.05$, $\alpha_2 = 0.7$, $\alpha_3 = 0.1$, $\alpha_4 = 0.05$, $\beta_1 = 0.7$, $\beta_2 = 0.6$, $\sigma_1 = 0.5$, $\sigma_2 = 0.5$, $\mu = 0.04$,

4.1.1. Strategy A: *The control that represents the efforts to raise religious awareness.* Given the importance of the efforts to raise religious awareness, early education on religion, advocating the conformity of behavior with the religious ideas, we propose an optimal strategy for this purpose. Hence, we activate the optimal control variable u which represents the efforts to raise religious awareness, early education on religion, advocating the conformity of behavior with the religious ideas for potential religious individuals. The figure (Fig 2) compares the evolution of individuals who are convinced of the religious ideas with and without control u in which the effect of the proposed efforts to raise religious awareness is proven to be positive in increasing the number of religious individuals who are convinced of the religious ideas and practise religious rites.

4.1.2. Strategy B: *The control that represents the efforts made on deep intellectual seminars.* When the number of the individuals who renounce religion and do not practise religious rites is so high, it is obligatory to resort to some strategies such as the efforts made through deep intellectual seminars in order to reduce the number of individuals who renounce religion. Therefore,

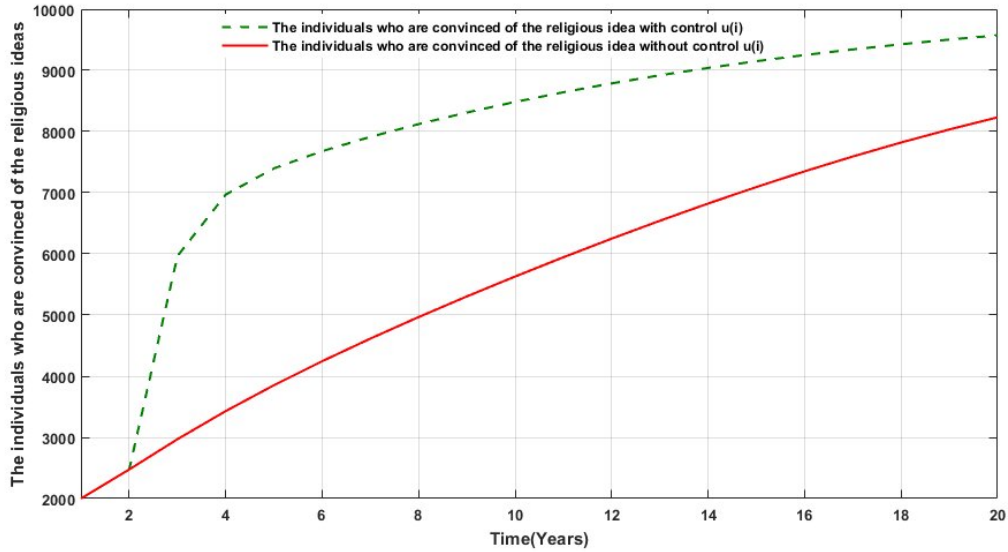


FIGURE 2. The evolution of the individuals who are convinced of the religious ideas with and without controls $u(i)$.

we propose an optimal strategy by using the optimal control v in the beginning. In spite of using the optimal control v , we observe a temporary decrease of the individuals who renounce religion (Figure3),(Figure4). The reason for this decrease is explained and justified by the influence of specialized scholars and thinkers on individuals who renounce religion. On the other hand, the impact of this strategy remains limited compared to the role of religious NGOs, unofficial preachers and missionaries, and civil society groups interested in religious issues.

4.1.3. Strategy C: *The control that represents the efforts to raise religious awareness and the efforts made on deep intellectual seminars.* In this strategy, we combine the two previous strategies to achieve better results. We notice that the number of potential religious individuals (Figure5) who renounce religion (Figure6) are decreased markedly and the number of religious individuals who are convinced of the religious ideas is increased significantly which leads to satisfactory results.

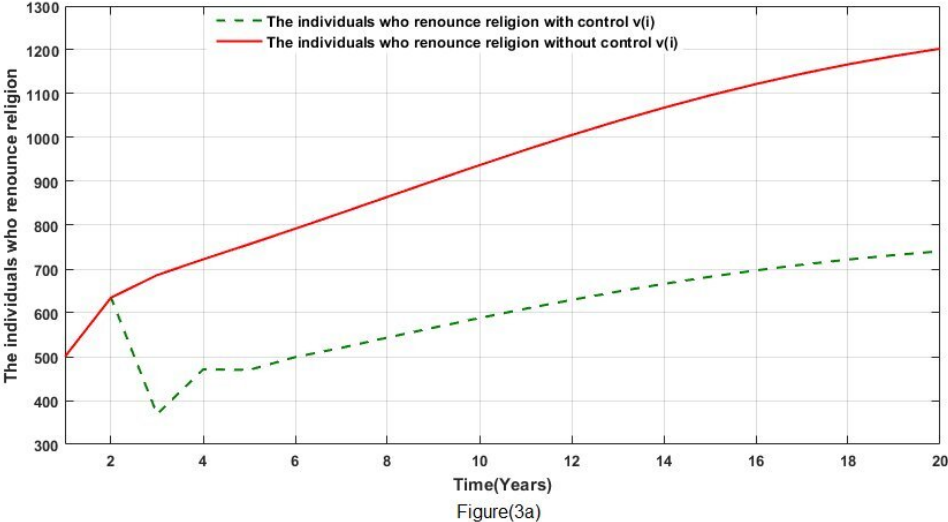


FIGURE 3. The evolution of the individuals who renounce religious ideas with and without controls $v(i)$.

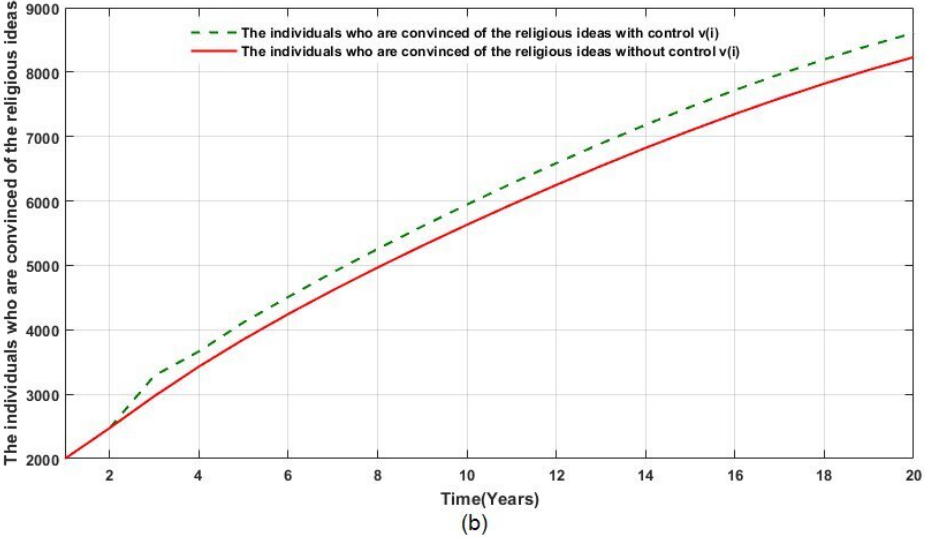


FIGURE 4. The evolution of the individuals who are convinced of the religious ideas with and without controls $v(i)$.

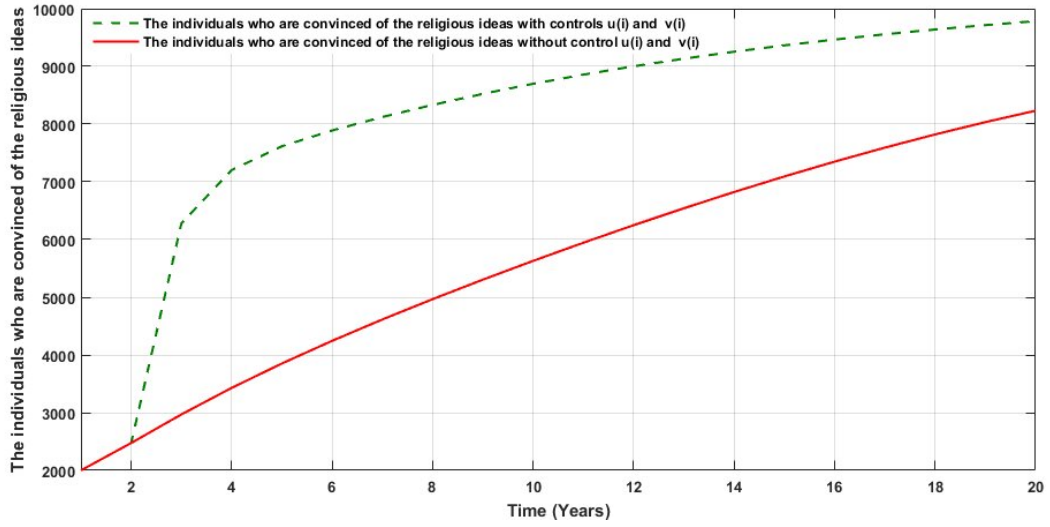


FIGURE 5. The evolution of the individuals who renounce the religious ideas with and without controls $u(i)$ and $v(i)$.

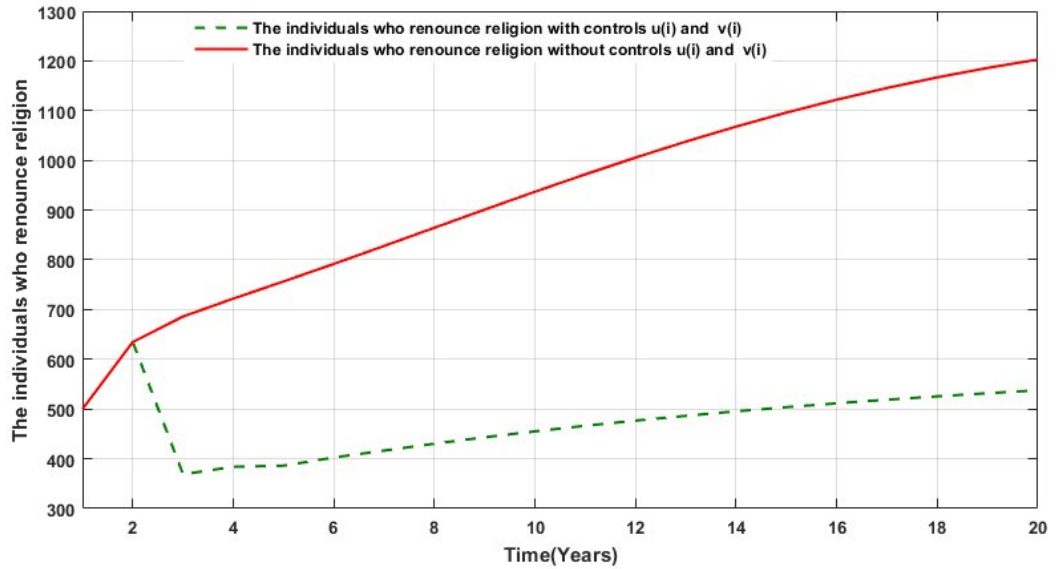


FIGURE 6. The evolution of the individuals who are convinced of the religious ideas with and without controls $u(i)$ and $v(i)$.

5. CONCLUSION

In this paper, we introduced a discrete modeling of the propagation of religious ideas in order to maximize the number of religious individuals who are convinced of the religious ideas and practise religion and minimize the individuals who renounce religion and do not practise religious rites. We also introduced two controls which respectively represent the efforts to raise religious awareness, early education on religion, advocating the conformity of behavior with the religious ideas and the efforts made on intellectual seminars and deep religious debates on the part of great scholars and thinkers. We applied the results of the control theory and we managed to obtain the characterisations of the optimal controls. The numerical simulation of the obtained results proved the effectiveness of the proposed control strategies.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] A. Kouidere, O. Balatif, H. Ferjouchia, A. Boutayeb, M. Rachik, Optimal Control Strategy for a Discrete Time to the Dynamics of a Population of Diabetics with Highlighting the Impact of Living Environment, *Discrete Dyn. Nat. Soc.* 2019 (2019), Article ID 6342169.
- [2] A. Labzai, O. Balatif, M. Rachik, Optimal Control Strategy for a Discrete Time Smoking Model with Specific Saturated Incidence Rate, *Discrete Dyn. Nat. Soc.* 2018 (2018), Article ID 5949303.
- [3] O. Balatif, A. Labzai, M. Rachik, A Discrete Mathematical Modeling and Optimal Control of the Electoral Behavior with regard to a Political Party, *Discrete Dyn. Nat. Soc.* 2018 (2018), Article ID 9649014.
- [4] V. Guibout and A. M. Bloch. A discrete maximum principle for solving optimal control problems. In 43rd IEEE Conference on Decision and Control, volume 2, pages 1806-1811 Vol.2, 2004.
- [5] C.L. Hwang, L.T. Fan, A Discrete Version of Pontryagin's Maximum Principle, *Oper. Res.* 15 (1) 1967, 139-146.

- [6] R.L. Montgomery, The Spread of Religions and Macrosocial Relations, *Soc. Anal.* 52 (1) (1991), 37-53.
- [7] P. Vermeer, J. Janssen, J. Hart, Religious Socialization and Church Attendance in the Netherlands from 1983 to 2007: a Panel Study. *Soc. Compass*, 58 (3) (2011), 373-392.
- [8] P. Vermeer, Religion and Family Life: An Overview of Current Research and Suggestions for Future Research. *Religions*, 5 (2014), 402–421.
- [9] <https://www.pewforum.org/2019/01/31/religions-relationship-to-happiness-civic-engagement-and-health-around-the-world>. 2019.
- [10] <https://www.pewresearch.org/fact-tank/2019/01/31/are-religious-people-happier-healthier-our-new-global-study-explores-this-question>. 2019.
- [11] L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Wiley, New York, 1962.
- [12] M.D. Rafal, W.F. Stevens, Discrete dynamic optimization applied to on-line optimal control. *AIChE J.* 14 (1) (1968), 85-91.
- [13] C.D. Rowe, An epidemic model of adolescent cigarette smoking, *J. Appl. Soc. Psychol.* 22 (4) (1992), 261–285.
- [14] D. Wandt, R. Hendon, B. Cathey, E. Lancaster, and R. Germick, Discrete time optimal control applied to pest control problems; *Involve*, 7 (4) (2014), 479-489. .
- [15] <http://www.globalreligiousfutures.org>.
- [16] D. Zhang, B. Shi, Oscillation and global asymptotic stability in a discrete epidemic model, *J. Math. Anal. Appl.* 278 (2003), 194-202.