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RELIABILITY AND AVAILABILITY ANALYSIS OF A 2-STATE REPAIRABLE SYSTEM WITH TWO TYPES OF FAILURE

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Abstract: This paper deals with a 2- state repairable complex system with two types of failure. Let failure rate and repair rate of [type1, type2] components are assumed to be exponentially distributed. The expressions of availability and reliability characteristics such as the system reliability, and the mean time to failure are derived. We used several cases to analysis graphically the effect of various system parameters on the availability system, reliability system and mean time to failure. We also investigate the sensitivity analysis for the system reliability with changes in a specific value of the system parameters.

Keywords: Availability, Reliability, Mean time to failure, Sensitivity analysis.

2010 Mathematics Subject Classification: 93A30.

1. Introduction

Studding the reliability of machine repair problem is very important in our life because it is widely used in industrial system and manufacturing system .any system becomes unreliable due to various reasons. In the traditional systems, the units of the system have only two states up and down. However, in many situations the units of the system can have finite number of states. Most reliability models assume that the up and down times of the components are exponentially distributed. This assumption leads to a Markovian model with constant transition rates. The analysis in such cases is relatively simple and the numerical results can be easily obtained.

In [1] stochastic analysis of a repairable system with three units and two repair facilities was introduced. In [2], reliability characteristic of cold-standby redundant system was introduced. In

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[3], some reliability parameters of a three state repairable system with environmental failure were evaluated. In [4], human error and common-cause failure modeling was established for a two-unit multiple system. In [5], reliability modeling of 2-out-of-3 redundant system is introduced subject to degradation after repair. Recently Reliability and availability analysis of a standby repairable system with degradation facility were studied in [6]. In. [7] Reliability measures of a degradable system with standby switching failures and reboot delay were introduced. In. [8] Reliability and availability analysis of a standby repairable system with two types of failure were studied. In this paper, we consider a system consists of a 2-state repairable unit with two types of failures. We develop the explicit expressions for availability function, reliability function and mean time to failure using Laplace transform techniques then we perform a parametric investigation which presents numerical results to analyze the effects of the various system parameters on the system reliability and system availability.

1.1.Notations

λ_1: the failure rate of type1.

λ_2 : the failure rate of type2.

μ_1 : the repair rate of type1.

μ_2 : the repair rate of type2.

$P_i(t)$: Probability for i = 0, 1, 2

0→normal state

1→failed state due to failure rate of type1.

2→failed state due to failure rate of type2.

$P_i^*(s)$: Laplace transform of $P_i(t)$.

$A(t)$: availability function of the system.

$R(t)$: reliability function of the system.

$MTTF$: mean time to system failure.

Laplace Transform of $P_i(t)$ are defined as:

$$P_i^*(s) = \int_0^\infty e^{-st} P_i(t)\,dt, i = 0,1,2.$$

1.2 Model description

We consider a system consists of a 2-state repairable unit with two types of failures. Let failure times of type1 and type2 are assumed to be exponentially distributed with parameter λ_1 and λ_2 respectively, repair rates oftype1 and type2 are assumed to be exponentially distributed with parameters μ_1 and μ_2 respectively. We also investigate the sensitivity analysis for the system reliability with changes in a specific value of the system parameters.

In this paper various states probabilities have been evaluated in the form of Laplace transform. The expressions of availability and reliability characteristics will be obtained in addition to we perform sensitivity analysis of system reliability with respected to system parameters.

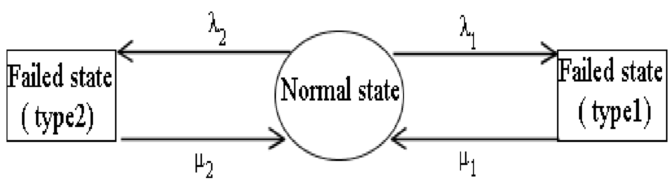

Figure1.System configuration diagram.

1.3. Mathematical formulation of the model

According to System configuration diagram in Fig.1, the difference-differential equations for this stochastic process which is continuous in time and discrete in space are given as follows.

$$\frac{dP_0(t)}{dt} = -[\lambda_1 + \lambda_2]P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \tag{1}$$

$$\frac{dP_1(t)}{dt} = -\mu_1 P_1(t) + \lambda_1 P_0(t) \tag{2}$$

$$\frac{dP_2(t)}{dt} = -\mu_2 P_2(t) + \lambda_2 P_0(t) \tag{3}$$

Initial conditions:

$$P_i(0) = \begin{cases} 1, & where \quad i = 0 \\ 0 & otherwise \end{cases}$$

Taking Laplace transform of equations $(1)-(3)$, we get

$$(\lambda_1 + \lambda_2 + s)P_0^*(s) - \mu_1 P_1^*(s) - \mu_2 P_2^*(s) = P_0(0) \tag{4}$$

$$(\mu_1 + s)P_1^*(s) - \lambda_1 P_0^*(s) = P_1(0) \tag{5}$$

$$(\mu_2 + s)P_2^*(s) - \lambda_2 P_0^*(s) = P_2(0) \tag{6}$$

Solving Equations $(4-6)$, we obtain:

$$P_0^*(s)=\frac{(s+\mu_2)(s+\mu_1)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (7)$$

$$P_1^*(s)=\frac{\lambda_1(s+\mu_2)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (8)$$

$$P_2^*(s)=\frac{\lambda_2(s+\mu_1)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (9)$$

By taking inverse Laplace transform of equations $(7-9)$, we get

$$P_0(t)=\frac{1}{d}\left[b+e^{-\frac{1}{2}tc}\left[(d-b)\cosh(\frac{1}{2}t\sqrt{c^2-4d})+\frac{(2ad-dc-cb)\sinh(\frac{1}{2}t\sqrt{c^2-4d})}{\sqrt{c^2-4d})}\right]\right] \qquad (10)$$

$where\ \ a=\mu_1+\mu_2\ ,\ b=\mu_1\mu_2\ ,\ c=\lambda_1+\lambda_2+\mu_1+\mu_2\ ,d=\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2.$

2. System Availability and Reliability

2.1. Availability analysis of the system

We find that $A(t)=\frac{1}{d}\left[b+e^{-\frac{1}{2}tc}\left[(d-b)\cosh(\frac{1}{2}t\sqrt{c^2-4d})+\frac{(2ad-dc-cb)\sinh(\frac{1}{2}t\sqrt{c^2-4d})}{\sqrt{c^2-4d}}\right]\right]$ (11)

The steady-state availability can be obtained from the following relation

$$A=\lim_{t\to\infty}A(t)=\frac{\mu_1\mu_2}{\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2} \qquad (12)$$

Special cases:

1- When $\lambda_1=0$ we find $A(t)=\frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}+\mu_2}{\lambda_2+\mu_2}$ and $A=\frac{\mu_2}{\lambda_2+\mu_2}$.

2- When $\lambda_2=0$ we find $A(t)=\frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}+\mu_1}{\lambda_1+\mu_1}$ and $A=\frac{\mu_1}{\lambda_1+\mu_1}$.

2.2. Reliability analysis of the system

To obtain the reliability function for this model, we assume that at least one of failed states [type1, type2] is absorbing state and the transition rate from this state equal to zero.

$$R(t)=\frac{1}{d}\left[b+e^{-\frac{1}{2}tc}\left[(d-b)\cosh(\frac{1}{2}t\sqrt{c^2-4d})+\frac{(2ad-dc-cb)\sinh(\frac{1}{2}t\sqrt{c^2-4d})}{\sqrt{c^2-4d})}\right]\right] \qquad (13)$$

As we know, we have two failed states this lead to three cases of reliability function.

1- Failed state [type1] is absorbing when $\mu_1 = 0$.

2- Failed state [type2] is absorbing when $\mu_2 = 0$.

3- Failed states [type1, type2] are absorbing when $\mu_1 = \mu_2 = 0$.

2.3. The mean time to failure

The mean time to system failure $MTTF$ can be obtained from the following relation.

$$MTTF = \int_0^\infty R(t)dt \qquad (14)$$

As mention above, reliability function has three cases so we find $MTTF$ has following cases:

1- When $\mu_1 = 0$ we find $MTTF = \frac{1}{\lambda_1}$.

2- When $\mu_2 = 0$ we find $MTTF = \frac{1}{\lambda_2}$.

3- When $\mu_1 = \mu_2 = 0$ we find $MTTF = \frac{1}{\lambda_1 + \lambda_2}$.

It is noticed that $MTTF$ depend only on failure rate that doesn't has repair.

2.4. Sensitivity analysis

In this section we use numerical example to perform sensitivity analysis for changes in the reliability of the system $R(t)$ from changes in system parameters $\lambda_1, \lambda_2, \mu_1$ and μ_2 in three cases of reliability.

1- When $\mu_1 = 0$

We setting λ_1=.0007, λ_2=.001, μ_1=0, μ_2=.03, Fig.2 reveals that the sensitivities of λ_2 and μ_2 on the $R(t)$, we observe that the sensitivity of λ_2 reverse the sign from negative to positive nearly at the same certain time when the sensitivity of μ_2 reverse the sign from positive to negative. We also observe that the influence of λ_1 on $R(t)$ be negative from Figure 3.

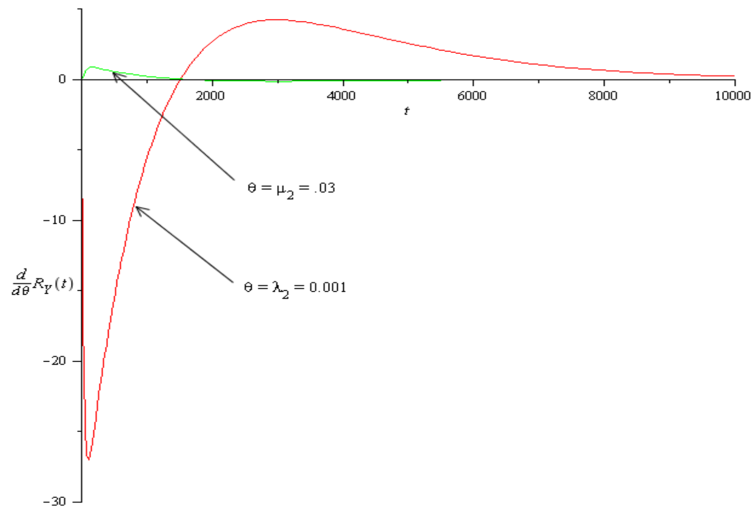

Figure 2.Sensitivity of system reliability with respect to μ_2 and λ_2 .

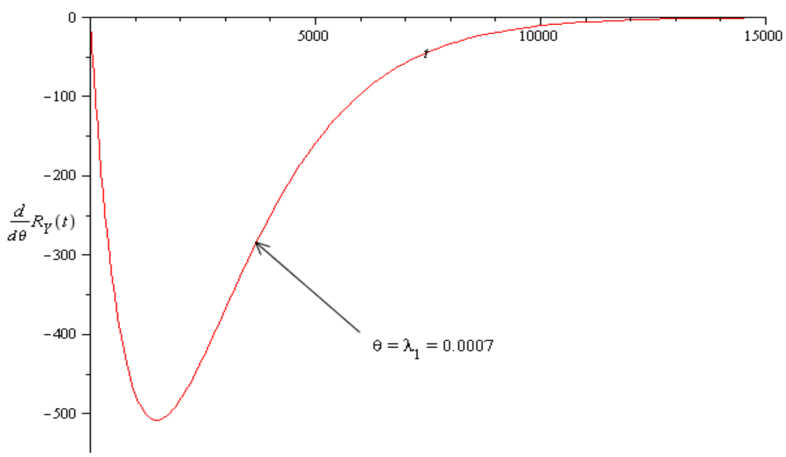

Figure 3.Sensitivity of system reliability with respect to λ_1 .

2- When $\mu_2 = 0$

We setting λ_1=.0007, λ_2=.001, μ_1=.05, μ_2=0, Fig.4 reveals that the sensitivities of λ_1 and μ_1 on the $R(t)$ have the same sensitivities of λ_2 and μ_2 on the $R(t)$ when $\mu_1 = 0$. From Fig.5 the influence of λ_2 on $R(t)$ has the same influence of λ_1 on $R(t)$ when $\mu_1 = 0$.

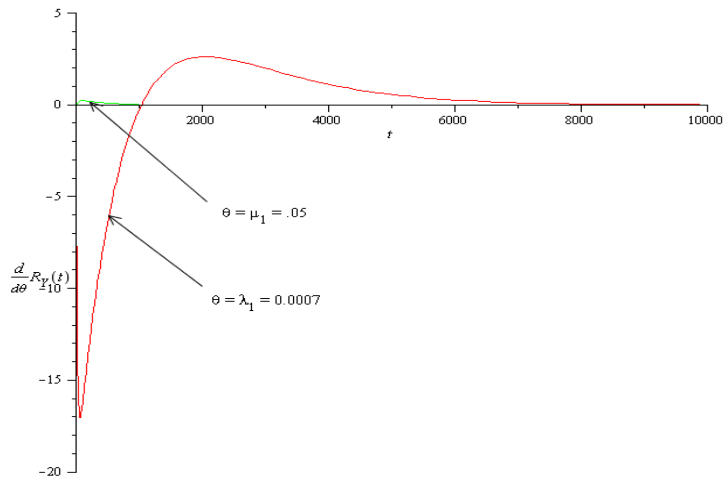

Figure 4.Sensitivity of system reliability with respect to μ_1 and λ_1.

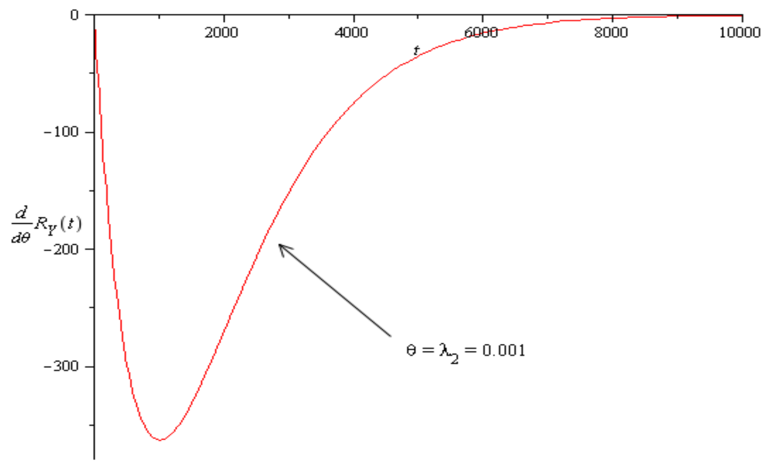

Figure 5.Sensitivity of system reliability with respect to λ_2.

3- When $\mu_1 = \mu_2 = 0$

We setting λ_1 =.0007, λ_2 =.001, μ_1 =0, μ_2 =0, in Fig.6 it is noticed that λ_1 and λ_2 have the same impact during time.

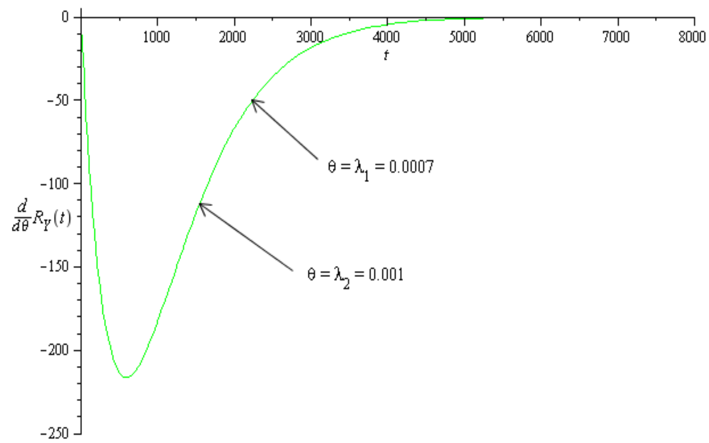

Figure 6.Sensitivity of system reliability with respect to λ_1 and λ_2 .

4. Conclusions

In this paper, a mathematical model was constructed for a repairable system with two types of failure. Availability, reliability and mean time to failure in addition to sensitivity analysis for the system reliability were obtained and the results were shown graphically by the aid of MAPLE program. Results indicate that the *MTTF* and sensitivity analysis for the system reliability depend on which of failed states [type1, type2] is absorbing.

Conflict of Interests

The authors declare that there is no conflict of interests.

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