ROOT SQUARE MEAN GRAPHS OF ORDER $\leq 5$

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Abstract. The concept of Root Square Mean labeling was introduced in [5]. In this paper we prove the Root Square Mean labeling of $C_n \bar{\delta} K_{1,m}$, $C_n \bar{\delta} K_{1,m}$, $K_n - e$ and graphs of order $\leq 5$.

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1. Introduction

All graphs in this paper are finite, simple and undirected graph $G = (V, E)$ with $p$ vertices and $q$ edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. Now we provide the definitions and theorems which are useful for the present study.

Definition 1.1: A graph $G = (V, E)$ with $p$ vertices and $q$ edges is called a Root square mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case $f$ is called a Root square Mean labeling of $G$.

Definition 1.2: The graph $G - e$ is obtained from $G$ by deleting the edge $e$ from $G$.

Definition 1.3: The union of two graphs $G_1$ and $G_2$ is the graph $G_1 \cup G_2$ with

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$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

**Definition 1.4:** A graph $C_n \overline{\delta} K_{1,m}$ is obtained from $C_n$ and $K_{1,m}$ by identifying any vertex of $C_n$ and the central vertex of $K_{1,m}$.

**Definition 1.5:** The graph $C_n \overline{\delta} K_{1,m}$ is obtained from $C_n$ and $K_{1,m}$ by identifying any vertex of $C_n$ and a pendent vertex of $K_{1,m}$ (i.e. non-central vertex of $K_{1,m}$).

**Theorem 1.6:** Any path is a Root Square Mean graph.

**Theorem 1.7:** Any cycle is a Root Square Mean graph.

**Theorem 1.8:** The complete graph $K_n$ is a Root Square mean graph if and only if $n \leq 4$.

**Theorem 1.9:** The graph $K_{1,n}$ is a Root Square mean graph if $n \leq 6$.

**Theorem 1.10:** Dragon $C_n \overline{\@} P_m$ is a Root Square Mean graph.

**Theorem 1.11:** The graph $C_m \cup P_n$ is a Root Square Mean graph.

**Remark 1.12:** If $p > q + 1$, then the graph $G$ is not a Root Square Mean graph, since there is no sufficient labels from $1, 2, ..., q + 1$ for all the vertices.

2. **Main Results**

**Theorem 2.1:** The graph $C_n \overline{\delta} K_{1,m}$ is a Root Square Mean graph if $m \leq 4$.

**Proof:**

**Case (i):** If $1 \leq m \leq 4$

Let $u_1, u_2, ..., u_n$ be the vertices of $C_n$ and $u, v_1, v_2, ..., v_m$ be the vertices of $K_{1,m}$. Let $u$ be the central vertex of $K_{1,m}$. Identify $u_n$ with $u$.

Define a function $f: V(C_n \overline{\delta} K_{1,m}) \to \{1, 2, ..., q + 1\}$ by

$f(u_i) = i, 1 \leq i \leq n - 1$

$f(u_n) = f(u) = n + 1$

$f(v_i) = n + i + 1, 1 \leq i \leq m$.

Then the edge labels are distinct. Hence $C_n \overline{\delta} K_{1,m}$ is a Root Square mean graph.

The labeling pattern of $C_6 \overline{\delta} K_{1,4}$ is shown below.
Case (ii): If $m > 4$, then we have the repetition of edge labels. The labeling pattern of $C_5 \delta K_{1,5}$ is shown below.

Here the edge labels of $(u, v_4)$ and $(u, v_5)$ are repeated. The same condition repeats when we take $u = n + 1$ and $m > 4$. Hence $C_n \delta K_{1,m}$ is not a Root Square mean graph for $m > 4$.

**Theorem 2.2:** The graph $C_n \delta K_{1,m}$ is a Root Square Mean graph if $m \leq 5$.

**Proof:**

Case (i): If $1 \leq m \leq 5$

Since $C_n \delta K_{1,1} = C_n \odot P_2$ and $C_n \delta K_{1,2} = C_n \odot P_3$, by theorem 1.10, $C_n \delta K_{1,1}$ and $C_n \delta K_{1,2}$ are Root Square mean graphs.

Let $u_1, u_2, \ldots, u_n$ be the vertices of $C_n$ and $u, v_1, v_2, \ldots, v_m$ be the vertices of $K_{1,m}$. Let $u$ be the central vertex of $K_{1,m}$. Identify $v_m$ with $u_n$. 

Define a function $f: V(C_n \bar{\delta} K_{1,m}) \to \{1,2, ... , q + 1\}$ by

$f(u_i) = i , 1 \leq i \leq n - 1$
$f(u_n) = f(v_m) = n + 1$
$f(u) = n + 2$
$f(v_i) = n + i + 2 , 1 \leq i \leq m - 1$.

Then the edge labels are distinct. Hence $C_n \bar{\delta} K_{1,m}$ is a Root Square mean graph.

The labeling pattern of $C_5 \bar{\delta} K_{1,5}$ is shown below.

![Figure 3]

**Case (ii):** If $m > 5$, then we have the repetition of edge labels.

The labeling pattern of $C_4 \bar{\delta} K_{1,6}$ is given below.

![Figure 4]

Here the edge labels of $(u, v_4)$ and $(u, v_5)$ are same. The same condition repeats when we take $u = n + 1$ and $m > 5$. Hence $C_n \bar{\delta} K_{1,m}$ is not a Root Square Mean graph if $m > 5$.

**Remark 2.3:**

If $n = 2$, $K_2 - e$ is a set of two isolated vertices and here $p = 2$ and $q + 1 = 0 + 1 = 1$. Since $p > q + 1$, by remark 1.12, $K_2 - e$ is not a Root Square mean graph.
**Remark 2.4:** If $n = 3$, $K_3 - e = P_2$, which is a path on two vertices, by theorem 1.6, $K_3 - e$ is a Root Square mean graph. The labeling pattern is shown below.

![Figure 5](image)

**Remark 2.5:** If $n = 4$, $K_4 - e$ is a Root Square mean graph and the labeling pattern is shown below.

![Figure 6](image)

**Remark 2.6:** If $n > 4$, $K_n - e$ is not a Root Square mean graph and the labeling pattern of $K_5 - e$ is shown below.

![Figure 7](image)

Here the labeling of the edges $(2,9)$ and $(4,9)$ are repeated. So $K_n - e$ is not a Root Square mean graph for $n > 4$. 
Theorem 2.7: The following graphs of order \( \leq 5 \) are not Root square mean graphs.

i) \( K_2^c, K_3^c, K_4^c, K_5^c \)

ii) \( K_5 \)

iii) \( K_5 - e \)

Proof:

- The graphs in case(i) are not Root Square mean graphs by remark 1.12.
- By theorem 1.8, the graph \( K_5 \) is not a Root Square mean graph.
- By remark 2.6, \( K_5 - e \) is not a Root Square mean graph.

Theorem 2.8: The following graphs of order \( \leq 5 \) are Root square mean graphs.

i) \( P_1, P_2, P_3, P_4, P_5 \)

ii) \( C_3, C_4, C_5 \)

iii) \( K_4 \)

iv) \( K_{1,3}, K_{1,4} \)

v) \( C_3 \cup P_2 \)

vi)
Proof:

- Since all the paths are Root Square mean graphs, by theorem 1.6, the graphs in case (i) are Root Square mean graphs.
- By theorem 1.7, the graphs in case (ii) are Root Square mean graphs.
- By theorem 1.8, $K_4$ is a Root Square mean graph.
- By theorem 1.9, $K_{1,3}, K_{1,4}$ are Root square mean graphs.
- The graphs in case (v) are Root Square mean graphs by theorem 1.11.
- By theorem 2.1 and remark 2.5, the graphs in case (vi) are Root Square mean graphs.

The remaining graphs of order $\leq 5$ are Root Square mean graphs by giving specific labeling assigned to the vertices of each such graphs. These graphs are classified according their size of $q$ and drawn with a specific Root Square mean labeling in the following table.
Remark:

The following table gives the number of graphs of order \( \leq 5 \) which are Root Square mean graph and not Root square mean graph.

<table>
<thead>
<tr>
<th>Order</th>
<th>Root Square mean</th>
<th>Not Root Square mean</th>
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<td>0</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
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Conflict of Interests

The authors declare that there is no conflict of interests.

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