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MATHEMATICAL MODELLING OF MOTORCYCLE SALE COMPETITION IN INDONESIA

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Abstract. Motorbike is a means of transportation that is widely used in Indonesia and other developing countries. In this paper, we apply the Lotka-Volterra competition mathematical model to describe the dynamics of motorcycle sale competition between two motorcycle producers in Indonesia. We also estimate the parameter values of the model using the particle swarm optimization method. We found that the mathematical model quite well to explain the annual motorcycle sales of both companies. The results of the model analysis show that the competition between the two motorcycle producers is pure competition, with the first producer dominates market share of motorcycle selling in Indonesia.

Keywords: motorcycle sale competition; mathematical model; Lotka-Volterra; parameter estimation; particle swarm optimization.

2010 AMS Subject Classification: 37N40.

1. INTRODUCTION

The population increase causes an increase in the need for transportation facilities. People need transportation to support their mobility. Motorbikes are a transportation mode that is widely used in Indonesia and other developing countries. The use of motorbikes in Indonesia

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is common because of their relatively low price. The motorcycle also has the advantage of being able to maneuver between traffic jams. The use of motorbikes also provides efficiency in travel costs. The use of motorbikes can also increase the population's mobility in terms of flexibility and shorten travel time. Furthermore, motorbikes can foster economic activity in the form of goods or goods delivery services. In the last ten years, online transportation services using motorbikes have even begun to develop in Indonesia.

Indonesia is the country with the most purchases of motorbikes in Southeast Asia. Also, Indonesia is the third-largest motor vehicle user in the world after India and China. Referring to Motorcycles data, motorcycle sales in Indonesia in 2019 reached 6.53 million units, an increase of 1.3% from the previous year. Japanese brands dominated motorcycles' sales in Indonesia and controlling more than 98% of Indonesia's motorcycle market [1]. Motorcycle sales in Indonesia are dominated by companies that are members of the Association of Indonesian Motorcycle Industry. This association consists of five companies, manufacturers of motorcycles with the Honda, TVS, Kawasaki, Suzuki, and Yamaha brands [2]. Based on data from the Association of Indonesian Motorcycle Industry, motorcycle sales in Indonesia in 2019 were dominated by Honda motorbikes and followed by motorcycles from Yamaha, Suzuki, Kawasaki, and TVS brands [3]. Although motorcycle sales in Indonesia have decreased in 2020 due to the Covid-19 pandemic, Indonesia's motorcycle sales are expected to increase in 2021.

The potential for motorcycle sales in Indonesia is large enough, so it encourages motorcycle manufacturers to produce motorcycles that attract consumer interest. Sales competition illustrates consumer trends in choosing motorbikes over a long period. Accurate predictions can help companies identify potential growth or potential decline in demand for a motorbike brand. Moreover, a company can also develop strategies to maintain and increase the market share of motorbike demand in the years to come. Accurate forecasts related to potential motorcycle sales are also a reference for investors in identifying investment targets.

The Lotka-Volterra type competition mathematical model has been used to describe the competition between species. In 1934, Gause applied the Lotka-Volterra competition model to explain competition between two Protozoa species, namely *Paramecium caudatum* and *Paramecium aurelia* species [4]. Recently, Novoa-Munoz et al. used the Lotka-Volterra competition model to

explain the competition dynamics between two lizards species, namely *Liolaemus cyanogaster* and *Liolaemus tenuis* species [5]. The Lotka-Volterra type competition model was also used to explain competition between companies in the struggle for share, including competition between the Korean stock markets [6], the retail industry in Taiwan [7], and commercial banks and rural banks competition in Indonesia [8].

In this paper, we apply the Lotka-Volterra type competition mathematical model to explain the competitive dynamics of motorcycle sales from two motorcycle producers in Indonesia. We also estimate the parameters of the model using the particle swarm optimization method. Moreover, we perform a stability analysis of the model by changing the estimation results' parameter values and finding the model's equilibria. We also perform a stability analysis of each equilibrium model, and we simulate the model to study the future competitive conditions of the two motor producers.

2. MATHEMATICAL MODEL OF MOTORCYCLE SALES COMPETITION

In this section, we present an application of the Lotka-Volterra type competition mathematical model to describe the dynamics of motorcycle sales competition in Indonesia. We examine the dynamics of competition between two motorcycle producers (companies) in Indonesia. Here, the first and second motorcycle producers are the market-leader producer and the market-leader main competitor. Suppose y_1 and y_2 represent the number of annual motorcycle sales of the first producer and the second producer in Indonesia. The mathematical model of the two motorbike producers' competition is constructed based on the following assumptions:

- (1) There are only two motorcycle producers (manufacturers) considered in the model.
- (2) Competition between the two motorcycle manufacturers and other manufacturers was ignored because the two companies' motorcycle sales were much higher than other motorbikes' sales.
- (3) In the absence of competition, the two motorcycle producers' sales grow following the logistic growth model.
- (4) Motorcycle variations are ignored in the mathematical model.
- (5) Sales of motorbikes for export are neglected.

- (6) The decline rate of motorcycle sales due to the competition is proportional to the two motorcycle producers' motorcycle sales.

Based on these assumptions, the competitive dynamics of the two motor manufacturers can be represented in the following differential equation system:

$$(1) \quad \frac{dy_1}{dt} = a_1 y_1 \left(1 - \frac{y_1}{K}\right) - b_1 y_1 y_2,$$

$$(2) \quad \frac{dy_2}{dt} = a_2 y_2 \left(1 - \frac{y_2}{K}\right) - b_2 y_1 y_2.$$

Here, parameters a_1 , a_2 are the growth rates of the first and second producer. Parameter K represents the maximum number of annual motor sales from both producers. Parameters b_1 , and b_2 denote the decline rates in the first producer and the second producer's annual sales of motorcycles, respectively. We assume that a_1 , a_2 , $K > 0$ and b_1 , $b_2 \geq 0$.

Equation (1) represents the rate of change in motorcycle sales from the first producer (the market-leader producer) per time unit. The number of the first producer's annual sales may increase due to the first producer's motorcycle sales growth rate. On the other hand, the first manufacturer's motorbikes' annual sales may decline due to competition between the first producer and the second producer.

Equation (2) describes the dynamics of the number of annual sales of motorcycles from the second producer (the first producer's main competitor) per time unit. The yearly motorcycle sale of the second producer could increase due to the company's growth rate. On the other hand, the second producer's annual motorcycle sale could decrease due to competition with the first producer.

3. PARAMETER ESTIMATION ON THE MATHEMATICAL MODEL OF MOTORCYCLE SELLING COMPETITION

In this section, we estimate the parameter values in the mathematical model of motorbike sales competition in Indonesia, as presented in equations (1) - (2). Because the analytic solution of the differential equation system in equations (1) - (2) is unknown, then we could apply heuristic methods such as genetic algorithms and particle swarm optimization method to determine the parameter values of the competition mathematical model [9, 10]. Here we use the particle

swarm optimization method as a parameter estimation method since the particle swarm optimization method is more robust than the genetic algorithm [10]. We use annual motorcycle sales data from 2006 ($t = 0$) to 2019 ($t = 13$). The annual sales data of motorcycles from the first and second producer were cited from the literature [11, 12, 13, 14, 15], and the data were presented in Figure 1 and Figure 2, respectively.

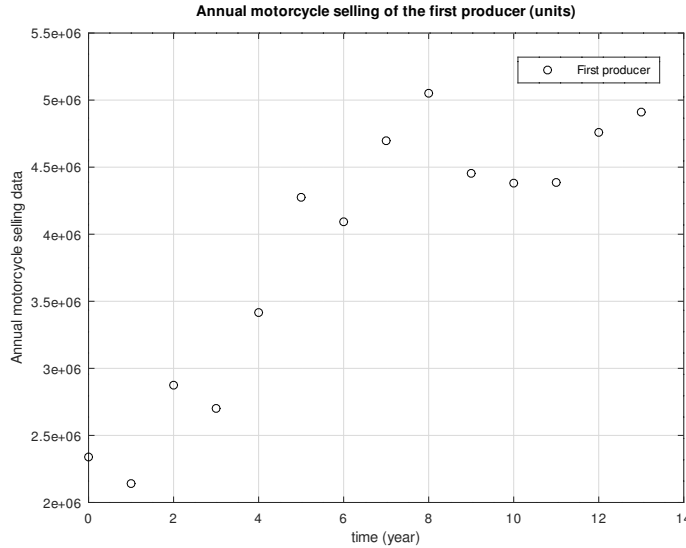


FIGURE 1. The annual motorcycle sales data of the first producer (the market leader producer)

The parameter value is selected in such a way that the parameter produces the smallest error value. In this article, we use MAPE (Mean Absolute Percentage Error) as an objective function. The MAPE (E) value for this problem is given by

$$E = \frac{1}{2n} \sum_{i=1}^n \left(\left| \frac{y_{1,i}^* - y_{1,i}}{y_{1,i}} \right| + \left| \frac{y_{2,i}^* - y_{2,i}}{y_{2,i}} \right| \right).$$

Here, n is the number of data, $y_{1,i}$, $y_{2,i}$ are the annual motorcycle sales data from the first and second producers in the year i , where and $i = 0, 1, 2, \dots, n$. Besides, $y_{1,i}^*$, $y_{2,i}^*$ are the predicted annual motorcycle sales quantities of the first and second producers in the year i .

The particle swarm optimization method was carried out as many as 50 trials with 300 iterations for each trial. We obtained the best objective function value (the smallest MAPE value) of 0.086222 or 8.622%. The best parameter values from the parameter estimation process are presented in Table 1.

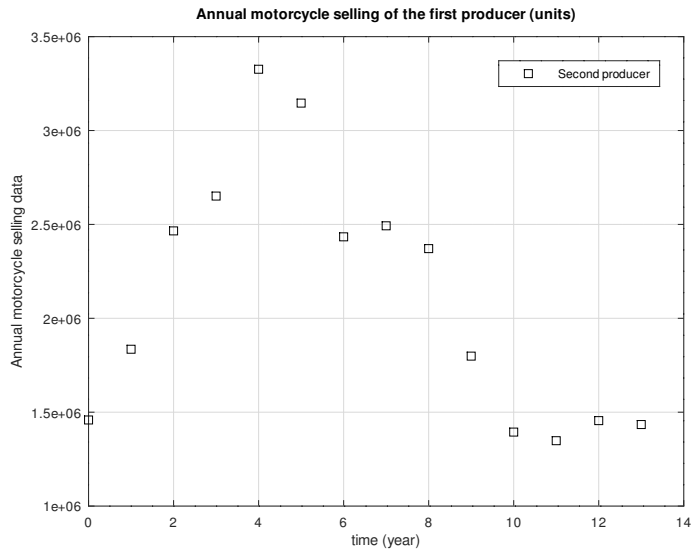


FIGURE 2. Annual motorcycle sales data for the second producer (the main competitor of market-leader producer)

TABLE 1. Best parameter values

Parameter	Parameter values
a_1	0.174947
b_1	0
K	5830774
a_2	1.194924
b_2	2.00262×10^{-7}

Figure 3 and Figure 4 show the comparison of the model prediction results and the data of the first and second producers, respectively.

Figure 3 shows the comparison between the data and the prediction results of motorbike sales in the first producer. The prediction results of motorbike sales tend to increase by around 5 million units of motorcycles/year. Figure 4 shows the comparison between the data and the prediction from the model of motorcycle sales model for the second producer. After experiencing an increase in sales until the fourth year, the prediction results of the second manufacturer's motorcycle sales have decreased to a sales figure of around 1.4 million units of motorcycles/year. Although there is quite a big difference between the data and the model prediction of the second

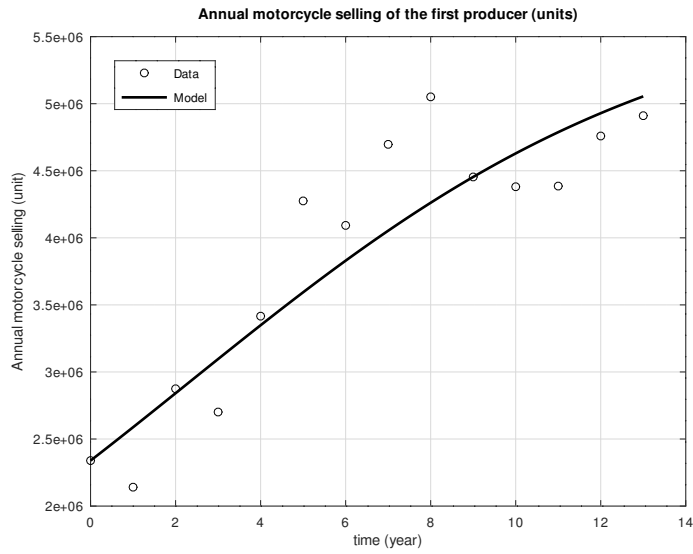


FIGURE 3. Comparison between the data and the predicted annual sales of motorcycles from the first producer

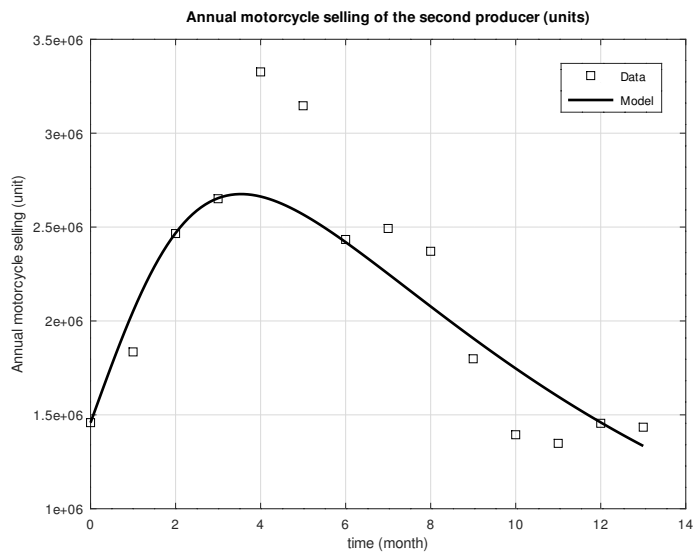


FIGURE 4. Comparison between the data and the predicted annual sales of motorcycles from the second producer

producer in the fourth and fifth years (2010 and 2011), the trend of the projection of motorbike sales in the second company is quite useful in describing the dynamics of growing and declining of the motorcycle sales from the company.

By substituting the parameter values in Table 1 into equations (1) -(2), the competitive mathematical model of motorcycle sales in Indonesia, between two producers, can be described in

the following form.

$$(3) \quad \frac{dy_1}{dt} = 0.174947y_1 \left(1 - \frac{y_1}{5830774}\right),$$

$$(4) \quad \frac{dy_2}{dt} = 1.194924y_2 \left(1 - \frac{y_2}{5830774}\right) - 2.00262 \times 10^{-7}y_1y_2.$$

The mathematical model of motorbike sales competition in equations (3)-(4) has four equilibria, namely:

- (1) The equilibrium of the two producers went bankrupt $P_1(y_1, y_2) = (0, 0)$.
- (2) The equilibrium for the bankruptcy of the second producer $P_2(y_1, y_2) = (5830774, 0)$.
- (3) The equilibrium for the bankruptcy of the first producer $P_3(y_1, y_2) = (0, 5830774)$.
- (4) The coexistence equilibrium $P_4(y_1, y_2) = (5830774, 132928)$.

By using the eigenvalues method, the equilibria P_1, P_2, P_3 are unstable, and the coexistence equilibrium P_4 is locally asymptotically stable. The coexistence equilibrium P_4 is asymptotically stable so that this equilibrium may occur in real situations. From a practical point of view, the motorcycle sales competition from the two producers is a pure competition with the domination of the first producer as the market-leader producer.

Next, we present numerical simulations to predict annual motorcycle sales and the dynamics of competition between the first and second companies for the future. We also perform numerical simulations by using parameter values in Table 1 and the initial value from the motorcycles sales data in 2006. Numerical simulations are carried out from $t = 0$ to $t = 30$ (years). The results of the numerical model simulation are presented in Figure 5.

Figure 5 uses the motorcycle sales data of the two manufacturers in 2006. The initial values used in the simulation process are 2339168 motor units and 1458561 motor units. The results show that the annual number of motorbikes from the first manufacturer tends to increase, while the yearly sales of motorbikes from the second manufacturer have decreased after 2010). The annual sales of the first producer reached more than 5 million units/year. The first producer should maintain the best efforts to hold and to increase its market share. On the other hand, the motorcycle annual sales of the second producer have the potential to decrease with sales figures of less than 1 million units of motorcycles/year in the twentieth year (2026). Without the right

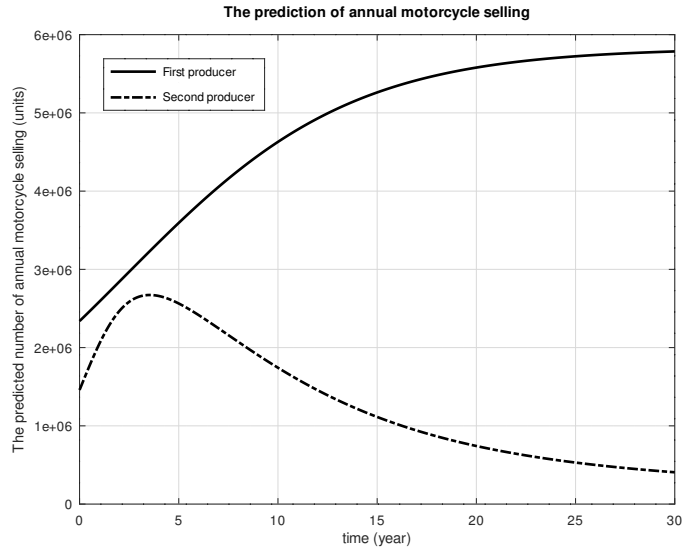


FIGURE 5. Dynamics of annual motorcycle sales of first and second producers

marketing strategy, the annual sales of the second motorcycle producer have the potential to decrease towards sales figures of less than 150 thousand units of motorbikes/year in the long term.

4. CONCLUSION

In this paper, the mathematical model of the Lotka-Volterra competition has been successfully applied to explain the dynamics of the motorcycle sales competition between two manufacturers in Indonesia. The Lotka-Volterra competition model adequately describes the annual motorcycle sales figures of both manufacturers. The results of the model analysis show that the competition between the two motorbike manufacturers is pure competition, with the first manufacturer controlling the motorcycle market in Indonesia.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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