Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2020, 2020:86
https://doi.org/10.28919/cmbn/5041
ISSN: 2052-2541

CONTROL OF INFORMATION DISSEMINATION IN ONLINE ENVIRONMENTS: OPTIMAL FEEDBACK CONTROL

ZINEB RACHIK[1], HAMZA BOUTAYEB[2], SARA BIDAH[2], OMAR ZAKARY[2,*], MOSTAFA RACHIK[2]

[1]System and Networks Team (ASYR) - Laboratory of Systems Engineering (LaGeS) Hassania School of Public Works EHTP Casablanca, Morocco

[2]Laboratory of Analysis, Modeling and Simulation (LAMS), Department of Mathematics and Computer Science, Hassan II University of Casablanca, Faculty of Sciences Ben M'Sik, Sidi Othman, BP 7955, Casablanca, Morocco

Abstract. In this article, we consider a discrete-time model describing the dissemination of information between members of any kind of online structure, it can be shared word-to-mouth via video calls and/ or chat or shares in online environments such as Facebook, WhatsApp and Twitter. The impact of information sharing is becoming more and more noticeable in the world, its potential has become clearer through controlling people's behavior and influencing their opinions. Thus, depending on the speed of the latter's spread and its impact, the reaction and measures taken by the state or authorities concerned varies.

In this direction, we chose to use a feedback control function depending on the number of convinced people who share the information at any time. Therefore, the reaction of the control depends on the relevance of this information. We provide sufficient conditions of the existence of such control. And based on a discrete version of Pontryagin's Maximum Principle we characterize the optimal control. Numerical simulations are carried out to study this scenario before and after the use of our control strategy, accompanied by discussions of results.

Keywords: dissemination; information; online environment; feedback control; optimal control.

2010 AMS Subject Classification: 68U35, 93C41, 94A15, 93B52.

---

*Corresponding author

E-mail address: zakaryma@gmail.com

Received September 21, 2020

1. INTRODUCTION

Nowadays, we can easily find a lot of information on the internet, and sometimes it reaches us without searching for it. Notwithstanding it is very difficult to pinpoint the real things. In some cases, one deems information from close people such as friends, family members, or apparently close friends to be a form of credibility. As a result, they can spread widely in seconds and pose risks to public opinion, which can have unforeseen and irreversible consequences.

Oral communication as a widespread phenomenon remains an essential way for people to share information and interact with each other [1], it has been revealed that about 2 billion people use the Internet every day and its many services. Socially defined networks [2]. It's in a networked community where we live [3]. The era of social media has facilitated the spread of unconfirmed information [4] further it spread rapidly [5]. With the rapid development of social networking sites and the proliferation of social media content, it has become easy to access user messages to a wide audience [6], hence, the understanding of the unconfirmed information spread mechanism in social networks , has become important because of its ability to disseminate information [7, 8]. The social network is the main tunnel of rumors or dissemination of information [9]. Technology has led to the production of information reproduction democracy, the prevalence of misinformation has increased significantly [10].

The purpose of disseminating information varies according to the goals and ideas of its promoters. Sometimes the goal could be economical, as one seeks to increase the demand for a specific product, such as the marketing method, or challenging the company's competitors by distorting facts, and unrealistic fictional story, or a story with a small portion of reality that can lead to a major source [11, 12, 13]. For the crisis for business organizations [14, 15, 16].

Moreover, with more and more people using online social media, politicians can no longer ignore their opinions [17]. Unconfirmed information can have a significant impact on people's lives, distort scientific facts, and influence political opinions [13, 10]. Once posted on social media, it is very difficult to report updates or fixes [18]. This ability to rapidly disseminate information and long-term social media presents unprecedented challenges in ensuring the quality and management of information [6].

People exchange information every day, but due to misunderstanding or forgetfulness, and sometimes to avoid embarrassing situations or to pass an object, a person may share unconfirmed or wrong information, which may lead to negative results. For instance, in January 2004, 14 people in Vietnam were admitted to regional hospitals with acute respiratory diseases. H5N1 avian influenza was detected in samples from 3 of these patients. These rumors could have pushed countries to impose restrictions on trade and travel with negative social and economic consequences for health [19]. Because of the serious consequences, the need to reduce the spread of some information and weaken their potential for damage has become increasingly important [5]. This makes information dissemination one of the most important topics discussed by researchers.

Some information affects human behavior that could be exploited by political activists, funders, companies, to advertise their new products and ideas. The goal of the campaign is to reach as many people as possible before the campaign deadline with the best available resources (such as money, business time, and people). The information has been an essential part of human interaction as long as people have had questions about their social environment, especially when we talk about rumors, a piece of unidentified information which can lead to a lot of gossips [20]. Researchers research information dissemination laws by creating models. The information is similar to the virus in the way it spreads between individuals, so current information dissemination patterns, including rumors, are mainly based on research findings on infectious disease models [21, 5].

Infectious disease modeling is a tool used to study disease spread mechanisms, predict the future course of an epidemic, and evaluate epidemic control strategies [20, 22, 23, 24]. Rumor models, in principle, are similar to biological epidemiological models as being treatable [1]. Epidemiological protocols have become an important raw material for disseminating information in networks. Glaring examples of so-called diffusion protocols [25].

Rumors can be interpreted as injury to the mind. The original version of the rumors was presented by Daley and Kendall (DK). Important models for DK are the Maki-Thompson (MK) model. In the past, these models have been widely used to study the spread of rumors. In the DK model, the closed and homogeneous population is divided into three groups: the ignorant, those who have not heard rumors yet; the spreaders, those who heard the rumors and are willing to spread them; the removed, those who heard the rumors and will not share it [20].

The authors in [26], assumed that there is communication between stiflers (removed), and this contact leads to loss of interest in spreading more rumors [26]. In [27], authors proposed a model for the dissemination of late rumors. The population is divided into three groups: ignorant (I), spreader (S) and stifler (R). While in [28], the authors proposed a specific dynamic model, based on the classic DK model. It is believed that people are eager to spread the word as long as it is new, once they have met other people who are familiar with the rumors, it is no longer interesting to share it. In their work, they assumed that two publishers meet, pass on the information, and it is possible to feel that the other person already knows this information, so they continue to spread the rumor without a bit of motivation [28].

Public opinion in a community or market can be shaped simply by influencing the individual beliefs of its members through rumors, and it can be spread in order to raise awareness, create momentum, slander others, cause panic or distract, affect elections, affect markets. Financial, etc [29, 10]. For example, China experienced social panic and instability for several days, and this was due to the spread of rumors that radiation in Japan could contaminate seawater and then sea salt. The salt buying craze in China is emerging after a rumor that iodized salt could help protect people from nuclear radiation leakage [30].

The transmission of information is often seen as social contagion processes [31, 30]. It is possible to apply dynamic population models to collect information about disseminating ideas and information, thus allowing the testing of social hypotheses [32, 33].

The similarities between the spread of the epidemic and the spread of information allowed the researcher to use epidemiological models to model information dissemination. In this article, we divide the population into three groups, which will make it possible to study the development of ignorant people (people who do not know the information), spreaders (people who are interested in this information, who find pleasure in sharing it), removed (people who see that this information lacks relevance and compatibility with their profiles, then they refuse to share it). In this work, we are using feedback control to control the dissemination of information, which means that the control we propose is the feedback of the number of spreaders. We use a discrete version of Pontryagin's Maximum Principle to describe optimal control, then simulate our results numerically to evaluate the effectiveness of this type of optimal control in reducing the number of spreaders and increasing the number of removals at a cost optimum.

2. Presentation of the Model

Information is easily spread, by all means, word of mouth, emails, phone calls, social networks, etc. With the help of all the advanced technologies that facilitate human communication, information spreads quickly. One of the most important factors in spreading information is the introduction of the "Share" button that accompanies any status update, link, video, or image posted. Content viewers (for example, friends of the creator and subscribers) are allowed to share the post. For example, on Facebook, if the content was originally posted publicly, anyone can view and share it [21]. Based on the ideas published in [21], we devise here a compartmental model to study the dissemination of information in online environments of N users (Facebook, WhatsApp or Tweeter groups or pages) by posting, sharing and discussing. In these online environments, when a user posts information (Text, image, video...), only his neighbors can see it and determine if that information needs to be shared again or not. If the information is so interesting and some neighbors decide to share it, neighbors of the author's neighbors can then see and re-share them again. The influence of the information then exceeded the local scope of the author and can be widely distributed on the network. On the other hand, if none of the original author's neighbors are attracted by this information, it will disappear soon and very few users will see it. When a user shares a post, the information is displayed on its homepage for a long time even if he does not care about it anymore, all his neighbors can always see the information he has shared. At the same time, if the neighbors have seen the post and do not share it immediately, they may lose interest gradually and ignore that information.

If a user notices that some information is repeated and shared by several of his neighbors, then he will discuss it with his friends through the chat tools or face to face, so he can determine the relevance of this information, and then decide to share it or not.

Our model consists of three compartments: Ignorants (I), Sharers or spreaders (S), and Removed people (R). The term "ignorant" means a person that does not know about the information. The word "Sharer" is used to denote that a person is attracted by the information and/or he finds it funny or interesting, then he decides to share it. The term “Removed” means a person who has seen the post and has decided not to share it. For example, because of irrelevance or for other personal reasons. We kept the term Removed from the classical SIR epidemiological model to denote individuals removed from the sharing system. All transmissions are modeled using the mass action principle, which accounts for the probability of transmission in contact between the different compartments.

Each information has the potential of sharing, but one can find some information not useful or does not fit the user interests, and then there is no need to share it. For example, if the information is about a concern of the public opinion (Raising costs of education, election cheats, public safety...), the probability of shares will be very important. Therefore, the potential relevance of the information will be taken into account and it will be defined based on the proportions of shares. Let’s define the potential relevance of the information by the average β_1 , while the potential irrelevance of the information is defined by the average β_2.

We assume that when information is shared, it is relevant. An Ignorant becomes a Sharer just after he shares the information at the rate $\frac{\beta_1 I_i S_i}{N}$. An Ignorant that decides not to share the information becomes Removed at a rate $\frac{\beta_2 I_i S_i}{N}$. A Sharer that contacts a Removed and he decides not to share the information anymore, becomes a Removed at a rate $\gamma\frac{S_i R_i}{N}$. Ignorant, Sharer and Removed individuals can leave the environment at a rate μI_i, μS_i, and μR_i, respectively. We assume that all new members are recruited as Ignorants at the constant rate Λ. All these interactions happen at the instant i.

The model resulting from these assumptions is governed by the following equations

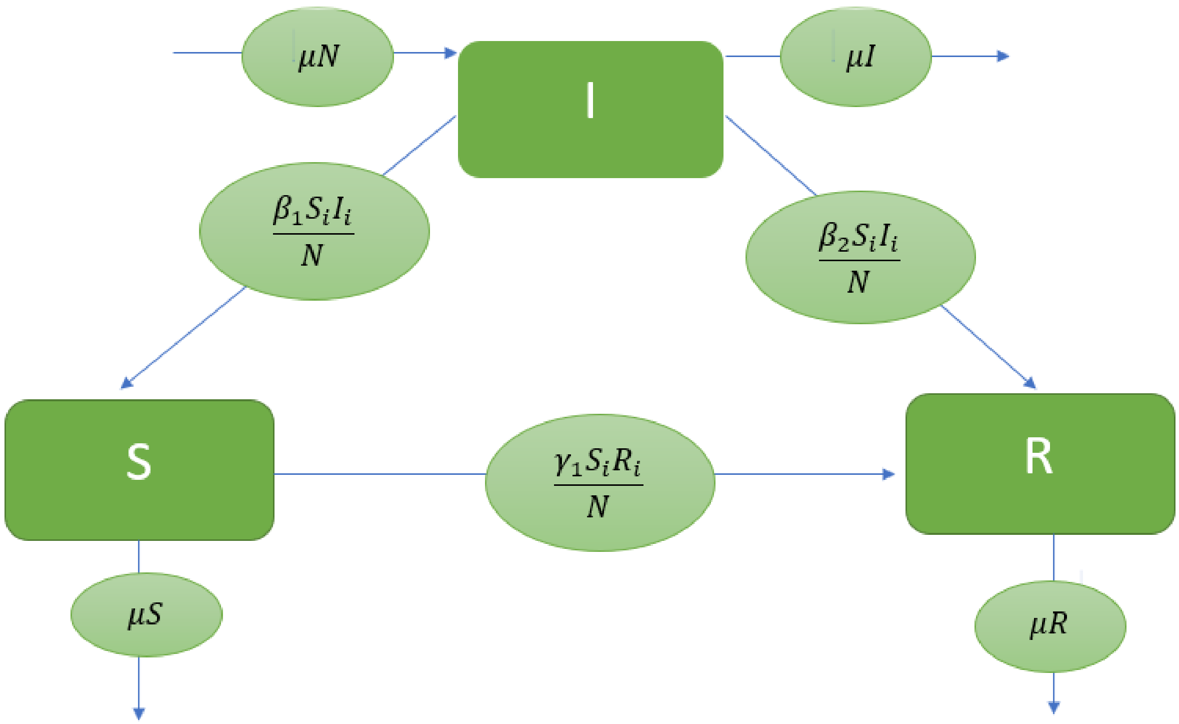
$$I_{i+1} = I_i + \Lambda - \mu I_i - \frac{\beta I_i S_i}{N} \tag{1}$$

$$S_{i+1} = S_i - \mu S_i + \frac{\beta_1 I_i S_i}{N} - \gamma\frac{S_i R_i}{N} \tag{2}$$

$$R_{i+1} = R_i - \mu R_i + \frac{\beta_2 I_i S_i}{N} + \gamma\frac{S_i R_i}{N} \tag{3}$$

Where $S_i > 0, I_i > 0$ and $R_i > 0$ for all i. For simplicity, we have put $\beta_1 + \beta_2 = \beta$, and without loss of generality, we assume that the number of new individuals is equal to the number of outgoing users, that is $\Lambda = \mu N$.

FIGURE 1. Flow chart for the model (1-3)



Parameter	description
Λ	Recruitment rate
β_1	Potential relevance rate of information
β_2	Potential irrelevance rate of information
γ	Sharer to Removed transition rate
μ	Exit rate

TABLE 1. Parameters description

Note that $N = N_i = I_i + S_i + R_i$, in fact

$$N_{i+1} = N_i + \Lambda - \mu N_i = N_i$$

A flow chart for the model is shown in Fig. 1, and parameters description can be found in Table 1.

3. The Optimal Control Problem

3.1. Presentation of the controls.

To eradicate the dissemination of information, some governments prefer to block all communications and can ban social media platforms [34]. But this strategy of control can lead to some protests or upsets. In seeking good interventions, governments began using social media to control the spread of some annoying information, by commenting on false information to correct it, or on real ones to confirm it.

In February 2019, some infections and deaths associated with the H1N1 virus were recorded, making the media the main platform for disseminating this information and leaving Moroccan society in a state of confusion and worry. This prompted the Ministry of Health to intervene on February 5, 2019, to respond to news from the social networks on deaths due to seasonal influenza. By issuing a joint press release with the WHO country office at Morocco on its official Facebook page [35] so that residents do not worry about the spread of the seasonal H1N1 virus [36]. This communication between people and government institutions builds trust, leads to stability and reduces anxiety.

By following this direction in the seek of control strategies, we propose a control strategy using a new feedback optimal control that will be interact in function of the number of spreaders. This control represents the effectiveness of comments and clarifications from official institutions such as government or any credible source. For example, the publication of the press release of the Moroccan Ministry of Health on its official Facebook page [35] and on its official web site [36]. This control can be also documents or some videos released in WhatsApp or on YouTube to aware people and/or to reveal the truth. As in the case of the companies in the Egyptian Market that reacted to rumors by publishing short videos explaining manufacturing processes and distributing documents confirming the quality and safety of their products [37].

The originality of this control is evident in that it depends on the number of participants, as the greater the number of posts, the greater the percentage of control over the publication of the necessary explanations and documents to limit the spread of this information by posts' delete and stopping shares. Where in the situation of rumors, after the truth was revealed, people will not only refrain from sharing that rumor but will also comment on others' posts by pointing to links to the truth, resulting in a lack of re-sharing and deleting posts. One thing that discourages re-sharing is when others comment on it by referring to external sources where the validity of the rumor is discussed. People who spread that rumor may try not to continue sharing it or stay away from these rumors if they know they are wrong [38]. Sometimes, Governments criminalize the re-sharing of some information on social networks due to the seriousness of the situation. In such situations, almost all group administrators delete all publications related to this information and block the publication or sharing of these contents on their pages.

Based on these facts, we introduce the feedback control variable $k_i = u_i S_i$ such that:

$$I_{i+1} = I_i + \Lambda - \mu I_i - \frac{\beta I_i S_i}{N} \tag{4}$$

$$S_{i+1} = S_i - \mu S_i + \frac{\beta_1 I_i S_i}{N} - \gamma \frac{S_i R_i}{N} - k_i S_i \tag{5}$$

$$R_{i+1} = R_i - \mu R_i + \frac{\beta_2 I_i S_i}{N} + \gamma \frac{S_i R_i}{N} + k_i S_i \tag{6}$$

Where $S_i > 0$, $I_i > 0$ and $R_i > 0$ for all i. For simplicity, we put $\beta_1 + \beta_2 = \beta$, $\Lambda = \mu N$.

3.2. Objective functional. The main objective of this study is to use a variable feedback control function, depending on output of the system. We use optimal control strategy to reduce the number of Sharers and increase the number of Removeds, and that with optimal costs of applying the control. Then, the problem is to minimize the objective functional given by

$$\begin{aligned} J(u,v) &= (\alpha_S S_{\mathcal{N}} - \alpha_R R_{\mathcal{N}}) \\ &+ \sum_{i=0}^{\mathcal{N}-1} \left(\alpha_S S_i - \alpha_R R_i + \frac{A}{2}(u_i)^2 \right) \end{aligned} \tag{7}$$

Where $A > 0$, $\alpha_S > 0$, $\alpha_R > 0$ are the weight constants of control, the sharers and removed, respectively, $u = (u_0, \ldots, u_{\mathcal{N}-1})$, and $\mathcal{N}$ is the final time of our strategy of control.

Our goal is to minimize Sharers, minimize the cost of applying controls and increase the number of Removed. In other words, we are seeking an optimal control u^* such that

$$J(u^*) = min\{J(u)/u \in \mathcal{U}\}$$

where $\mathcal{U}$ is the control set defined by

$$\mathcal{U} = \{u / u_{min} \leq u_i \leq u_{max}, i = 0, \ldots, \mathcal{N} - 1\} \tag{8}$$

such that

$$0 < u_{min} < u_{max} < 1$$

3.3. Sufficient conditions.

Theorem 1. *There exists an optimal control $u^* \in \mathscr{U}$ such that*

$$J(u^*) = min\{J(u)/u \in \mathscr{U}\}$$

subject to the control system (4)-(6) and initial conditions.

Proof. Since the parameters of the system are bounded and there are a finite number of time steps, that is I, S, and R are uniformly bounded for all u in the control set $\mathscr{U}$, thus $J(u)$ is also bounded for all $u \in \mathscr{U}$. Which implies that $\inf_{u\in\mathscr{U}} J(u)$ is finite, and there exists a sequence $u^n \in \mathscr{U}$ such that

$$\lim_{n\to+\infty} J(u^n) = \inf_{u\in\mathscr{U}} J(u)$$

and corresponding sequences of states I^n, S^n and R^n.

Since there is a finite number of uniformly bounded sequences, there exists $u^* \in \mathscr{U}$ and I^*, S^* and R^* such that, on a sequence,

$$u^n \to u^*$$

$$I^n \to I^*$$

$$S^n \to S^*$$

$$R^n \to R^*$$

Finally, due to the finite dimensional structure of the system (4)-(6) and the objective function $J(u)$, u^* is an optimal control with corresponding states I^*, S^*, and R^*. Which complete the proof. □

3.4. Necessary conditions.

By using a discrete version of the Pontryagin's maximum principle [39, 40, 41, 42], we derive necessary conditions for our optimal controls. For this purpose, we define the Hamiltonian as:

$$
\begin{aligned}
\mathscr{H}(i) &= \alpha_S S_i - \alpha_R R_i + \frac{A}{2}(u_i)^2 \\
&+ \zeta_{1,i+1}\left[I_i + \Lambda - \mu I_i - \frac{\beta I_i S_i}{N}\right] \\
&+ \zeta_{2,i+1}\left[S_i - \mu S_i + \frac{\beta_1 I_i S_i}{N} - \gamma\frac{S_i R_i}{N} - u_i S_i^2\right] \\
&+ \zeta_{3,i+1}\left[R_i - \mu R_i + \frac{\beta_2 I_i S_i}{N} + \gamma\frac{S_i R_i}{N} + u_i S_i^2\right]
\end{aligned} \tag{9}
$$

Theorem 2. *Given optimal controls u^* and solutions I^*, S^* and R^*, there exists $\zeta_{k,i}$, $i = 0 \ldots \mathscr{N} - 1$, $k = 1,2,3$, the adjoint variables satisfying the following equations:*

$$
\begin{aligned}
\Delta\zeta_{1,i} &= -\left[\zeta_{2,i+1}\left(\frac{S_i\,\beta_1}{N}\right) + \zeta_{3,i+1}\left(\frac{S_i\,\beta_2}{N}\right)\right. \\
&\quad -\zeta_{1,i+1}\left(\mu + \frac{S_i\,(\beta_1+\beta_2)}{N} - 1\right)\Big]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\Delta\zeta_{2,i} &= -\left[\alpha_S + \zeta_{3,i+1}\left(2u_i S_i + \frac{I_i\,\beta_2}{N} + \frac{R_i\,\gamma}{N}\right)\right. \\
&\quad -\zeta_{2,i+1}\left(\mu + 2S_i u_i - \frac{I_i\,\beta_1}{N} + \frac{R_i\,\gamma}{N} - 1\right) \\
&\quad \left. -\frac{I_i\,\zeta_{1,i+1}\,(\beta_1+\beta_2)}{N}\right]
\end{aligned} \tag{11}
$$

$$
\Delta\zeta_{3,i} = -\left[\zeta_{3,i+1}\left(\frac{S_i\,\gamma}{N} - \mu + 1\right) - \alpha_R - \frac{S_i\,\gamma\,\zeta_{2,i+1}}{N}\right] \tag{12}
$$

where $\zeta_{1,N} = 0$, $\zeta_{2,N} = \alpha_S$, $\zeta_{3,N} = -\alpha_R$ are the transversality conditions. In addition

$$
\begin{aligned}
u_i^* &= min\left\{max\left\{u_{min}, \frac{S_i^2\,(\zeta_{2,i+1} - \zeta_{3,i+1})}{A}\right\}, u_{max}\right\} \\
&\quad , i = 0, \ldots, \mathscr{N} - 1
\end{aligned} \tag{13}
$$

Proof. Using the discrete version of the Pontryagin's maximum principle [39, 40], we obtain the following adjoint equations:

$$\begin{aligned}
\Delta\zeta_{1,i} &= -\frac{\partial \mathscr{H}}{\partial I_i} \\
&= -\Big[\zeta_{2,i+1}\left(\frac{S_i\,\beta_1}{N}\right)+\zeta_{3,i+1}\left(\frac{S_i\,\beta_2}{N}\right) \\
&\quad -\zeta_{1,i+1}\left(\mu+\frac{S_i\,(\beta_1+\beta_2)}{N}-1\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\Delta\zeta_{2,i} &= -\frac{\partial \mathscr{H}}{\partial S_i} \\
&= -\Big[\alpha_S+\zeta_{3,i+1}\left(2u_iS_i+\frac{I_i\,\beta_2}{N}+\frac{R_i\,\gamma}{N}\right) \\
&\quad -\zeta_{2,i+1}\left(\mu+2S_iu_i-\frac{I_i\,\beta_1}{N}+\frac{R_i\,\gamma}{N}-1\right) \\
&\quad -\frac{I_i\,\zeta_{1,i+1}\,(\beta_1+\beta_2)}{N}\Big] \\
\Delta\zeta_{3,i} &= -\frac{\partial \mathscr{H}}{\partial R_i} \\
&= -\left[\zeta_{3,i+1}\left(\frac{S_i\,\gamma}{N}-\mu+1\right)-\alpha_R-\frac{S_i\,\gamma\,\zeta_{2,i+1}}{N}\right]
\end{aligned}$$

With $\zeta_{1,N}=0$, $\zeta_{2,N}=\alpha_S$, $\zeta_{3,N}=-\alpha_R$. To obtain the optimality conditions we take the variation with respect to controls (u_i and v_i) and set it equal to zero

$$\frac{\partial \mathscr{H}}{\partial u_i}=A\,u_i-S_i^2\,\zeta_{2,i+1}+S_i^2\,\zeta_{3,i+1}=0$$

Then we obtain the optimal control pair

$$u_i=\frac{S_i^2\,(\zeta_{2,i+1}-\zeta_{3,i+1})}{A}$$

By the bounds in $\mathscr{U}$ of the controls in the definitions (8), it is easy to obtain u_i^* in the following form

$$\begin{aligned}
u_i^* &= min\left\{max\left\{u_{min},\frac{S_i^2\,(\zeta_{2,i+1}-\zeta_{3,i+1})}{A}\right\},u_{max}\right\} \\
&\quad ,\, i=0,\ldots,\mathscr{N}-1
\end{aligned}$$

□

I_0	S_0	R_0	Λ	β_1	β_2	γ	μ
1000	10	10	μN	0.081	0.0310	0.002	2.6×10^{-4}

TABLE 2. Parameters values utilized for the resolution of the discrete systems (1-3) and (4-6), and then leading to simulations obtained from Figure 2 to Figure 4, with the initial conditions I_0, S_0, R_0.

4. NUMERICAL SIMULATIONS

We now present numerical simulations associated with the above-mentioned optimal control problem. We write code in MATLAB^{TM} and simulated our results using data from Table 2. The optimality systems are solved based on an iterative discrete scheme that converges following an appropriate test similar to the one related to the Forward-Backward Sweep Method (FBSM). The state system with an initial guess is solved forward in time and then the adjoint system is solved backward in time because of the transversality conditions. Afterward, we update the optimal control values using the values of state and co-state variables obtained at the previous steps. Finally, we execute the previous steps until a tolerance criterion is reached.

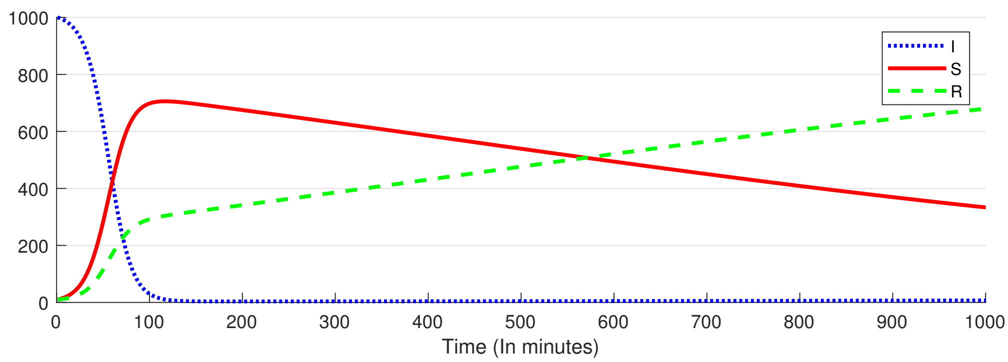
In all the simulations bellow, the minutes were used as a time unit. Because the spread of information occurs faster in time. We focus here on information that is more appealing and has the potential to be shared.

Without loss of generality, and as an example, we have chosen as a studied population, a group (In Facebook, Tweeter, WhatsApp ...) with 1000 members, that can be considered as the ignorant group.

At the initial time $i=0$, ten sharers, and ten removeds are introduced into this group with information which can be a posted video or image or text.

All parameters of the table 2 are chosen to get a situation in which the number of sharers rises above 700 individuals of the population and the removed group remains small. In this situation, it can be shown that our proposed strategy of optimal control is very efficient to reduce the number of sharers and thus the amount of the information while it increases the number of the removed population and that with an optimal cost.

FIGURE 2. Dynamic of the system (1-3) without the control: Ignorants (I), Sharers (S) and Removeds (R).



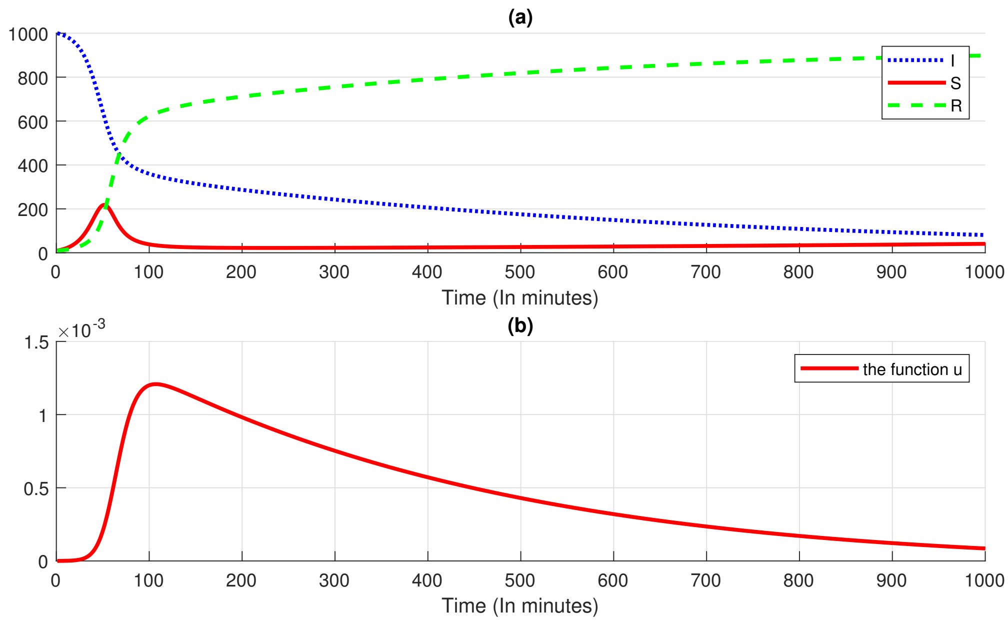
In Fig.2 it can be seen that about 100 minutes from the injection of the information, there is no more ignorant. Which means that the information reaches almost all the members of the group. We talk then about an explosion of the information. In the case of false information, this situation can lead to serious economic and/or political damages. Because it can be seen from this figure that the more the number of sharers is big the more of the amount of the information is huge. Thus, the proliferation of information can not be stopped, consequently, it can spread out to external groups and reaches other spreaders in other places.

The reason for low participants is that the information has already reached all members of the group and may have reached other groups resulting in a loss of interest in sharing this information.

Fig. 3 shows the dynamic of I, S and R in the model (4-6) with the control function $k_i = u_i S_i$. Where it can be seen that the number of sharers remains very small, while the number of the removed people rise to about 100 % of the members. Making comments and explanations on social networks could be another way to ensure people's safety in an emergency. More people reading official comments, namely correct information or interpretations, will probably deal with the subject rationally. For example, after the posting of explanatory videos by the companies involved in the Egyptian market [37], people stopped sharing this information. As a result, these companies have succeeded in gaining people confidence again.

To achieve these optimal results by following our control strategy, we suggest using it in the first 24 hours of information appearing to bring forward the peak of sharers. When those

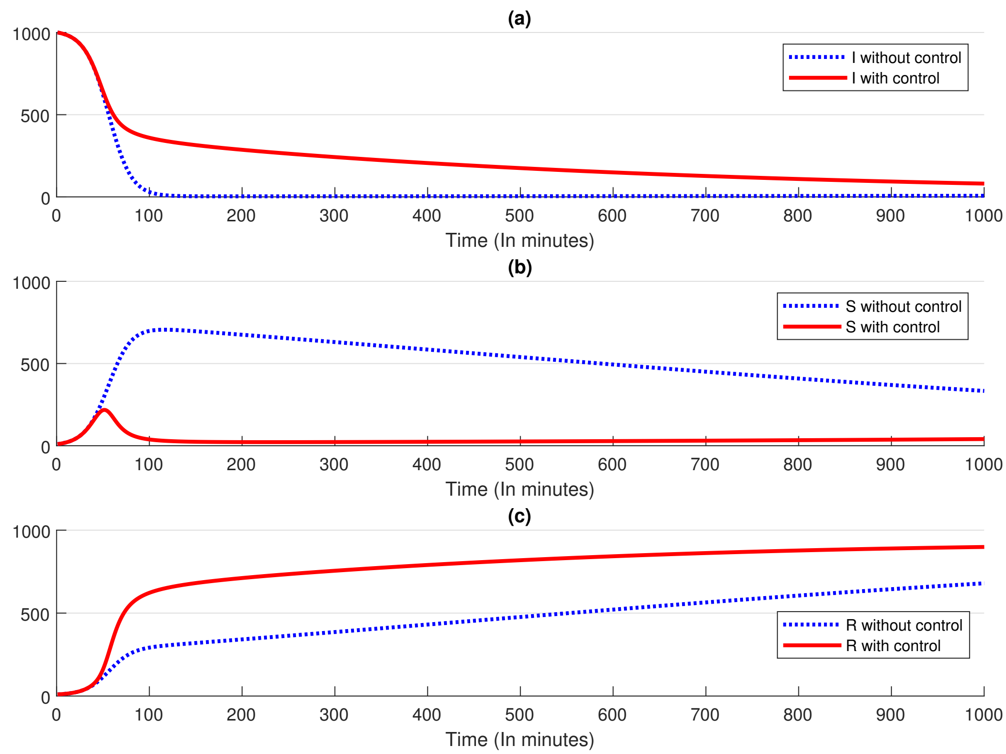
FIGURE 3. Dynamic of the system (4-6) without the control: (a) Ignorants (I), Sharers (S) and Removeds (R). (b) The control function u.



concerned do not provide more explanatory information, people can be left feeling that there is something wrong, which leads to a lot of gossips. In the case of government rumors, some information can build trust and strengthen social stability.

Fig.4 shows the comparison of the different states of the proposed model with and without the control function. It can be seen from that figure in (a) that after using the optimal control the number of ignorant people decreases very quickly compared to the case when there is no control. This fact can be explained by the efficiency of the control approach we utilized here in making all members of the group aware. Sub-figure (b) shows the comparison between the number of sharers with and without the control, where it can be seen that when using this control we can bring forward the peak of sharing and avoid the spread of the information. The number of shares does not exceed 200 members when the control is used, compared to the case when there is no control where this number reaches about 700 individuals to begin decreasing slowly up to the end of the simulation with a value exceeds 400 members. In the sub-figure (c) we can see the comparison between the number of removed with and without the use of the optimal control, where it can be seen that when the control is utilized the number of removed rises quickly by

FIGURE 4. Comparison between the dynamic of the system (1-3) and the system (4-6) with control: (a) Ignorants, (b) Sharers, and (c) Removeds.



about the first 50 minutes to exceed 600 individuals to continue growing slightly, compared to the case when there is no control it grows slightly from the beginning of the simulation.

From these figures, we can see the efficiency of the optimal control that we propose in this paper, in reducing the number of shares and increase the number of removed individuals with an optimal cost.

5. Conclusion

In this article, we considered a discrete-time model describing the dissemination of information between members of any kind of online structure, it can be shared word-to-mouth via video calls and/ or chat or shares in online environments such as Facebook, WhatsApp and Twitter. We have chosen to use a feedback control function depending on the number of convinced people who share the information at any time. Which means that the reaction of the control depends on the relevance of this information. We provided sufficient conditions of the existence of the optimal control, then we used a discrete version of Pontryagin's Maximum Principle to characterize this optimal control. Numerical simulations are carried out to study this scenario before and after the use of our control strategy, accompanied by discussions of results.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] K. Kandhway, J. Kuri, Optimal control of information epidemics modeled as maki thompson rumors, Commun. Nonlinear Sci. Numer. Simul. 19 (12) (2014), 4135–4147.

[2] N. Fountoulakis, K. Panagiotou, T. Sauerwald, Ultra-fast rumor spreading in social networks, in: Proceedings of the twenty-third annual ACM-SIAM symposium on Discrete Algorithms, SIAM, 2012, pp. 1642–1660.

[3] R. Paluch, X. Lu, K. Suchecki, B. K. Szymański, J. A. Hołyst, Fast and accurate detection of spread source in large complex networks, Sci. Rep. 8 (2018), 2508.

[4] S. Kwon, M. Cha, K. Jung, Rumor detection over varying time windows, PloS One. 12 (2017), e0168344.

[5] L. Zhao, J. Wang, R. Huang, Immunization against the spread of rumors in homogenous networks, PloS One. 10 (2015), e0124978.

[6] Y. Liu, S. Xu, Detecting rumors through modeling information propagation networks in a social media environment, IEEE Trans. Comput. Soc. Syst. 3 (2) (2016) 46–62.

[7] F. Chierichetti, S. Lattanzi, A. Panconesi, Rumor spreading in social networks, Theor. Comput. Sci. 412 (24) (2011), 2602–2610.

[8] D. H.-L. Goh, A. Y. Chua, H. Shi, W. Wei, H. Wang, E. P. Lim, An analysis of rumor and counter-rumor messages in social media, in: International Conference on Asian Digital Libraries, Springer, 2017, pp. 256–266.

[9] M. Tang, X. Mao, Z. Guessoum, H. Zhou, Rumor diffusion in an interests-based dynamic social network, Sci. World J. 2013 (2013), 824505.

[10] C. Wang, Z. X. Tan, Y. Ye, L. Wang, K. H. Cheong, N.-g. Xie, A rumor spreading model based on information entropy, Sci. Rep. 7 (2017), 9615.

[11] S. Bidah, O. Zakary, M. Rachik, H. Ferjouchia, Mathematical modeling of public opinions: Parameter estimation, sensitivity analysis, and model uncertainty using an agree-disagree opinion model, Abstr. Appl. Anal. 2020 (2020), 1837364.

[12] S. Bidah, O. Zakary, M. Rachik, Stability and global sensitivity analysis for an agree-disagree model: Partial rank correlation coefficient and latin hypercube sampling methods, Int. J. Differ. Equ. 2020 (2020), 5051248.

[13] S. Bidah, O. Zakary, M. Rachik, Modeling and control of the public opinion: An agree-disagree opinion model, Int. J. Differ. Equ. in press.

[14] W. T. Coombs, S. J. Holladay, Helping crisis managers protect reputational assets: Initial tests of the situational crisis communication theory, Manage. Commun. Quart. 16 (2) (2002), 165–186.

[15] G. Chen, New Challenges from Popular Politics: NGOs, Commercial Organizations, Social Media, and Civic Society, in: The Politics of Disaster Management in China, Palgrave Macmillan US, New York, 2016: pp. 93?108.

[16] A. Zubiaga, M. Liakata, R. Procter, K. Bontcheva, P. Tolmie, Towards detecting rumours in social media, in: Workshops at the Twenty-Ninth AAAI Conference on Artificial Intelligence, 2015.

[17] M. Takayasu, K. Sato, Y. Sano, K. Yamada, W. Miura, H. Takayasu, Rumor diffusion and convergence during the 3.11 earthquake: a twitter case study, PLoS One. 10 (4) (2015), e0121443.

[18] N. M. Jones, R. R. Thompson, C. D. Schetter, R. C. Silver, Distress and rumor exposure on social media during a campus lockdown, Proc. Nat. Acad. Sci. 114 (44) (2017), 11663–11668.

[19] G. Samaan, M. Patel, B. Olowokure, M.C. Roces, H. Oshitani, the World Health Organization Outbreak Response Team, Rumor Surveillance and Avian Influenza H5N1, Emerg. Infect. Dis. 11 (2005), 463?466.

[20] F. Roshani, Y. Naimi, Effects of degree-biased transmission rate and nonlinear infectivity on rumor spreading in complex social networks, Phys. Rev. E. 85 (3) (2012), 036109.

[21] H. Boutayeb, S. Bidah, O. Zakary, M. Rachik, A new simple epidemic discrete-time model describing the dissemination of information with optimal control strategy, Discr. Dyn. Nat. Soc. 2020 (2020), 7465761.

[22] O. Zakary, S. Bidah, M. Rachik, The impact of staying at home on controlling the spread of covid-19: Strategy of control, Mexican J. Biomed. Eng. 42 (2020), 10–26.

[23] O. Zakary, S. Bidah, M. Rachik, H. Ferjouchia, Mathematical model to estimate and predict the covid-19 infections in morocco: Optimal control strategy, J. Appl. Math. 2020 (2020), 9813926.

[24] B. Sara, Z. Omar, T. Abdessamad, R. Mostafa, F. Hanane, Parameters' estimation, sensitivity analysis and model uncertainty for an influenza a mathematical model: case of morocco, Commun. Math. Biol. Neurosci. 2020 (2020), Article ID 57.

[25] Y. Rabani, Proceedings of the twenty-third annual ACM-SIAM symposium on Discrete Algorithms, SIAM, 2012.

[26] Y. Moreno, M. Nekovee, A. F. Pacheco, Dynamics of rumor spreading in complex networks, Phys. Rev. E. 69 (6) (2004), 066130.

[27] H. Laarabi, A. Abta, M. Rachik, J. Bouyaghroumni, Stability analysis of a delayed rumor propagation model, Differ. Equ. Dyn. Syst. 24 (4) (2016), 407–415.

[28] L. Huo, C. Ma, Optimal control of rumor spreading model with consideration of psychological factors and time delay, Discrete Dynamics in Nature and Society 2018 (2018), 9314907.

[29] L. Zhao, J. Wang, Y. Chen, Q. Wang, J. Cheng, H. Cui, Sihr rumor spreading model in social networks, Physica A. Stat. Mech. Appl. 391 (7) (2012), 2444–2453.

[30] T. Lin, C. Fan, C. Liu, J. Zhao, et al., Optimal control of a rumor propagation model with latent period in emergency event, Adv. Differ. Equ. 2015 (2015), 54.

[31] K. Kawachi, Deterministic models for rumor transmission, Nonlinear Anal., Real World Appl. 9 (5) (2008), 1989–2028.

[32] L. M. Bettencourt, A. Cintrón-Arias, D. I. Kaiser, C. Castillo-Chávez, The power of a good idea: Quantitative modeling of the spread of ideas from epidemiological models, Physica A. Stat. Mech. Appl. 364 (2006), 513–536.

[33] C.-Y. Sang, S.-G. Liao, Modeling and simulation of information dissemination model considering user's awareness behavior in mobile social networks, Physica A. Stat. Mech. Appl. 537 (2020), 122639.

[34] F. Jin, E. Dougherty, P. Saraf, Y. Cao, N. Ramakrishnan, Epidemiological modeling of news and rumors on twitter, in: Proceedings of the 7th Workshop on Social Network Mining and Analysis, ACM, 2013, p. 8.

[35] M. of Health, Ministere de la sante du royaume du maroc page officielle, https://www.facebook.com/ministere.sante.ma/.

[36] W. c. o. i. M. Ministry of Health, La grippe saisonniere actuelle au maroc : aucune raison de s'inquieter, https://www.sante.gov.ma/Pages/Communiques.aspx?IDCom=316 (2019).

[37] A. A. Abdelkader, H. E. D. F. Mohamed, The impact of rumors on social media sites in behavior of purchasing food products in the egyptian market, Int. J. Customer Relation. Market. Manage. 9 (2018), 19–35.

[38] A. Friggeri, L.A. Adamic, D. Eckles, J. Cheng. Rumor cascades. In: Proceedings of the 8th international conference on Weblogs and Social Media, Ann Arbor, Michigan, USA, pp 101–110. (2014).

[39] L. Pontryagin, V. Boltyansky, R. Gamkrelidze, E. Mischenko, The mathematical theory of optimal processes. intern. publ, Inc., NY. (1962).

[40] S. P. Sethi, What is optimal control theory? in: Optimal Control Theory, Springer, 2019, pp. 1–26.

[41] O. Zakary, M. Rachik, I. Elmouki, On the analysis of a multi-regions discrete sir epidemic model: an optimal control approach, Int. J. Dyn. Control. 5 (3) (2017), 917–930.

[42] O. Zakary, M. Rachik, I. Elmouki, A new epidemic modeling approach: Multi-regions discrete-time model with travel-blocking vicinity optimal control strategy, Infect. Dis. Model. 2 (2017), 304–322.