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## MATHEMATICAL MODELING AND OPTIMAL CONTROL STRATEGY FOR THE OBESITY EPIDEMIC

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**Abstract.** The aim of this work is to propose a discrete mathematical model to study the behavioral dynamics of a population affected by the disease of obesity. Thus, the population under study is divided into six compartments: susceptible (S), exposed (E), slightly obese (I1), moderately obese (I2), very obese (I3), and recovered (R). To fight this disease, we used four controls: Awareness through education and media, food and sports programs, medical treatment with drugs, and treatment with surgical intervention. The discrete time Pontryagin maximum principle is used to characterize the optimal controls. The numerical simulation via MATLAB confirms the performance of theoretical results.

**Keywords:** obesity epidemic; mathematical modeling; optimal control.

**2010 AMS Subject Classification:** 93C55, 93B05, 65K05, 49N10, 97A40, 37M05, 37N25, 37N35.

### 1. INTRODUCTION

Obesity is an abnormal or excessive accumulation of fat mass initially caused by an energy imbalance. Since this imbalance modifies the state of health of the individual, it leads to the

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appearance of several physical diseases such as diabetes, high blood pressure, sleep apnea, other psychological and social consequences like depression, low self-esteem, discrimination, isolation etc[3]. Over the past decades, obesity has reached high epidemic rates[15]. It has become a global problem. The WHO report states[18]that there are over a billion overweight adults, at least 300 million of them are obese. To measure an individual's excess weight, there are several direct and indirect measurement methods. Fat mass is often measured indirectly through the Body Mass Index (BMI), which is weight (in kg) divided by height (in meters) squared. It is easily calculated and provides an overall assessment of the build and the amount of fat in the body. It is for this reason that it has become the international benchmark in clinical and epidemiological practice. Following various epidemiological studies on mortality, the WHO has defined BMI thresholds for which increases in the risk of mortality can be observed. From these BMI thresholds, the WHO has defined abnormal weight states, which are associated with health risks. These so-called abnormal states, in terms of excess weight, include states of overweight (BMI between 25 and 30), obesity (BMI greater than 30) and morbid obesity (BMI greater than 40)[3].

Some of the most important factors causing obesity include high consumption of high calorie foods, excessive nutrition, fast food and inactivity i.e. less physical activity to allow the body to get rid of excess calories. The normal comparison between body fat and weight for people is 18-23% for men and 25-30% for women[17]. According to estimates made by the WHO in 2011, excess weight is the fifth risk factor for death worldwide. The number of associated adult deaths would thus be of the order of 2.8 million each year. In addition, obesity is responsible for 44% of deaths linked to diabetes, 23% linked to ischemic heart disease and a considerable percentage (7-41%) linked to certain cancers[14]. And in more recent estimates, the prevalence of obesity more than doubled globally between 1980 and 2014. Indeed, in 2014, over 1.9 billion adults were overweight, and over 600 million were obese, which represents 39% and 13% of the global adult population respectively. According to the report published by the Global Consortium Against Obesity, a partner of the WHO, obesity is the second reason for hospitalization, and it is found that more than half of people who die from COVID-19 are overweight[16]. In Morocco, according to a survey by the High Commission for Planning (HCP)[10], 10.3 million Moroccan

adults, of which 63.1% of women are in a situation of obesity or pre-obesity. This survey reveals that in 10 years, severe and morbid obesity increased by 7.3% per year between 2001 and 2011. To understand the dynamics of obesity, there are some works based on compartmental models that are widely used in epidemiology, among these contributions, we cite : [8, 9, 11]. We note that researchers in most of these studies focus on continuous-time models described by differential equations. It is noteworthy that, in recent years, more and more attention has been paid to separate time models [1, 2, 6, 7, 13, 19]. The reasons for adopting discrete modeling are as follows : First, statistical data is collected separately at times (day, week, month or year). Therefore, it is more direct, more precise and more timely to describe the disease using discrete-time models from continuous-time models. Second, the use of discrete time models can avoid certain computational complications such as the choice of functional space and the regularity of the solution. Third, the numerical simulation from continuous time models is obtained in a discretionary way.

Based on the above reasons, we will develop a discrete temporal model to study the dynamics of the population infected with overweight and obesity. In addition, in order to find the best strategy to reduce the number of those infected with overweight and various types of obesity. We will use four control strategies : awareness programs through media and education, promoting healthy diets and physical activity, pharmacological and/or surgical treatments with psychological support.

In this paper, we construct a discrete  $SEI_kR$  Mathematical Obesity Model ( $k=1,2,3$ ). In Section 2, the mathematical model is proposed. In Section 3, we investigate the optimal control problem for the proposed discrete mathematical model. Section 4 consists of numerical simulation through MATLAB. The conclusion is given in Section 5.

## 2. FORMULATION OF THE MATHEMATICAL MODEL

**2.1. Description of the Model.** In this section, we present a discrete  $SEI_kR$  Mathematical Obesity Model. The population under investigation is divided into six compartments :

**The compartment S:** represents people who do not have obesity yet, but might become overweight in the future. This compartment is increased by the recruitment of individuals at

rate  $\Lambda$  and it is decreased by the rates  $\beta_1 \frac{S_k E_k}{N}$ ,  $\beta_2 S_k M$ . Some of the people in this compartment vacate at a constant death rate of  $\mu$  due to the total natural death rate  $\mu S_k$ .

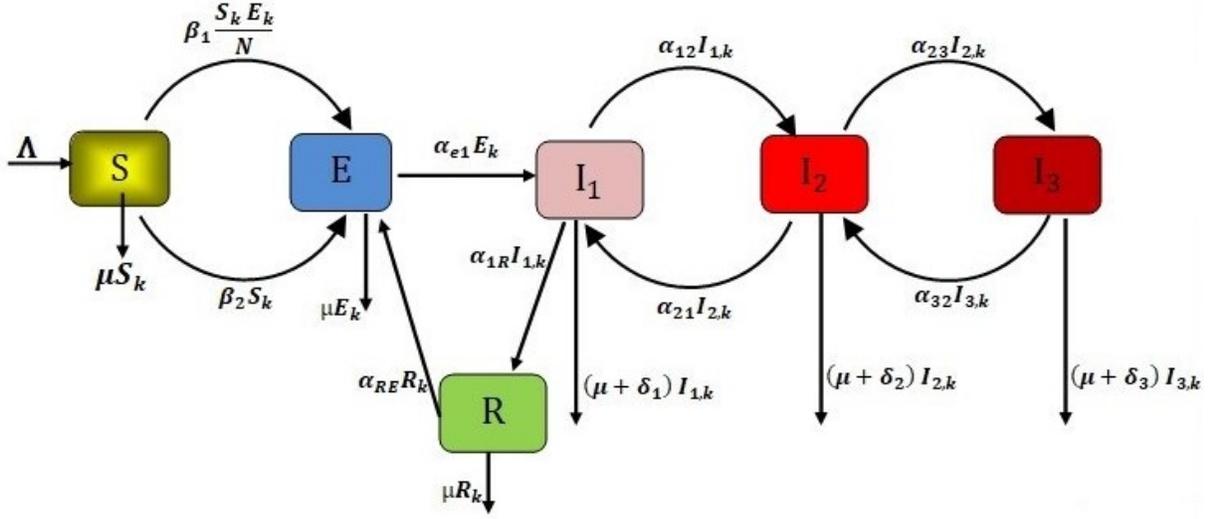
**The compartment E:** represents people who are at greater risk of being obese without symptoms. This compartment is increased by  $\beta_1 \frac{S_k E_k}{N}$ ,  $\beta_2 S_k M$  and  $\alpha_{RE} R_k$ . Here  $\beta_1$ ,  $\beta_2$  are respectively social and non-social rates.  $\alpha_{RE}$  is the rate of recovered healthy weight individuals who are disposed to weight regain. This compartment is decreased by the natural death  $\mu E_k$  and by the rate  $\alpha_{e1} E_k$  (represents the persons who are more likely to become overweight).

**The compartment I<sub>1</sub>:** consists of the individuals affected by overweight (their body mass index (BMI) is between 25 and 30). Their number increases by the rate  $\alpha_{e1} E_k$  and by the rate  $\alpha_{21} I_{2k}$  (people who have recovered from obesity, but still remain overweight). This compartment is decreased by the natural death  $\mu I_{1k}$ , by death due to overweight  $\delta_1 I_{1k}$  and the rates  $\alpha_{1R} I_{1k}$  and  $\alpha_{12} I_{1k}$  (which respectively represent the recovered persons and overweight people who become obese).

**The compartment I<sub>2</sub>:** consists of the individuals affected by obesity (their body mass index (BMI) is between 30 and 40). Their number increases by the rates  $\alpha_{12} I_{1k}$  and  $\alpha_{32} I_{3k}$  (people who have recovered from morbid obesity, but still remain obese). This compartment is decreased by the natural death  $\mu I_{2k}$ , by death due to obesity  $\delta_2 I_{2k}$  and the rates  $\alpha_{21} I_{2k}$  and  $\alpha_{23} I_{2k}$  (the obese people who become extremely obese).

**The compartment I<sub>3</sub>:** comprises the individuals affected by extreme obesity (their body mass index (BMI) is greater than 40). Their number increases by the rates  $\alpha_{23} I_{2k}$ . This compartment is decreased by the natural death  $\mu I_{3k}$ , by death due to extreme obesity  $\delta_3 I_{3k}$  and the rate  $\alpha_{32} I_{3k}$  (people who have recovered from extreme obesity, but still remain obese).

**The compartment R:** represents overweight individuals who lose a sufficient weight that returns them back to normal BMI (25 kg/m<sup>2</sup>) and thus they are considered recovered. Their number increases by the rates  $\alpha_{1R} I_{1k}$ . This compartment is decreased by the natural death  $\mu R_k$ , and by  $\alpha_{RE} R_k$ . The following diagram will demonstrate the flow directions of individuals among the compartments. These directions are going to be represented by directed arrows (see Figure 1).


 FIGURE 1. The flow between the six compartments  $SEI_1I_2I_3R$ 

The total population size at time  $k$  is denoted by  $N_k$  with  $N_k = S_k + E_k + I_{1k} + I_{2k} + I_{3k} + R_k$ .

**2.2. Model Equations.** By the addition of the rates at which individuals enter the compartment and also by subtracting the rates at which people vacate the compartment, we obtain a difference equation for the rate at which the individuals of each compartment change over discrete time. Hence, we present the obese infection model by the following system of difference equations:

$$(1) \quad \begin{cases} S_{k+1} = \Lambda + (1 - \mu)S_k - \beta_1 \frac{S_k}{N} E_k - \beta_2 S_k \\ E_{k+1} = (1 - \mu)E_k + \beta_1 \frac{S_k}{N} E_k + \beta_2 S_k - \alpha_{e1} E_k + \alpha_{RE} R_k \\ I_{1,k+1} = (1 - \mu - \delta_1) I_{1,k} + \alpha_{e1} E_k + \alpha_{21} I_{2,k} - \alpha_{1R} I_{1,k} - \alpha_{12} I_{1,k} \\ I_{2,k+1} = (1 - \mu - \delta_2) I_{2,k} + \alpha_{12} I_{1,k} + \alpha_{32} I_{3,k} - \alpha_{21} I_{2,k} - \alpha_{23} I_{2,k} \\ I_{3,k+1} = (1 - \mu - \delta_3) I_{3,k} + \alpha_{23} I_{2,k} - \alpha_{32} I_{3,k} \\ R_{k+1} = (1 - \mu) R_k - \alpha_{RE} R_k + \alpha_{1R} I_{1,k} \end{cases}$$

where  $S_0 \geq 0, E_0 \geq 0, I_{1,0} \geq 0, I_{2,0} \geq 0, I_{3,0} \geq 0$ , and  $R_0 \geq 0$  are the given initial states.

### 3. THE OPTIMAL CONTROL PROBLEM

The control strategies we adopt consist of an awareness program through media and education, encouraging the practice of sport and adopting a healthy diet, treatment and psychological

support. Our main goal in adopting these strategies is to minimize the number of obese individuals and overweight individuals during the time steps  $k= 0$  to  $T$  and also minimizing the cost spent to apply the four strategies.

In this model, we include the four controls  $u_k$ ,  $v_{1k}$ ,  $v_{2k}$ , and  $v_{3k}$  which consecutively represent the awareness program through media and education, encouraging the practice of sport and adopting a healthy diet, medical treatment with drugs, and surgical intervention along with psychological support. Thus, the controlled mathematical system is given by the following system of difference equations:

$$(2) \quad \begin{cases} S_{k+1} = \Lambda + (1 - \mu)S_k - \beta_1(1 - u_k)\frac{S_k E_k}{N} - \beta_2(1 - u_k)S_k \\ E_{k+1} = (1 - \mu)E_k + \beta_1(1 - u_k)\frac{S_k E_k}{N} + \beta_2(1 - u_k)S_k - \alpha_{e1}E_k + \alpha_{RE}R_k \\ I_{1,k+1} = (1 - \mu - \delta_1)I_{1,k} + \alpha_{e1}E_k + \alpha_{21}I_{2,k} - \alpha_{1R}I_{1,k} - \alpha_{12}I_{1,k} - v_{1,k}I_{1,k} \\ I_{2,k+1} = (1 - \mu - \delta_2)I_{2,k} + \alpha_{12}I_{1,k} + \alpha_{32}I_{3,k} - \alpha_{21}I_{2,k} - \alpha_{23}I_{2,k} - v_{2,k}I_{2,k} \\ I_{3,k+1} = (1 - \mu - \delta_3)I_{3,k} + \alpha_{23}I_{2,k} - \alpha_{32}I_{3,k} - v_{3,k}I_{3,k} \\ R_{k+1} = (1 - \mu)R_k - \alpha_{RE}R_k + \alpha_{1R}I_{1,k} + v_{1,k}I_{1,k} + v_{2,k}I_{2,k} + v_{3,k}I_{3,k} \end{cases}$$

Where  $S_0 \geq 0, E_0 \geq 0, I_{1,0} \geq 0, I_{2,0} \geq 0, I_{3,0} \geq 0$ , and  $R_0 \geq 0$  are the given initial states.

There are four controls  $u_k = (u_0, u_1, \dots, u_{T-1})$ ,  $v_{1,k} = (v_{1,0}, v_{1,1}, \dots, v_{1,T-1})$ ,  $v_{2,k} = (v_{2,0}, v_{2,1}, \dots, v_{2,T-1})$  and  $v_{3,k} = (v_{3,0}, v_{3,1}, \dots, v_{3,T-1})$ . The first control can be interpreted as the proportion to be adopted to awareness programs through media and education. So, we note that  $(1 - u_k)\frac{S_k E_k}{N}$  is the proportion of the susceptible people who are protected from contacting exposed people at time step  $k$ . The second control can be interpreted as the proportion of individuals who are advised to eat healthy and to be physically active. We observe that  $v_{1,k}I_{1,k}$  is the proportion of the individuals who cease contacting overweight people and who will transform into the recovered individuals at time step  $k$ . The third control can be interpreted as the proportion of individuals to be subjected to treatment with drugs. So, we note that  $v_{2,k}I_{2,k}$  is the proportion of the individuals who will move from the class of obese people towards the class of the individuals who recover from obesity at time step  $k$ . The fourth control can be interpreted as the proportion of individuals who will get surgical and psychological treatment along with follow-up. So, we observe that  $v_{3,k}I_{3,k}$  is the proportion of the individuals who recover from extreme obesity at time step  $k$ .

The challenge that we face here is how to minimize the objective functional:

$$\begin{aligned}
 J(u, v_1, v_2, v_3) &= A_T E_T + B_{1,T} I_{1,T} + B_{2,T} I_{2,T} + B_{3,T} I_{3,T} + \\
 (3) \quad &+ \sum_{k=0}^{T-1} (A_k E_k + B_{1,k} I_{1,k} + B_{2,k} I_{2,k} + B_{3,k} I_{3,k}) \\
 &+ \sum_{k=0}^{T-1} \left( \frac{d}{2} c_k u_k^2 + \sum_{i=1}^3 \left( \frac{1}{2} a_i D_{i,k} v_{i,k}^2 \right) \right),
 \end{aligned}$$

Where the parameters  $A_k > 0$ ,  $B_{1,k} > 0$ ,  $B_{2,k} > 0$ ,  $B_{3,k} > 0$ ,  $C_k > 0$ ,  $E_k > 0$  and  $D_{i,k} > 0$  are the cost coefficients, they are selected to weigh the relative importance of  $S_k$ ,  $E_k$ ,  $I_{1,k}$ ,  $I_{2,k}$ ,  $I_{3,k}$ ,  $u_k$ ,  $v_{1,k}$ ,  $v_{2,k}$  and  $v_{3,k}$  at time  $k$ .  $T$  is the final time.

In other words, we seek the optimal controls  $u_k^*$ ,  $v_{1,k}^*$ ,  $v_{2,k}^*$ , and  $v_{3,k}^*$  such that:

$$(4) \quad J(u_k^*, v_{1,k}^*, v_{2,k}^*, v_{3,k}^*) = \min_{(u, v_1, v_2, v_3) \in U_{ad}^4} J(u_k, v_{1,k}, v_{2,k}, v_{3,k}),$$

Where  $U_{ad}$  is the set of admissible controls defined by:

$$\begin{aligned}
 U_{ad} &= \{u_k = (u_0, u_1, \dots, u_{T-1}), v_{i,k} = (v_{i,0}, v_{i,1}, \dots, v_{i,T-1}) \mid a \leq u_k \leq b \\
 (5) \quad &\text{for } i = 1, 2, 3 : m_i \leq v_{i,k} \leq n_i ; k = 0, 1, 2, \dots, T-1\}
 \end{aligned}$$

The sufficient condition for the existence of the optimal controls  $(u, v_1, v_2, v_3)$  for the problem (2-3) comes from the following theorem.

**Theorem 1.** *There exist the optimal controls  $(u^*, v_1^*, v_2^*, v_3^*)$  such that:*

$$(6) \quad J(u^*, v_1^*, v_2^*, v_3^*) = \min_{(u, v_1, v_2, v_3) \in U_{ad}^4} J(u, v_1, v_2, v_3)$$

*subject to the control system (2) with initial conditions.*

*Proof.* Since the coefficients of the state equations are bounded and there is a finite number of time steps,  $S = (S_0, S_1, \dots, S_T)$ ,  $E = (E_0, E_1, \dots, E_T)$ ,  $I_1 = (I_{1,0}, I_{1,1}, \dots, I_{1,T})$ ,  $I_2 = (I_{2,0}, I_{2,1}, \dots, I_{2,T})$ ,  $I_3 = (I_{3,0}, I_{3,1}, \dots, I_{3,T})$  and  $R = (R_0, R_1, \dots, R_T)$  are uniformly bounded for all  $(u, v_1, v_2, v_3)$  in the control set  $U_{ad}$ ; thus  $J(u, v_1, v_2, v_3)$  is bounded for all  $(u, v_1, v_2, v_3) \in U_{ad}^4$ . Since  $J(u, v_1, v_2, v_3)$  is bounded,  $\inf_{(u, v_1, v_2, v_3) \in U_{ad}^4} J(u, v_1, v_2, v_3)$  is finite, and there exists a sequence  $(u^j; v_1^j; v_2^j; v_3^j) \in U_{ad}^4$  such that  $\lim_{j \rightarrow +\infty} J(u^j; v_1^j; v_2^j; v_3^j) = \inf_{(u, v_1, v_2, v_3) \in U_{ad}^4} J(u, v_1, v_2, v_3)$

and corresponding sequences of states  $S^j, E^j, I_1^j, I_2^{-j}, I_3^j$  and  $R^j$ . Since there is a finite number of uniformly bounded sequences, there exist  $(u^*, v_1^*, v_2^*, v_3^*) \in U_{ad}^4$  and  $S^*, E^*, I_1^*, I_2^*, I_3^*$  and  $R^* \in \mathbb{R}^{T+1}$  such that on a subsequence,  $(u^j; v_1^j; v_2^j; v_3^j) \rightarrow (u^*, v_1^*, v_2^*, v_3^*)$ ,  $S^j \rightarrow S^*$ ,  $E^j \rightarrow E^*$ ,  $I_1^j \rightarrow I_1^*$ ,  $I_2^j \rightarrow I_2^*$ ,  $I_3^j \rightarrow I_3^*$  and  $R^j \rightarrow R^*$ . Finally, due to the finite dimensional structure of system (2) and the objective function  $J(u, v_1, v_2, v_3)$ ;  $(u^*, v_1^*, v_2^*, v_3^*)$  is an optimal control with corresponding states  $S^*, E^*, I_1^*, I_2^*, I_3^*$  and  $R^*$ . Therefore  $\inf_{(u, v_1, v_2, v_3) \in U_{ad}^4} J(u, v_1, v_2, v_3)$  is achieved.  $\square$

We apply the discrete version of Pontryagin's Maximum Principle[4, 5, 6, 7, 12, 13]. The key idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts the problem of finding the control to optimize the objective functional subject to the state of difference equation with initial condition to find the control to optimize the Hamiltonian point by point (with respect to the control).

We have the Hamiltonian  $H_k$  at time step  $k$ , defined by:

$$(7) \quad H_k = A_k E_k + B_{1,k} I_{1,k} + B_{2,k} I_{2,k} + B_{3,k} I_{3,k} + \frac{C_k}{2} u_k^2 + \frac{a_1}{2} v_{1,k}^2 + \frac{a_2}{2} v_{2,k}^2 + \frac{a_3}{2} v_{3,k}^2 + \sum_{i=1}^6 \zeta_{i,k+1} f_{i,k+1},$$

where  $f_{i,k+1}$  is the right side of the system of difference equations (2) of the  $i^{th}$  state variable at time step  $k+1$ .

**Theorem 2.** *Given the optimal controls  $(u^*, v_1^*, v_2^*, v_3^*) \in U_{ad}^4$  and the solutions  $S^*, E^*, I_1^*, I_2^*, I_3^*$  and  $R^*$  of the corresponding state system (2), there exist adjoint functions  $\zeta_{1,k}, \zeta_{2,k}, \zeta_{3,k}, \zeta_{4,k}, \zeta_{5,k}$  and  $\zeta_{6,k}$  satisfying*

$$(8) \quad \begin{aligned} \zeta_{1,k} &= \zeta_{1,k+1}(1 - \mu) + (\beta_2 + \beta_1 \frac{E_k}{N})(1 - u_k)(\zeta_{2,k+1} - \zeta_{1,k+1}) \\ \zeta_{2,k} &= A_k + \zeta_{2,k+1}(1 - \mu) + \beta_1(1 - u_k)(\zeta_{2,k+1} - \zeta_{1,k+1}) \frac{S_k}{N} + \alpha_{e1}(\zeta_{3,k+1} - \zeta_{2,k+1}). \\ \zeta_{3,k} &= B_{1,k} + \zeta_{3,k+1}(1 - \mu - \delta_1 + \alpha_{1R} - v_{1,k}) + \zeta_{4,k+1} \alpha_{12} + \zeta_{6,k+1}(\alpha_{1R} + v_{1,k}). \\ \zeta_{4,k} &= B_{2,k} + \zeta_{3,k+1} \alpha_{21} + \zeta_{4,k+1}(1 - \mu - \delta_2 - \alpha_{21} - \alpha_{23} - v_{2,k}) + \zeta_{6,k+1} v_{2,k}. \\ \zeta_{5,k} &= B_{3,k} + \zeta_{4,k+1} \alpha_{32} + \zeta_{5,k+1}(1 - \mu - \delta_3 - \alpha_{32} - v_{3,k}) + \zeta_{6,k+1} v_{3,k}. \end{aligned}$$

$$\zeta_{6,k} = \zeta_{2,k+1}\alpha_{RE} + \zeta_{6,k+1}(1 - \mu - \alpha_{RE}).$$

With the transversality conditions at time  $T$ .  $\zeta_{1,T} = \zeta_{6,T} = 0$ ,  $\zeta_{2,T} = A_T$ ,  $\zeta_{3,T} = B_{1,T}$ ,  $\zeta_{4,T} = B_{2,T}$  and  $\zeta_{5,T} = B_{3,T}$ .

Furthermore, for  $k = 0, 1, 2, \dots, T-1$  the optimal controls  $u_k^*$ ,  $v_{1,k}^*$ ,  $v_{2,k}^*$  and  $v_{3,k}^*$  are given by:

$$(9) \quad \begin{aligned} u_k^* &= \min \left[ b; \max \left( a, \frac{1}{C_k} \left[ (\zeta_{2,k+1} - \zeta_{1,k+1}) (\beta_2 + \beta_1 \frac{E_k}{N}) S_k \right] \right) \right] \\ v_{1,k}^* &= \min \left[ n_1; \max \left( m_1, \frac{1}{D_{1k}} (\zeta_{6,k+1} - \zeta_{3,k+1}) I_{1,k} \right) \right] \\ v_{2,k}^* &= \min \left[ n_2; \max \left( m_2, \frac{1}{D_{2k}} [\zeta_{4,k+1} - \zeta_{6,k+1}] I_{2,k} \right) \right] \\ v_{3,k}^* &= \min \left[ n_3; \max \left( m_3, \frac{1}{D_{3k}} [(\zeta_{5,k+1} - \zeta_{6,k+1}) I_{3,k}] \right) \right] \end{aligned}$$

*Proof.* The Hamiltonian at time step  $k$  is given by :

$$(10) \quad \begin{aligned} H_k &= A_k E_k + B_{1,k} I_{1,k} + B_{2,k} I_{2,k} + B_{3,k} I_{3,k} + \frac{C_k}{2} u_k^2 + \frac{a_1}{2} v_{1,k}^2 + \frac{a_2}{2} v_{2,k}^2 + \frac{a_3}{2} v_{3,k}^2 + \zeta_{1,k+1} f_{1,k+1} \\ &\quad + \zeta_{2,k+1} f_{2,k+1} + \zeta_{3,k+1} f_{3,k+1} + \zeta_{4,k+1} f_{4,k+1} + \zeta_{5,k+1} f_{5,k+1} + \zeta_{6,k+1} f_{6,k+1} \\ &= A_k E_k + B_{1,k} I_{1,k} + B_{2,k} I_{2,k} + B_{3,k} I_{3,k} + \frac{C_k}{2} u_k^2 + \frac{a_1}{2} v_{1,k}^2 + \frac{a_2}{2} v_{2,k}^2 + \frac{a_3}{2} v_{3,k}^2 \\ &\quad + \zeta_{1,k+1} \left[ \Lambda + (1 - \mu) S_k - \beta_1 (1 - u_k) \frac{S_k E_k}{N} - \beta_2 (1 - u_k) S_k \right] \\ &\quad + \zeta_{2,k+1} \left[ (1 - \mu) E_k + \beta_1 (1 - u_k) \frac{S_k E_k}{N} + \beta_2 (1 - u_k) S_k - \alpha_{e1} E_k + \alpha_{RE} R_k \right] \\ &\quad + \zeta_{3,k+1} \left[ (1 - \mu - \delta_1) I_{1,k} + \alpha_{e1} E_k + \alpha_{21} I_{2,k} - \alpha_{1R} I_{1,k} - v_{1,k} I_{1,k} \right] \\ &\quad + \zeta_{4,k+1} \left[ (1 - \mu - \delta_2) I_{2,k} + \alpha_{12} I_{1,k} + \alpha_{32} I_{3,k} - \alpha_{21} I_{2,k} - \alpha_{23} I_{2,k} - v_{2,k} I_{2,k} \right] \\ &\quad + \zeta_{5,k+1} \left[ (1 - \mu - \delta_3) I_{3,k} + \alpha_{23} I_{2,k} - \alpha_{32} I_{3,k} - v_{3,k} I_{3,k} \right] \\ &\quad + \zeta_{6,k+1} \left[ (1 - \mu) R_k - \alpha_{RE} R_k + \alpha_{1R} I_{1,k} + v_{1,k} I_{1,k} + v_{2,k} I_{2,k} + v_{3,k} I_{3,k} \right] \end{aligned}$$

For  $k = 0, 1, \dots, T-1$  the optimal controls  $u_k$ ,  $v_{1,k}$ ,  $v_{2,k}$  and  $v_{3,k}$  can be solved from the optimality condition,

$$(11) \quad \frac{\partial H_k}{\partial u_k} = 0, \quad \frac{\partial H_k}{\partial v_{1,k}} = 0, \quad \frac{\partial H_k}{\partial v_{2,k}} = 0 \quad \text{and} \quad \frac{\partial H_k}{\partial v_{3,k}} = 0$$

That are,

$$\begin{aligned}
 \frac{\partial H_k}{\partial u_k} &= C_k u_k - (\zeta_{2,k+1} - \zeta_{1,k+1}) (\beta_2 + \beta_1 \frac{E_k}{N}) S_k = 0 \\
 \frac{\partial H_k}{\partial v_{1,k}} &= D_{1,k} a_1 v_{1,k} - (\zeta_{6,k+1} - \zeta_{3,k+1}) I_{1,k} = 0 \\
 \frac{\partial H_k}{\partial v_{2,k}} &= D_{2,k} a_2 v_{2,k} - (\zeta_{4,k+1} - \zeta_{6,k+1}) I_{2,k} = 0 \\
 \frac{\partial H_k}{\partial v_{3,k}} &= D_{3,k} a_3 v_{3,k} - (\zeta_{4,k+1} - \zeta_{6,k+1}) I_{3,k} = 0
 \end{aligned}
 \tag{12}$$

So, we have

$$\begin{aligned}
 u_k &= \frac{1}{C_k} \left[ (\zeta_{2,k+1} - \zeta_{1,k+1}) (\beta_2 + \beta_1 \frac{E_k}{N}) S_k \right] \\
 v_{1,k} &= \frac{1}{a_1 D_{1,k}} [(\zeta_{6,k+1} - \zeta_{3,k+1}) I_{1,k}] \\
 v_{2,k} &= \frac{1}{a_2 D_{2,k}} [(\zeta_{4,k+1} - \zeta_{6,k+1}) I_{2,k}] \\
 v_{3,k} &= \frac{1}{a_3 D_{3,k}} [(\zeta_{4,k+1} - \zeta_{6,k+1}) I_{3,k}]
 \end{aligned}
 \tag{13}$$

By the bounds in  $U_{ad}$  of the controls, it is easy to obtain  $u_k^*$ ,  $v_{1,k}^*$ ,  $v_{2,k}^*$  and  $v_{3,k}^*$  in the form of (9). □

#### 4. SIMULATION

In this section, we present the results obtained by solving numerically the optimality system. This system consists of the state system, adjoint system, initial and final time conditions and the controls characterization.

In this formulation, there were initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with separated boundary conditions at time steps  $k = 0$  and  $k = T$ . We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization. We continue until convergence of successive iterates is achieved.

**4.1. Discussion.** In this section, we study and analyse numerically the effects of optimal control strategies such as awareness programs through media and education, treatment and psychological support along with follow-up for the obese and extremely obese people. The numerical solution of the model (2) is executed using Matlab with the following parameter values and initial values of state variable in table (1).

$S_0$	$E_0$	$I_{1,0}$	$I_{2,0}$	$I_{3,0}$	$R_0$	$\Lambda$	$\beta_1$	$\beta_2$	$\mu$
$5.10^3$	$3.10^3$	$1,5.10^3$	$1.10^3$	$2.10^3$	$1.10^3$	$5.10^2$	0.45	0.5	0.1
$\delta_1$	$\delta_2$	$\delta_3$	$\alpha_{12}$	$\alpha_{21}$	$\alpha_{32}$	$\alpha_{32}$	$\alpha_{1R}$	$\alpha_{e1}$	$\alpha_{RE}$
0.025	0.04	0.2	0.1	0.05	0.05	0.05	0.7	0.05	0.05

Table (1)

**4.2. Some objectives of the proposed control strategies.** The proposed control strategies in this work help to achieve several objectives:

**4.2.1. Objective A:** *Protecting and warning people who are most vulnerable to obesity and overweight individuals from becoming obese.* Awareness programs through media and education have a vital role in preventing obesity. Preventing obesity is not just about preventing people of normal weight from becoming obese, it is also supposed to prevent them becoming overweight, curing obesity for those who are already overweight and preventing the recovered from being overweight again. We provide an optimal strategy for this. Therefore, we activate the optimal control variable  $u$  which represents awareness programs for overweight people and obese people.

Figure 2 compares the evolution of overweight individuals without control, only with  $u$  control, and with four controls :  $u, v_1, v_2$  and  $v_3$  in which the effect of awareness programs offered through media and education has been shown to be positive in reducing overweight gain and preventing people of normal weight from contacting obese individuals. In addition to the role of awareness in protecting against obesity, Figure 2 shows the effect of the other measures to reduce the number of overweight people. Those measures are as follows : adopting a healthy diet, exercising or doing daily physical activity, hospitalization with medication or surgical intervention with psychological follow-up.

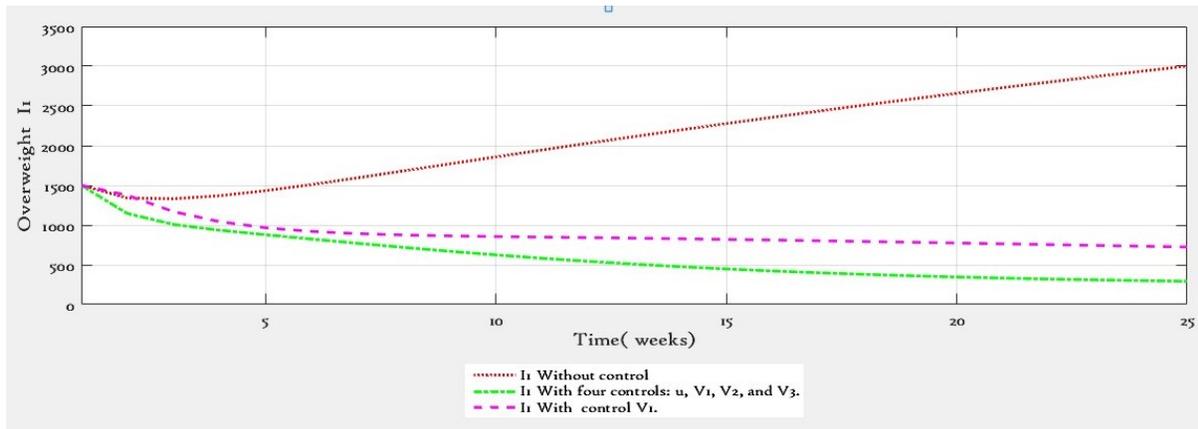


FIGURE 2. The evolution of  $I_1$  with and without controls

**4.2.2. Objective B : Decreasing the number of obese people.** In order to reduce the number of obese people, we adopt the optimal control  $v_1$ . It entails carrying out awareness campaigns and awareness through various media means, where calls are made to amend a healthy diet free of fat, staying away from night eating, exercising continuously, especially running for a half an hour a day.

Figure 3 shows the clear effect of these measures and their apparent effectiveness in reducing the number of obese people.

When we add the other strategies for optimal obesity control, figure 3 also shows the important role of awareness and medical intervention in reducing the number of obese people.

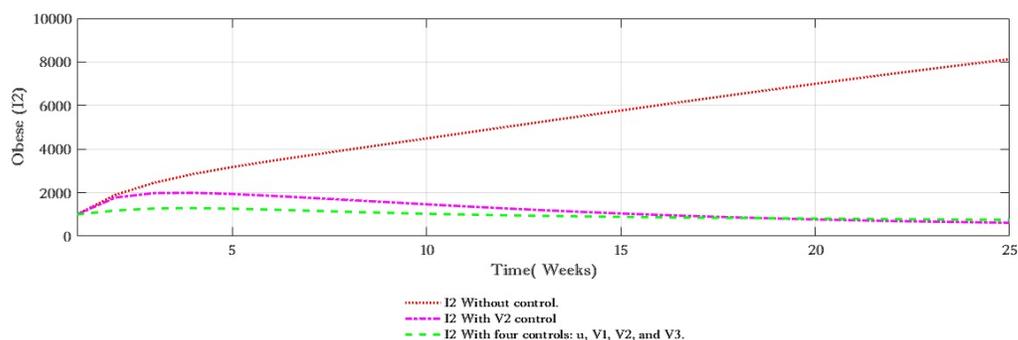


FIGURE 3. The evolution of  $I_2$  with and without controls

**4.2.3. Objective C : Medical treatment of severe obesity.** If the treatment of obesity and overweight sometimes succeeds with the approach of exercising and following healthy diets, many people face many difficulties in treating obesity by these means despite their commitment to them. Therefore, we suggest the surgical procedure  $v_3$ . These surgeries have achieved amazing results in treating obesity and getting rid of excess weight and fat in many people. They have spread among people quickly due to their efficiency and ability to enable patients to treat obesity and reach the ideal weight they want. Figure 4 shows the decrease in the number of obese people after undergoing surgery only. This figure shows that the number of obese individuals decreases significantly when surgical intervention is accompanied by the adoption of a healthy diet followed by exercise.

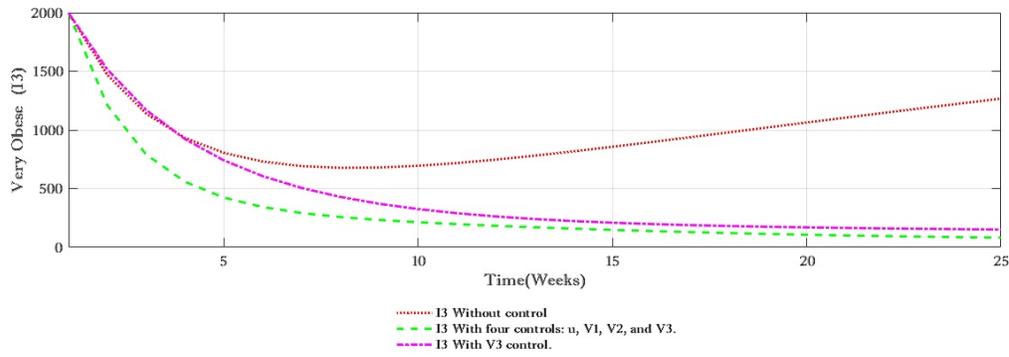


FIGURE 4. The evolution of  $I_3$  with and without controls

**4.2.4. Objective D : Increasing the number of people recovered from obesity.** Figure 5 shows the effectiveness of the four control strategies  $u$ ,  $v_1$ ,  $v_2$  and  $v_3$  proposed in this article to fight the obesity epidemic. This figure shows an increase in the number of people who recovered after adopting the suggested strategies, namely awareness, healthy eating and physical activity, medical treatment with drugs, and medical surgery.

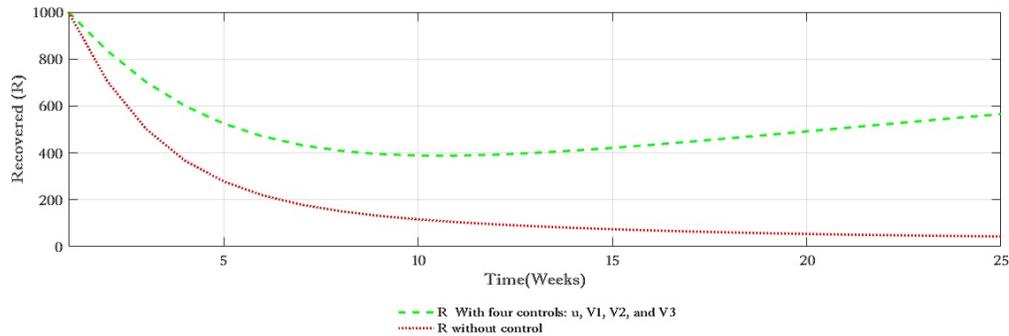


FIGURE 5. The evolution of R with and without controls

## 5. CONCLUSION

In this article, we have introduced discrete obesity modeling in order to minimize the number of overweight individuals, obese people and increase the number of people recovered from extreme obesity. We have also introduced four controls which respectively represent awareness programs through education and the media, encouraging the practice of sport and adopting a healthy diet, medical treatment with drugs, and medical surgery. We have applied the results of the control theory and we succeeded in obtaining the characterizations of the optimal controls. Numerical simulation of the obtained results showed the effectiveness of the proposed control strategies.

## CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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