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GLOBAL ATTRACTIVITY OF A SINGLE SPECIES STAGE-STRUCTURED MODEL WITH FEEDBACK CONTROL AND INFINITE DELAY

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Abstract. In this paper, a single species stage-structured model with feedback control and infinite delay is considered in this paper. By applying the comparison theorem of differential equations, sufficient condition is obtained for the extinction of the system; By using an iterative method, sufficient condition is obtained for the global asymptotic stability of the positive equilibrium of the model. Our results show that the death rate of mature species plays important role on the persistent and stability property of the system.

Keywords: Global stability; Stage-structured; Infinite delay; Equilibrium; Iterative method.

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1. Introduction

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Consider the following single species stage-structured model with infinite delay and feedback control:

$$
\begin{aligned}
\dot{x}_1(t) &= \alpha x_2(t) - \gamma x_1(t) - \alpha e^{-\gamma\tau} x_2(t-\tau), \\
\dot{x}_2(t) &= \alpha e^{-\gamma\tau} x_2(t-\tau) - d x_2(t) - \beta x_2^2(t) - c x_2(t) \int_{-\infty}^{t} K_1(t-s) u(s) ds, \\
\dot{u}(t) &= -a u(t) + b \int_{-\infty}^{t} K_2(t-s) x_2(s) ds,
\end{aligned}
\tag{1.1}
$$

where the coefficients $\alpha, \gamma, \beta, a, b, c$ are all positive constants. The delay kernels $K_i : [0, +\infty) \to (0, +\infty), i = 1, 2$ are continuous functions such that

$$
\int_0^{+\infty} K_i(s) ds = 1. \tag{1.2}
$$

We shall consider (1.1) together with the initial conditions

$$
x_i(s) = \phi_i(s),\ u(s) = \psi(s),\ s \in (-\infty, 0],\ i = 1, 2; \tag{1.3}
$$

where $\phi_i, \psi \in BC^+$ and

$$
BC^+ = \{\phi \in C((-\infty, 0], [0, +\infty)) : \phi(0) > 0 \text{ and } \phi \text{ isbounded}\},\ i = 1, 2.
$$

It is well known that by the fundamental theory of functional differential equations [1], system (1.1) has a unique solution $col(x_1(t), x_2(t), u(t))$ satisfying the initial condition (1.3). We easily prove $x_i(t) > 0$ for all $i = 1, 2$ and $u(t) > 0$ in maximal interval of existence of the solution. In this paper, the solution of system (1.1) satisfying the initial conditions (1.3) is said to be positive.

There are considerable works on the study of ecosystem with feedback controls, and many excellent results have been obtained([1]-[28]). However, seldom did scholars consider the influence of feedback controls on the stage-structured ecosystem([27]-[28]).

Based on the famous single species model proposed by Aiello and Freedman, Ding and

Cheng[27] proposed the following single species stage-structured model with feedback control:

$$\begin{aligned}
\dot{x}_1(t) &= \alpha x_2(t) - \gamma x_1(t) - \alpha e^{-\gamma\tau} x_2(t-\tau), \\
\dot{x}_2(t) &= \alpha e^{-\gamma\tau} x_2(t-\tau) - \beta x_2^2(t) - c x_2(t) u(t), \\
\dot{u}(t) &= -au(t) + b x_2(t).
\end{aligned} \tag{1.4}$$

In [27], it was shown that if the inequality

$$a\beta > bc \tag{1.5}$$

holds, then $x_1(t) \to x_1^*, x_2(t) \to x_2^*, u(t) \to u^*$ as $t \to +\infty$.

Already, there are several papers studied the dynamic behaviors of ecosystem with both feedback control and time delays. For example, P. X. Weng[8] and Fan *et al*[6] studied the dynamic behaviors of a n-species competition system with infinite feedback control, their results showed that delay has no influence on the existence of positive periodic solution of the system; F. D. Chen[5] studied the stability property of a n-species pure-delay type competition system with feedback controls, his results imply that the delays play an important role on the stability of the system. Z. Li *et al.* [19] studied an autonomous Lotka-Volterra competitive system with infinite delays and feedback controls. If the Lotka-Volterra competitive system is globally stable, then they showed that the feedback controls only change the position of the unique positive equilibrium and retain the stable property, and delay has no influence on the stability property of the system. Recently, Y. P. Liu *et al.* [28] proposed a stage structured predator-prey system with feedback control and infinite delay, they studied the persistent property and the existence of positive periodic solution of the system, however, they did not investigate the stability property of the system.

On the other hand, in their series papers, Chen *et al.* ([31]-[33]) studied the dynamic behaviors of the following stage-structured predator-prey system (stage structure for both predator and prey).

$$
\begin{aligned}
\dot{x}_1(t) &= r_1(t)x_2(t) - d_{11}x_1(t) - r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1),\\
\dot{x}_2(t) &= r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1) - d_{12}x_2(t) - b_1(t)x_2^2(t) - c_1(t)x_2(t)y_2(t),\\
\dot{y}_1(t) &= r_2(t)y_2(t) - d_{22}y_1(t) - r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2),\\
\dot{y}_2(t) &= r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2) - d_{21}y_2(t) - b_2(t)y_2^2(t) + c_2(t)y_2(t)x_2(t),
\end{aligned}
\tag{1.6}
$$

where $x_1(t)$ and $x_2(t)$ denote the densities of the immature and mature prey species at time t, respectively; $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities of predator species at time t, respectively; $r_i(t)$, $b_i(t)$, $c_i(t)(i=1,2)$ are all continuous functions bounded above and below by positive constants for all $t \geq 0$. $d_{ij}, \tau_i, i, j = 1,2$ are all positive constants, d_{12} and d_{21} are the death rate of the mature prey species and mature predator species, respectively. The main different assumption between the works of Ding and Cheng [27] and Chen *et al.* [2] is that the latter one considering the death rate of the mature species. By using the comparison theorem of differential equation, they investigated the partial survival and extinction property of the system [2]. Their results showed that that small birth rate of the immature prey and predator species and large death rate of the mature prey and predator species will lead to the broken of the system, in the sense that both prey and predator species are will be driven to extinction. They also showed that for system (1.6), the extinction of prey species could not lead to the extinction of predator species, which seems very interesting and ridiculous.

Since the death rate of mature species plays important roles on the persistent property of system (1.6), it seems interesting to study a feedback control stage structure system with the death rate of mature spcies, this motivated us to propose and study the dynamic behaviors of system (1.1). We will focus on the stability property of the system, following are the main results of this paper:

Theorem 1.1. *Let $col(x_1(t), x_2(t), u(t))$ be a solution of (1.1) and (1.3). Assume that the coefficients of system (1.1) satisfy the inequality $\alpha e^{-\gamma\tau} \leq d$, the kernels $K_i(t), i = 1,2$ are positive functions and satisfy (1.2). Then the boundary equilibrium $E_0(0,0,0)$ of system* (1.1) *is globally* *attractive, that is,*

$$\lim_{t\to+\infty} x_i(t) = 0,\ \ i = 1,2,\ \ \lim_{t\to+\infty} u(t) = 0.$$

Theorem 1.2. *Let $col(x_1(t), x_2(t), u(t))$ be a solution of (1.1) and (1.3). Assume that the coefficients of system (1.1) satisfy the inequality (1.5) and $\alpha e^{-\gamma\tau} > d$, the kernels $K_i(t), i = 1,2$ are positive functions and satisfy (1.2). Then the unique interior equilibrium $E^*(x_1^*, x_2^*, u^*)$ of system* (1.1) *is globally attractive, that is,*

$$\lim_{t\to+\infty} x_i(t) = x_i^*,\ \ i = 1,2,\ \ \lim_{t\to+\infty} u(t) = u^*.$$

Remark 1.1. Theorem 1.1 shows that delays are harmless for the stability of the interior equilibrium point.

We will give a strict prove of this theorem in the next section. In Section 3, an example is presented to illustrate the feasibility of our main result.

2. Proof of the Main results

The equilibria of system (1.1) satisfies the following equations

$$\begin{cases} \alpha x_2 - \gamma x_1 - \alpha e^{-\gamma\tau} x_2 = 0, \\ \alpha e^{-\gamma\tau} x_2 - d x_2 - \beta x_2^2 - c x_2 u = 0, \\ -au + b x_2 = 0. \end{cases} \tag{2.1}$$

Solving above equations, the equilibria of system (1.1) are $E_0(0,0,0)$ and $E^*(x_1^*, x_2^*, u^*)$, where

$$x_1^* = \frac{a\alpha(\alpha e^{-\gamma\tau} - d)(1 - e^{-\gamma\tau})}{\gamma(a\beta + bc)},\ x_2^* = \frac{a(\alpha e^{-\gamma\tau} - d)}{a\beta + bc},\ u^* = \frac{b(\alpha e^{-\gamma\tau} - d)}{a\beta + bc}.$$

Obviously, if $\alpha e^{-\gamma\tau} > d$ hold, then E^* is positive equilibrium.

Concerned with the positive solution of system (2.1), similarly to the analysis of Theorem 1 in [27], we have

Lemma 2.1. *Solutions of system (1.1) with initial condition (1.3) are positive for all $t > 0$.*

Lemma 2.2 ([29]) *Consider the following equations:*

$$\begin{aligned} x'(t) &= bx(t-\delta) - a_1 x(t) - a_2 x^2(t), \\ x(t) &= \phi(t) > 0,\ -\delta \le t \le 0, \end{aligned}$$

and assume that $b, a_2 > 0, a_1 \ge 0$ *and* $\delta \ge 0$ *are constants, then:*
(i) If $b \ge a_1$*, then* $\lim\limits_{t\to+\infty} x(t) = \dfrac{b-a_1}{a_2}$;
(ii) If $b \le a_1$*, then* $\lim\limits_{t\to+\infty} x(t) = 0$.

Following Lemma 2.1 is Lemma 3 of Francisco Montes de Oca and Miguel Vivas [30].

Lemma 2.3. *Let* $x : R \to R$ *be a bounded nonnegative continuous function, and let* $k : [0,+\infty) \to (0,+\infty)$ *be a continuous kernel such that* $\int_0^\infty k(s)ds = 1$. *Then*

$$\begin{aligned} \liminf_{t\to+\infty} x(t) &\le \liminf_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)ds \\ &\le \limsup_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)ds \le \limsup_{t\to+\infty} x(t). \end{aligned}$$

As a direct corollary of Lemma 2.2 of Chen, Yang and Chen [18], we have

Lemma 2.4. *Suppose that*

$$\frac{dx}{dt} \le -ax + b,$$

where a, b *are positive constants, then*

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}.$$

Suppose that

$$\frac{dx}{dt} \ge -ax + b,$$

where a, b *are positive constants, then*

$$\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}.$$

Proof of Theorem 1.1. By the second equation of system (1.1), we have

$$\dot{x}_2(t) \le \alpha e^{-\gamma\tau} x_2(t-\tau) - dx_2 - \beta x_2^2(t). \tag{2.2}$$

Since $\alpha e^{-\gamma\tau} \leq d$, it follows Lemma 2.2 (ii) that

$$\lim_{t\to+\infty} x_2(t) = 0. \tag{2.3}$$

From (2.3) and Lemma 2.3 that

$$\lim_{t\to+\infty} \int_{-\infty}^{t} K_2(t-s)x_2(s)ds = 0. \tag{2.4}$$

therefore, for any enough small positive constant $\varepsilon > 0$, there exists T such that

$$\int_{-\infty}^{t} K_2(t-s)x_2(s)ds < \varepsilon,\ \ x_2(t) < \varepsilon \ \text{ for all } \ t \geq T. \tag{2.5}$$

Thus, for $t > T$, it follows from the third equation of system (1.1) that

$$\dot{u}(t) \leq -au(t) + b\varepsilon. \tag{2.6}$$

Applying Lemma 2.4 to above differential inequality leads to

$$\limsup_{t\to+\infty} u(t) \leq \frac{b\varepsilon}{a}. \tag{2.7}$$

On the other hand, from the positivity of $u(t)$, one has

$$\liminf_{t\to+\infty} u(t) \geq 0. \tag{2.8}$$

Setting $\varepsilon \to 0$, from (2.7) and (2.8) we have

$$\lim_{t\to+\infty} u(t) = 0. \tag{2.9}$$

Noting that the first equation of system (1.1) is equivalent to

$$x_1(t) = \int_{t-\tau}^{t} \alpha e^{-\gamma(t-s)} x_2(s)ds, \tag{2.10}$$

from (2.9) and (2.5), similarly to the analysis of (2.6)-(2.9), one could easily see that

$$\lim_{t\to+\infty} x_1(t) = 0. \tag{2.11}$$

(2.3), (2.9) and (2.11) show that $E_0(0,0,0)$ is globally attractive. This ends the proof of Theorem 1.1.

Proof of Theorem 1.2. Condition (1.5) together with $\alpha e^{-\gamma\tau} > d$ leads to

$$\alpha e^{-\gamma\tau} - d > c \cdot \frac{b}{a} \cdot \frac{(\alpha e^{-\gamma\tau} - d)}{\beta},$$

and so there exists a enough small positive constant $\varepsilon > 0$ such that

$$m_1^{(1)} \overset{\text{def}}{=} \frac{\alpha e^{-\gamma\tau} - d - c\Big(\frac{b}{a}\big(\frac{(\alpha e^{-\gamma\tau}-d)}{\beta} + \varepsilon\big) + \varepsilon\Big)}{\beta} - \varepsilon > 0; \tag{2.12}$$

$$m_2^{(1)} \overset{\text{def}}{=} \frac{b}{a} m_1^{(1)} - \varepsilon > 0. \tag{2.13}$$

By the second equation of system (1.1), we have

$$\dot{x}_2(t) \le \alpha e^{-\gamma\tau} x_2(t-\tau) - d x_2(t) - \beta x_2^2(t).$$

From Lemma 2.2, it follows that

$$\limsup_{t\to+\infty} x_2(t) \le \frac{\alpha e^{-\gamma\tau} - d}{\beta}. \tag{2.14}$$

From (2.14) and Lemma 2.3 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_2(t-s) x_2(s) ds \le \limsup_{t\to+\infty} x_2(t) \le \frac{\alpha e^{-\gamma\tau} - d}{\beta}. \tag{2.15}$$

Hence, for $\varepsilon > 0$ defined by (2.12), it follows from (2.14)-(2.15) that there exists a $T_1' > 0$ such that

$$\begin{aligned} & x_2(t) < \frac{\alpha e^{-\gamma\tau} - d}{\beta} + \varepsilon \overset{\text{def}}{=} M_1^{(1)}, \ \text{ for } \ t > T_1'; \\ & \int_{-\infty}^{t} K_2(t-s) x_2(s) ds < \frac{\alpha e^{-\gamma\tau} - d}{\beta} + \varepsilon \overset{\text{def}}{=} M_1^{(1)}, \ \text{ for } \ t > T_1'. \end{aligned} \tag{2.16}$$

By the third equation and (2.16), we have

$$\begin{aligned} \dot{u}(t) &= -au(t) + b\int_{-\infty}^{t} K_2(t-s) x_2(s) ds \\ &\le -au(t) + bM_1^{(1)}, \ \text{for all } \ t \ge T_1'. \end{aligned} \tag{2.17}$$

From Lemma 2.4 it follows that

$$\limsup_{t\to+\infty} u(t) \le \frac{bM_1^{(1)}}{a}. \tag{2.18}$$

From (2.18) and Lemma 2.3, we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s) u(s) ds \le \limsup_{t\to+\infty} u(t) \le \frac{bM_1^{(1)}}{a}. \tag{2.19}$$

Hence, for $\varepsilon > 0$ defined by (2.12), it follows from (2.18)-(2.19) that there exists a $T_1 > T_1'$ such that

$$
\begin{aligned}
&u(t) < \frac{bM_1^{(1)}}{a} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)}, \ \text{ for } \ t > T_1; \\
&\int_{-\infty}^{t} K_1(t-s)u(s)ds < \frac{bM_1^{(1)}}{a} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)}, \ \text{ for } \ t > T_1.
\end{aligned} \tag{2.20}
$$

For $t > T_1$, from the second equation of (1.1) and (2.20), we have

$$
\begin{aligned}
\dot{x}_2(t) &= \alpha e^{-\gamma\tau} x_2(t-\tau) - dx_2(t) - \beta x_2^2(t) - cx_2(t)\int_{-\infty}^{t} K_1(t-s)u(s)ds \\
&\geq \alpha e^{-\gamma\tau} x_2(t-\tau) - dx_2(t) - \beta x_2^2(t) - cM_2^{(1)} x_2(t).
\end{aligned} \tag{2.21}
$$

From (2.12) one could see that $\alpha e^{-\gamma\tau} > cM_2^{(1)}$, therefore, applying Lemma 2.2 to (2.21) leads to

$$
\liminf_{t\to+\infty} x_2(t) \geq \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(1)}}{\beta}. \tag{2.22}
$$

From (2.22) and Lemma 2.3, we have

$$
\liminf_{t\to+\infty} \textstyle\int_{-\infty}^{t} K_2(t-s)x_2(s)ds \geq \liminf_{t\to+\infty} x_2(t) \geq \dfrac{\alpha e^{-\gamma\tau} - d - cM_2^{(1)}}{\beta}. \tag{2.23}
$$

That is, for $\varepsilon > 0$ be defined by (2.12), there exists a $T_2' > T_1$ such that

$$
\begin{aligned}
&x_2(t) > \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(1)}}{\beta} - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)} > 0, \ \text{ for } \ t > T_2'; \\
&\int_{-\infty}^{t} K_2(t-s)x_2(s)ds > \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(1)}}{\beta} - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)} > 0, \ \text{for } \ t > T_2'.
\end{aligned} \tag{2.24}
$$

It follows from (2.24) and the third equation of system (1.1) that

$$
\begin{aligned}
\dot{u}(t) &= -au(t) + b\int_{-\infty}^{t} K_2(t-s)x_2(s)ds \\
&\geq -au(t) + bm_1^{(1)}, \ \text{ for } \ t > T_2'.
\end{aligned} \tag{2.25}
$$

Therefore, by Lemma 2.4 and (2.25), we have

$$
\liminf_{t\to+\infty} u(t) \geq \frac{bm_1^{(1)}}{a}. \tag{2.26}
$$

From (2.26) and Lemma 2.3, we have

$$
\liminf_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s)u(t-s)ds \geq \liminf_{t\to+\infty} u(t) \geq \frac{bm_1^{(1)}}{a}. \tag{2.27}
$$

That is, for $\varepsilon > 0$ be defined by (2.13), there exists a $T_2 > T_2'$ such that

$$\begin{aligned} & u(t) > \frac{bm_1^{(1)}}{a} - \varepsilon \overset{\text{def}}{=} m_2^{(1)} > 0, \ \text{ for } \ t > T_2; \\ & \int_{-\infty}^{t} K_1(t-s)u(s)ds > \frac{bm_1^{(1)}}{a} - \varepsilon \overset{\text{def}}{=} m_2^{(1)} > 0, \ \text{ for } \ t > T_2. \end{aligned} \tag{2.28}$$

From (2.28) and the second equation of system (1.1), we have

$$\begin{aligned} \dot{x}_2(t) &= \alpha e^{-\gamma\tau} x_2(t-\tau) - dx_2(t) - \beta x_2^2(t) - cx_2(t) \int_{-\infty}^{t} K_1(t-s)u(s)ds \\ &\le \alpha e^{-\gamma\tau} x_2(t-\tau) - dx_2(t) - \beta x_2^2(t) - cm_2^{(1)} x_2(t), \text{ for } \ t > T_2. \end{aligned} \tag{2.29}$$

It follows from (2.12) and (2.22) that

$$\alpha e^{-\gamma\tau} - d - cm_2^{(1)} > 0. \tag{2.30}$$

Therefore, by applying Lemma 2.2 to (2.29), it follows that

$$\limsup_{t\to+\infty} x_2(t) \le \frac{\alpha e^{-\gamma\tau} - d - cm_2^{(1)}}{\beta}. \tag{2.31}$$

From (2.31) and Lemma 2.3 we have

$$\limsup_{t\to+\infty} \textstyle\int_{-\infty}^{t} K_2(t-s)x_2(s)ds \le \limsup\limits_{t\to+\infty} x_2(t) \le \dfrac{\alpha e^{-\gamma\tau} - d - cm_2^{(1)}}{\beta}. \tag{2.32}$$

Hence, for $\varepsilon > 0$ be defined by (2.12), there exists a $T_3' > T_2$ such that

$$\begin{aligned} & x_2(t) < \frac{\alpha e^{-\gamma\tau} - d - cm_2^{(1)}}{\beta} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)}, \ \text{ for } \ t > T_3'; \\ & \int_{-\infty}^{t} K_2(t-s)x_2(s)ds < \frac{\alpha e^{-\gamma\tau} - d - cm_2^{(1)}}{\beta} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)}, \ \text{ for } \ t > T_3'. \end{aligned} \tag{2.33}$$

(2.33) together with the third equation of system (1.1) leads to

$$\dot{u}(t) \ \le \ -au(t) + bM_1^{(2)}, \text{ for all } \ t \ge T_3'. \tag{2.34}$$

From Lemma 2.4 it follows that

$$\limsup_{t\to+\infty} u(t) \le \frac{bM_1^{(2)}}{a}. \tag{2.35}$$

From (2.35) and Lemma 2.3 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s)u(s)ds \le \frac{bM_1^{(2)}}{a}. \tag{2.36}$$

Hence, for $\varepsilon > 0$ be defined by (2.12), it follows from (2.35)-(2.36) that there exists a $T_3 > T_3'$ such that

$$\begin{aligned} &u(t) < \frac{bM_1^{(2)}}{a} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)}, \text{ for } t > T_3; \\ &\int_{-\infty}^{t} K_1(t-s)u(s)ds < \frac{bM_1^{(2)}}{a} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)}, \text{ for } t > T_3. \end{aligned} \tag{2.37}$$

For $t > T_3$, from the second equation of (1.1) and (2.37), we have

$$\dot{x}_2(t) \geq \alpha e^{-\gamma\tau} x_2(t-\tau) - d - \beta x_2^2(t) - cM_2^{(2)} x_2(t). \tag{2.38}$$

From (2.12), $M_1^{(2)} < M_1^{(1)}$, and the definition of $M_2^{(2)}$, one could see that $\alpha e^{-\gamma\tau} > cM_2^{(2)}$, Therefore, by applying Lemma 2.2 to (2.38), it follows that

$$\liminf_{t\to+\infty} x_2(t) \geq \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(2)}}{\beta}. \tag{2.39}$$

From (2.39) and Lemma 2.3, we have

$$\liminf_{t\to+\infty} \int_{-\infty}^{t} K_2(t-s)x_2(s)ds \geq \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(2)}}{\beta}. \tag{2.40}$$

That is, for $\varepsilon > 0$ be defined by (2.12), there exists a $T_4' > T_3$ such that

$$\begin{aligned} &x_2(t) > \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(2)}}{\beta} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)} > 0, \text{ for } t > T_4'; \\ &\int_{-\infty}^{t} K_2(t-s)x_2(s)ds > \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(2)}}{\beta} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)} > 0, \text{ for } t > T_4'. \end{aligned} \tag{2.41}$$

It follows from (2.41) and the third equation of system (1.1) that

$$\dot{u}(t) \geq -au(t) + bm_1^{(2)}, \text{ for } t > T_4'. \tag{2.42}$$

Therefore, by Lemma 2.4 and (2.13), we have

$$\liminf_{t\to+\infty} u(t) \geq \frac{bm_1^{(2)}}{a}. \tag{2.43}$$

From (2.43) and Lemma 2.3, we have

$$\liminf_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s)u(s)ds \geq \frac{bm_1^{(2)}}{a}. \tag{2.44}$$

That is, for $\varepsilon > 0$ defined by (2.13), there exists a $T_4 > T_4^{'}$ such that

$$\begin{aligned} & u(t) > \frac{bm_1^{(2)}}{a} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)} > 0, \text{ for } t > T_4; \\ & \int_{-\infty}^{t} K_1(t-s)u(s)ds > \frac{bm_1^{(2)}}{a} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)} > 0, \text{ for } t > T_4. \end{aligned} \tag{2.45}$$

Obviously,

$$\begin{aligned} & M_1^{(2)} = \frac{\alpha e^{-\gamma\tau} - d - cm_2^{(1)}}{\beta} + \frac{\varepsilon}{2} < \frac{\alpha e^{-\gamma\tau}}{\beta} + \varepsilon = M_1^{(1)}; \\ & M_2^{(2)} = \tfrac{bM_1^{(2)}}{a} + \tfrac{\varepsilon}{2} < \tfrac{bM_1^{(1)}}{a} + \varepsilon = M_2^{(1)}; \\ & m_1^{(2)} = \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(2)}}{\beta} - \frac{\varepsilon}{2} > \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(1)}}{\beta} - \varepsilon = m_1^{(1)}; \\ & m_2^{(2)} = \frac{bm_1^{(2)}}{a} - \frac{\varepsilon}{2} > \frac{bm_1^{(1)}}{a} - \varepsilon = m_2^{(1)}. \end{aligned} \tag{2.46}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}, i = 1,2, n = 1,2,\cdots$, such that for $n \geq 2$

$$\begin{aligned} & M_1^{(n)} = \frac{\alpha e^{-\gamma\tau} - d - cm_2^{(n-1)}}{\beta} + \frac{\varepsilon}{n}; \\ & M_2^{(n)} = \tfrac{bM_1^{(n)}}{a} + \tfrac{\varepsilon}{n}; \\ & m_1^{(n)} = \frac{\alpha e^{-\gamma\tau} - d - cM_2^{(n)}}{\beta} - \frac{\varepsilon}{n}; \\ & m_2^{(n)} = \frac{bm_1^{(n)}}{a} - \frac{\varepsilon}{n}. \end{aligned} \tag{2.47}$$

By induction, one could show that sequences $M_i^{(n)}, i = 1,2$ are strictly decreasing, and sequences $m_i^{(n)}, i = 1,2$ are strictly increasing. Also

$$m_1^{(n)} < x_2(t) < M_1^{(n)}, \;\; m_2^{(n)} < u(t) < M_2^{(n)}, \quad \text{for } t \geq T_{2n}, \;\; i = 1,2.$$

Therefore,

$$\lim_{t\to+\infty} M_1^{(n)} = \bar{x}_2, \quad \lim_{t\to+\infty} M_2^{(n)} = \bar{u}, \;\; \lim_{t\to+\infty} m_1^{(n)} = \underline{x}_2, \quad \lim_{t\to+\infty} m_2^{(n)} = \underline{u}. \tag{2.48}$$

Letting $n \to +\infty$ in (2.47), we obtain

$$\begin{aligned}
&\beta\overline{x}_2 = \alpha e^{-\gamma\tau} - d - c\underline{u};\\
&\overline{u} = \tfrac{b}{a}\overline{x}_2;\\
&\beta\underline{x}_2 = \alpha e^{-\gamma\tau} - d - c\overline{u};\\
&\underline{u} = \tfrac{b}{a}\underline{x}_2.
\end{aligned} \tag{2.49}$$

Solving (2.49), we obtain

$$\overline{x}_2 = \underline{x}_2 = x_2^* = \frac{a(\alpha e^{-\gamma\tau} - d)}{a\beta + bc},\ \overline{u} = \underline{u} = u^* = \frac{b(\alpha e^{-\gamma\tau} - d)}{a\beta + bc},$$

that is

$$\lim_{t\to+\infty} x_2(t) = x_2^* \quad \lim_{t\to+\infty} u(t) = u^*. \tag{2.50}$$

Noting that the first equation of system (1.1) is equivalent to

$$x_1(t) = \int_{t-\tau}^{t} \alpha e^{-\gamma(t-s)} x_2(s)ds, \tag{2.51}$$

from (2.50)-(2.51), one could easily see that

$$\lim_{t\to+\infty} x_1(t) = x_1^* = \frac{a\alpha(\alpha e^{-\gamma\tau} - d)(1 - e^{-\gamma\tau})}{\gamma(a\beta + bc)}. \tag{2.52}$$

Thus, the unique interior equilibrium $E^*(x_1^*, x_2^*, u^*)$ is globally attractive. This completes the proof of Theorem 1.1.

3. Examples

The following example shows the feasibility of our main result.

Example 3.1. Consider the following system

$$\begin{aligned}
\dot{x}_1(t) &= 2x_2(t) - x_1(t) - 2e^{-1}x_2(t-1),\\
\dot{x}_2(t) &= 2e^{-1}x_2(t-1) - dx_2(t) - x_2^2(t) - x_2(t)\int_{-\infty}^{t} e^{-(t-s)}u(s)ds,\\
\dot{u}(t) &= -3u(t) + \int_{-\infty}^{t} e^{-(t-s)}x_2(s)ds.
\end{aligned} \tag{3.1}$$

Here, corresponding to system (1.1), we set $\alpha = 2, \gamma = \beta = b = c = \tau = 1, a = 3,\ K_i(t) = e^{-t}$. One could easily see that $a\beta = 3 > 1 = bc$. Obviously (1.5) holds, then it follows from Theorems 1.1 that if $d \geq 2e^{-1}$ hold, $E_0(0,0,0)$ is globally attractive, and if $d < 2e^{-1}$ hold, then from Theorem 1.2 the unique interior equilibrium $E^*\left(\frac{3(1-e^{-1})}{e}, \frac{3}{2}e^{-1}, \frac{1}{2}e^{-1}\right)$ of system (3.1) is globally attractive.

4. Discussion

Stimulated by the works of Ding and Cheng[27] and Chen et al[31, 32, 33], we propose a single species model with feedback control and infinite delays. The main difference between the model of [27] and system (1.1) is that we incorporate the death rate of mature species to the system.

Theorem 1.1 and 1.2 show that the death rate of mature species plays important role on the persistent and stability property of the system. Indeed, Ding and Cheng [27] had showed that for the system (1.4), $E_0(0,0,0)$ is unstable, which means that the species could not be driven to extinction. However, our result Theorem 1.1 shows that if the death rate d of mature species is enough large, then the system will outbreak in the sense that the species will be driven to the extinction.

It is also interesting that from (2.1) we know that condition $\alpha e^{-\gamma\tau} > d$ is needed to ensure the existence of the positive equilibrium. Then the essential condition of Theorem 1.2 is (1.5), which is coincidence with the condition to ensure the global attractivity of the positive equilibrium of system (1.4). It's in this sense that our Theorem 1.2 can be seen as the generalization of the main result of Ding and Cheng [27] to the infinite delay case.

Conflict of Interests

The authors declare that there is no conflict of interests.

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