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PRIORITY DISCIPLINED QUEUING MODELS WITH FUZZY PARAMETERS

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Abstract. This work constructs the membership functions of the system characteristics of Priority Disciplined Queuing models with Fuzzy parameters (preemptive priority, non preemptive priority) in which the arrival rate, service rate are fuzzy numbers. The α – cut approach is used to transform a fuzzy queue into a family of conventional crisp queues in this context. Approximate method of Extension is used to define membership functions of the performance measures of priority queuing system. We propose a fuzzy nature in *FMi/FFi/1* queuing system and arrivals and services from a single server follows a Poisson process with fuzzy parameters. In this paper we study the models with different service times across different classes. Numerical example is illustrated to check the validity of the proposed method.

Keywords: fuzzy subset theory; Queuing theory; priority- discipline; fuzzy system model.

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1. Introduction

Queuing models have wider applications in service organization as well as

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manufacturing firms, in that various customers are serviced by various types of servers according to specific queue discipline [1] within the context of traditional queuing theory, the inter arrival times and service times are required to follow certain distributions. Queuing models considered have had the property that unit proceed to service on a first come - first served basis. This is obviously not only the manner of service and there are many alternatives such as last come - first served, selection in random order and selection by priority.

In priority schemes, customers with the highest priority are selected for service ahead of those with lower priority, independent of their time of arrival into the system. There are two further refinements possible in priority situation, namely preemption and non-preemption. In preemptive cases the customer with the highest priority is allowed to enter service immediately even if another with lower priority is already present in service when the higher customer arrives to the system. In addition, a decision has to be made whether to continue the preempted customers service from the point of preemption when resumed or to start a new. The priority discipline is said to be non-preemptive if there is no interruption and the highest priority customer just goes to the head of the queue to wait his turn.

In practical, the priority queuing model, the input data arrival rate, service rate are uncertainly known. Uncertainty is resolved by using fuzzy set theory. Hence the classical queuing model with priority discipline will have more application if it is expanded using fuzzy models.

Fuzzy queuing models have been described by such researchers like Li and Lee [12], Buckley[3], Negi and Lee [13], Kao et al [11], Chen [5,6] are analyzed fuzzy queues using Zadeh' s extension principle [16]. Kao et al constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming. Recently, Chen [5] developed (FM/FM/1) : (∞ / FCFS) and ($FM/FM^{[k]}/1$) : (∞ / FCFS). Also fuzzy priority disciplined models are described by Groenevelt, and Altman [8], Harrison and Zhang [9], Kao, Li and Chen[10].

2. Priority queuing models

Consider a priority queuing system with single server, infinite calling population, in which the rate of arrival λ and rate of service μ. To establish the priority discipline queuing model, we must compare the average total cost of inactivity for the three cases: no priority discipline, Preemption priority, non-preemptive priority discipline, which are denoted respectively by C, C^1 and C^2.

(a) No priority queuing model:

Average total cost of inactivity when there is no priority discipline C

C = ($C_1 \lambda_1$+ $C_2 \lambda_2$)Wq, where Wq = $\frac{\lambda\mu^2}{2(1-\rho)}$ with $\mu = 0.3\ \mu_1 + 0.7\ \mu_2$, $\rho = \lambda\mu < 1$.

(b) Preemption priority queuing model:

Average total cost of inactivity when there is Preemption priority C^1

$C^1 = \sum_{i=1}^{2} c_i \lambda_i w_{q,i} = c_1\lambda_1 w_{q,1} + c_2\lambda_2 w_{q,2}$ with $W_{q,1} = \frac{\lambda_1\mu_1^2 + \lambda_2\mu_2^2}{2(1-\sigma_1)(1-\sigma_2)}$,

$W_{q,2} = \frac{\lambda_1\mu_1^2 + \lambda_2\mu_2^2}{2(1-\sigma_2)}$, $\sigma_1 = \lambda_1\mu_1 + \lambda_2\mu_2 = \lambda\mu$ and $\sigma_2 = \lambda_2\mu_2$

(c)Non- Preemption priority queuing model:

Average total cost of inactivity when there is non- Preemption priority C^2

$C^2 = \sum_{i=1}^{2} c_i \lambda_i w_{q,i} = c_1\lambda_1 w_{q,1} + c_2\lambda_2 w_{q,2}$ with $W_{q,1} = \frac{\mu_1(1-\sigma_1) + \frac{\lambda_1\mu_1^2 + \lambda_2\mu_2^2}{2}}{2(1-\sigma_1)(1-\sigma_2)} - \mu_1$,

$W_{q,2} = \frac{\mu_2(1-\sigma_2) + \frac{\lambda_2\mu_2^2}{2}}{(1-\sigma_2)} - \mu_2$, $\sigma_1 = \lambda_1\mu_1 + \lambda_2\mu_2 = \lambda\mu$ and $\sigma_2 = \lambda_2\mu_2$

Comparison of the three total costs shows which of the priority discipline minimizes the average total cost function of inactivity.

3. Fuzzy priority queuing models

Fuzzy priority queues are described by fuzzy set theory. This paper develops fuzzy priority queuing model in which the input source arrival rate and service rate are uncertain parameters. To establish the priority discipline fuzzy queuing model, we must compare the average total cost of inactivity for the three cases: no priority discipline, Preemption priority, non-preemptive priority discipline, which are denoted respectively by $\tilde{C}$, $\tilde{C}^1$ and $\tilde{C}^2$. Approximate methods of extension are propagating fuzziness for continuous valued mapping determined the membership functions for the output variable. We followed the following interval analysis arithmetic for fuzzy operations.

Interval analysis arithmetic

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$I_1 = [a, b], a \leq b$; $I_2 = [c, d], c \leq d$.

Define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷] symbolically the operation.

$I_1 * I_2 = [a, b] * [c, d]$ represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$[a, b] + [c, d] = [a + c, b + d]$

$[a, b] - [c, d] = [a - d, b - c]$

$[a, b] \bullet [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$

$[a, b] \div [c, d] = [a, b] \bullet \left[\frac{1}{d}, \frac{1}{c}\right]$ provided that $0 \notin [c, d]$

$\alpha[a, b] = \begin{cases} [\alpha a, \alpha b] \text{ for } \alpha > 0 \\ [\alpha b, \alpha a] \text{ for } \alpha < 0 \end{cases}$ where ac, ad, bc, bd, are arithmetic products and $\frac{1}{d}, \frac{1}{c}$ are quotients.

4. Solution procedure

Decisions relating the priority discipline for a queuing system are mainly based for a cost function. $C_i = \sum_{i=1}^{p} C_i L_i$ Where Ci is the unit cost of inactivity for units in class i, L_i is the average length in the system for unit of class i.

Let us consider a queuing model with two unit classes arrive at α_1 of arrivals belong to one of the classes, and α_2 are in the other class. The average arrival rate at the system follows a Poisson process, is approximately known and is given by the triangular fuzzy number $\tilde{\lambda}$, the service rates from a single server are distributed according to the triangular fuzzy number $\tilde{\mu}_A$, $\tilde{\mu}_B$ with membership function $\mu_{\tilde{\lambda}}$, $\mu_{\tilde{\mu}}$ respectively.

$$\mu_{\tilde{\lambda}} = \begin{cases} \dfrac{\tilde{\lambda} - a}{b - a}, & a \le \tilde{\lambda} \le b \\ \dfrac{c - \tilde{\lambda}}{c - b}, & b \le \tilde{\lambda} \le c \\ 0 & else\ where \end{cases} \qquad \mu_{\tilde{\mu}} = \begin{cases} \dfrac{\tilde{\mu} - a_1}{b_1 - a_1}, & a_1 \le \tilde{\mu} \le b_1 \\ \dfrac{c_1 - \tilde{\mu}_1}{c_1 - b_1}, & b_1 \le \tilde{\mu} \le c_1 \\ 0 & elsewhere \end{cases}$$

The possible distribution of unit cost of inactivity for unit in the same class, in established by a triangular fuzzy numbers $\tilde{C}_A$, $\tilde{C}_B$ with membership function.

$$\mu_{\tilde{C}_A} = \begin{cases} \dfrac{\tilde{c}_A - a_2}{b_2 - a_2}, & a_2 \le \tilde{c}_A \le b_2 \\ \dfrac{c_2 - \tilde{c}_A}{c_2 - b_2}, & b_2 \le \tilde{c}_A \le c_2 \\ 0 & else\ where \end{cases} \qquad \mu_{\tilde{C}_B} = \begin{cases} \dfrac{\tilde{c}_B - a_3}{b_3 - a_3}, & a_3 \le \tilde{c}_B \le b_3 \\ \dfrac{c_3 - \tilde{c}_B}{c_3 - b_3}, & b_3 \le \tilde{c}_B \le c_3 \\ 0 & elsewhere \end{cases}$$

We choose three values of α viz, 0, 0.5 and 1. For instance when α = 0, we obtain four intervals as follows.

$\tilde{\lambda}_0$ = [a, c] ; $\tilde{\mu}_0$ = [a_1, c_1] ; $\tilde{C}_{A,0}$ = [a_2, c_2] ; $\tilde{C}_{B,0}$ = [a_3, c_3]

Similarly when, α = 0.5, 1, we obtain 8 intervals and it is denoted by $\tilde{\lambda}_{0.5}$, $\tilde{\mu}_{0.5}$,

$\tilde{C}_{A,0.5}$, $\tilde{C}_{B,0.5}$, $\tilde{\lambda}_1$, $\tilde{\mu}_1$, $\tilde{C}_{A,1}$, $\tilde{C}_{B,1}$.

The average total cost of inactivity in three situation (i) No priority discipline (ii) Preemptive priority discipline (ii) Non-preemptive priority discipline are calculate for different α level values. Interval arithmetic is used for computational efficiency.

(i) Average cost of inactivity when there is no priority discipline.

$$\tilde{C}_0 = (\tilde{C}_{1,0} \quad \tilde{\lambda}_{1,0} \quad + \quad \tilde{C}_{2,0} \quad \tilde{\lambda}_{2,0}) \left(\frac{\tilde{\lambda}_0 \tilde{\mu}_0{}^2}{2(1 - \tilde{\lambda}_0 \tilde{\mu}_0)} \right)$$

$$\tilde{C}_{0.5} = (\tilde{C}_{1,0.5} \; \tilde{\lambda}_{1,0.5} \quad + \quad \tilde{C}_{2,0.5} \quad \tilde{\lambda}_{2,0.5}) \left(\frac{\tilde{\lambda}_{0.5} \tilde{\mu}_{0.5}{}^2}{2(1 - \tilde{\lambda}_{0.5} \tilde{\mu}_{0.5})} \right)$$

$$\tilde{C}_1 = (\tilde{C}_{1,1} \quad \tilde{\lambda}_{1,1} \quad + \quad \tilde{C}_{2,1} \quad \tilde{\lambda}_{2,1}) \left(\frac{\tilde{\lambda}_1 \tilde{\mu}_1{}^2}{2(1 - \tilde{\lambda}_1 \tilde{\mu}_1)} \right)$$

(ii) Average total cost of inactivity when there is a preemptive discipline.

$$\tilde{\mathrm{C}}_0^1 = \tilde{c}_{1,0} \alpha_{1,0} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,0} \tilde{\mu}_{1,0}{}^2 + \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0}{}^2}{2(1 - \tilde{\lambda}_0 \tilde{\mu}_0)(1 - \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0})} \right) + \tilde{c}_{2,0} \alpha_{2,0} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,0} \tilde{\mu}_{1,0}{}^2 + \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0}{}^2}{2(1 - \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0})} \right)$$

$$\tilde{\mathrm{C}}_{0.5}^1 = \tilde{c}_{1,0.5} \alpha_{1,0.5} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,0.5} \tilde{\mu}_{1,0.5}{}^2 + \tilde{\lambda}_{2,0.5} \tilde{\mu}_{2,0.5}{}^2}{2(1 - \tilde{\lambda}_{0.5} \tilde{\mu}_{0.5})(1 - \tilde{\lambda}_{2,0.5} \tilde{\mu}_{2,0.5})} \right) + \tilde{c}_{2,0.5} \alpha_{2,0.5} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,0.5} \tilde{\mu}_{1,0.5}{}^2 + \tilde{\lambda}_{2,0.5} \tilde{\mu}_{2,0.5}{}^2}{2(1 - \tilde{\lambda}_{2,0.5} \tilde{\mu}_{2,0.5})} \right)$$

$$\tilde{\mathrm{C}}_1^1 = \tilde{c}_{1,1} \alpha_{1,1} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,1} \tilde{\mu}_{1,1}{}^2 + \tilde{\lambda}_{2,1} \tilde{\mu}_{2,1}{}^2}{2(1 - \tilde{\lambda}_1 \tilde{\mu}_1)(1 - \tilde{\lambda}_{2,1} \tilde{\mu}_{2,1})} \right) + \tilde{c}_{2,1} \alpha_{2,1} \tilde{\lambda} \left(\frac{\tilde{\lambda}_{1,1} \tilde{\mu}_{1,1}{}^2 + \tilde{\lambda}_{2,1} \tilde{\mu}_{2,1}{}^2}{2(1 - \tilde{\lambda}_{2,1} \tilde{\mu}_{2,1})} \right)$$

(iii) Average total cost of inactivity when there is a non-preemptive discipline.

$$\tilde{\mathrm{C}}_0^2 = \tilde{c}_{1,0} \alpha_{1,0} \tilde{\lambda} \left(\frac{\tilde{\mu}_{1,0}(1 - \tilde{\lambda}_0 \tilde{\mu}_0) + \frac{\tilde{\lambda}_{1,o} \tilde{\mu}_{1,0}{}^2 + \tilde{\lambda}_{2,o} \tilde{\mu}_{2,o}{}^2}{2}}{(1 - \tilde{\lambda}_0 \tilde{\mu}_0)(1 - \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0})} - \tilde{\mu}_{1,0} \right) + \tilde{c}_{2,0} \alpha_{2,0} \tilde{\lambda} \left(\frac{\tilde{\mu}_{2,0}(1 - \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0}) + \frac{\tilde{\lambda}_{2,o} \tilde{\mu}_{2,o}{}^2}{2}}{(1 - \tilde{\lambda}_{2,0} \tilde{\mu}_{2,0})} - \tilde{\mu}_{2,0} \right)$$

$$\tilde{\mathrm{C}}_{.5}^2 = \tilde{c}_{1,.5} \alpha_{1,.5} \tilde{\lambda} \left(\frac{\tilde{\mu}_{1,.5}(1 - \tilde{\lambda}_{.5} \tilde{\mu}_{.5}) + \frac{\tilde{\lambda}_{1,.5} \tilde{\mu}_{1,.5}{}^2 + \tilde{\lambda}_{2,.5} \tilde{\mu}_{2,.5}{}^2}{2}}{(1 - \tilde{\lambda}_{.5} \tilde{\mu}_{.5})(1 - \tilde{\lambda}_{2,.5} \tilde{\mu}_{2,.5})} - \tilde{\mu}_{1,.5} \right)$$

$$+\tilde{c}_{2,.5}\alpha_{2,.5}\tilde{\lambda}\left(\frac{\tilde{\mu}_{2,.5}(1-\tilde{\lambda}_{2,.5}\tilde{\mu}_{2,.5})+\frac{\tilde{\lambda}_{2,.5}\tilde{\mu}_{2,.5}{}^{2}}{2}}{(1-\tilde{\lambda}_{2,.5}\tilde{\mu}_{2,.5})}-\tilde{\mu}_{2,.5}\right)$$

$$\tilde{C}_1^2=\tilde{c}_{1,1}\alpha_{1,1}\tilde{\lambda}\left(\frac{\tilde{\mu}_{1,1}(1-\tilde{\lambda}_1\tilde{\mu}_1)+\frac{\tilde{\lambda}_{1,1}\tilde{\mu}_{1,1}{}^{2}+\tilde{\lambda}_{2,1}\tilde{\mu}_{2,1}{}^{2}}{2}}{(1-\tilde{\lambda}_1\tilde{\mu}_1)(1-\tilde{\lambda}_{2,1}\tilde{\mu}_{2,1})}-\tilde{\mu}_{1,1}\right)+\tilde{c}_{2,1}\alpha_{2,1}\tilde{\lambda}\left(\frac{\tilde{\mu}_{2,1}(1-\tilde{\lambda}_{2,1}\tilde{\mu}_{2,1})+\frac{\tilde{\lambda}_{2,1}\tilde{\mu}_{2,1}{}^{2}}{2}}{(1-\tilde{\lambda}_{2,1}\tilde{\mu}_{2,1})}-\tilde{\mu}_{2,1}\right)$$

Comparison of the three total costs shows which of the priority discipline is preferable.

5. Numerical Example

Consider a centralized parallel processing system in which jobs arrive in two class with utilization of 30% and 70%. Jobs arrive at this system in accordance with a Poisson process and the service times follow a possibility distribution. Both the group arrival rate $\tilde{\lambda}$ = (0.05,0.06,0.07) and service rate of two unit classes $\tilde{\mu}_A$ = (12,15,16) and $\tilde{\mu}_B$ = (9,10,12) per minute are triangular fuzzy numbers . The possibility distribution of unit cost of inactivity for units of the two classes are triangular fuzzy numbers $\tilde{C}_A$ = (10,16,18) and $\tilde{C}_B$ = (5,6,9) respectively. The system manager wants to evaluate the total cost of inactivity when there is no priority discipline, preemptive priority discipline, non-preemptive priority discipline in the queue.

Table 1. The total costs of inactivity

Priority Disciplines	**α = 0**	**α = 0.5**	**α = 1**
No priority discipline : $\tilde{C}_\alpha$	(1.576 , 65.713)	(3.247 , 16.821)	(6.91 , 6.91)
Preemptive priority discipline: $\tilde{C}_\alpha^1$	(1.7212 , 81.721)	(3.727 , 20.496)	(8.41 , 8.41)
Non- Preemptive priority discipline: $\tilde{C}_\alpha^2$	(1.707,176.71)	(1.68 ,37.299)	(10.709,10.709)

Comparison of the three total costs shows which of the priority disciplines minimizes the average total cost function of inactivity. Even though they are overlapping fuzzy numbers, so minimum average total cost of inactivity is achieved by the fuzzy queuing model without priorities. We conclude that the optimum selection of a priority discipline for the fuzzy queuing model that we studied entails without priorities, i.e. the priority based on a first-in, first-out discipline, in which class A units will be assigned a higher priority and class B units will be assigned lower priority.

5. Conclusion

Fuzzy set theory has been applied to a number of queuing system to provide broader application in many fields. In this paper we apply measures to uncertainty of the initial information when some of the parameters of the models are fuzzy. The method proposed enables reasonable solution to be for each case, with different level of possibility, ranging from the most pessimistic to the most optimistic scenario. This paper also provides more information to help design fuzzy priority discipline queuing system.

Conflict of Interests

The author declares that there is no conflict of interests.

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