ON (L,M)-FUZZY SOFT TOPOLOGICAL SPACES

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Abstract. In this paper, the concepts of (L,M)-fuzzy soft topological spaces, (L,M)-fuzzy soft base and (L,M)-fuzzy soft filter spaces were introduced and their properties were studied, where L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice. Also, the relationships between these concepts were investigated.

Keywords: (L,M)-fuzzy soft topological spaces; (L,M)-fuzzy soft base; (L,M)-fuzzy soft filter spaces.

2010 AMS Subject Classification: 54A40, 03E72, 03G10, 06A15.

1. Introduction

In 1999, D. Molodtsov [29] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Also, he applied this theory to several directions (see, for example, [30],[31],[32]). The soft set theory has been applied to many different fields (see, for example, [1],[2],[6],[7],[10],[11], [21],[27],[33],[44],[39],[45]). Later, some researchers (see, for
example, [3], [8], [19], [20], [28], [34], [40], [46]) introduced and studied the notion of soft
topological spaces.

Šostak introduced a new definition of fuzzy topology in 1985 [41], which we will call”fuzzy
topology on Šostak sense.” According to Šostak [41], these definitions, a fuzzy topology is a
crisp subfamily of family of fuzzy sets and fuzziness in the concept of openness of a fuzzy set
has not been considered, which seems to be a drawback in the process of fuzzification of the
concept of topological spaces.

In this paper, we introduce the concepts of \((L,M)\)-fuzzy soft topological spaces and \((L,M)\)-
fuzzy soft filter spaces in Šostak sense. We study their properties and discuss the relationships
between these concepts.

2. Preliminaries

**Definition 2.1** [13]. Let \((L,\leq)\) be a poset.

1. L is called a lattice, if \(a \lor b \in L, a \land b \in L\) for any \(a, b \in L\).
2. L is called a complete lattice, if \(\lor S \in L, \land S \in L\) for any \(S \subseteq L\).
3. L is called distributive, if \(a \lor (b \land c) = (a \lor b) \land (a \lor c), a \land (b \lor c) = (a \land b) \lor (a \land c)\) for
any \(a, b, c \in L\).
4. L is called a complete distributive lattice (resp. a distributive lattice), if L is a complete
lattice (resp. a lattice) and distributive.

**Definition 2.2** [13]. Let L be a lattice with top element \(1_L\) and bottom element \(0_L\) and let
\(a, b \in L\). Then b is called a complement element of a, if \(a \lor b = 1_L, a \land b = 0_L\). If \(a \in L\) has a
complement element, then it is unique. We denote the complement element of a by \(a'\).

**Definition 2.3** [13]. Let \((L,\leq)\) be a poset. Then

1. L is called a Boolean lattice, if (i) L is a distributive lattice; (ii) L has 0\(_L\) and 1\(_L\); (iii) each
   \(a \in L\) has the complement \(a' \in L\).
2. L is called a complete Boolean lattice, if (i) L is a complete distributive lattice; (ii) L has
   0\(_L\) and 1\(_L\); (iii) each \(a \in L\) has the complement \(a' \in L\).
Definition 2.4 [14],[15],[35],[42]. A triple \((L, \leq, \odot)\) is called a strictly two-sided commutative quantale (stsc-quantale, for short) if and only if it satisfies the following conditions:

1. \((L, \leq, \lor, \land, 1, 0)\) is a completely distributive lattice where 1 is the universal upper bound and 0 is the universal lower bound.
2. \((L, \odot)\) is a commutative semigroup.
3. \(x = x \odot 1\) for each \(x \in L\).
4. \(\odot\) is distributive over arbitrary joins, i.e. \((\bigvee_{i \in \Gamma} a_i) \odot b = \bigvee_{i \in \Gamma} (a_i \odot b)\).

Let \((L, \leq, \odot)\) be a stsc-quantale. Then for each \(x, y \in L\) we define \((x \odot y) \leq z \Longleftrightarrow x \leq (y \rightarrow z)\). The it satisfies Galois correspondence. i.e. \((x \odot y) \leq z\) if and only if \(x \leq (y \rightarrow z)\).

Definition 2.5 [37]. Let \(E\) be a set of parameters, \(X\) be an initial universe. A pair \((f, E)\) is called a fuzzy soft set over \(X\), if \(f\) is a mapping given by \(f : E \rightarrow I^X\). We also denote \((f, E)\) by \(f_E\). The set of all fuzzy soft set is denoted by \(FS(X, E)\).

Definition 2.6 [26]. A fuzzy soft set \(f_E\) on \(X\) is called a null fuzzy soft set and denoted by \(\tilde{0}\) if \(f_e = \overline{0}\), for each \(e \in E\).

Definition 2.7 [4]. A fuzzy soft set \(f_E\) on \(X\) is called an absolute fuzzy soft set and denoted by \(\tilde{1}\) if \(f_e = \overline{1}\), for each \(e \in E\).

Definition 2.8 [25]. Let \(E\) be a set of parameters, \(X\) be an initial universe, \(L\) be a complete Boolean lattice and \(A \subseteq E\). An \(L\)-fuzzy soft set \(f_A\) over \((X, E)\) is a mapping \(f_A : E \rightarrow L^X\) such that \(f_A(e) = \overline{0}\) for all \(e \not\in A\). The set of all \(L\)-fuzzy soft set over \((X, E)\) is denoted by \(L-FS(X, E)\).

In other words, an \(L\)-fuzzy soft set \(f_E\) over \(X\) is a parameterized family of \(L\)-fuzzy sets in the universe \(X\). If \(L = [0, 1]\), then every \(L\)-fuzzy soft set is a fuzzy soft set.

Definition 2.9 [25]. Let \(f_A, g_B \in L-FS(X, E)\). Then
(1) \( f_A \) is said to be fuzzy soft subset of \( g_B \), denoted by \( f_A \subseteq g_B \) if \( f_A(e) \subseteq g_B(e) \) for all \( e \in E \), that is \( f_A(e)(x) \leq g_B(e)(x) \) for all \( e \in E \), and for all \( x \in X \).

Two \( L \)-fuzzy soft sets \( f_A \) and \( g_B \) over \((X,E)\) are said to be equal, denoted by \( f_A \cong g_B \) if \( f_A \subseteq g_B \) and \( g_B \subseteq f_A \).

(2) The union of \( f_A \) and \( g_B \) is also \( L \)-fuzzy soft set \( h_C \), defined by \( h_C(e)(x) = f_A(e)(x) \lor g_B(e)(x) \) for all \( e \in E \), where \( C = A \cup B \). Here we write \( h_C = f_A \cup g_B \).

(3) The intersection of \( f_A \) and \( g_B \) is also \( L \)-fuzzy soft set \( h_C \), defined by \( h_C(e)(x) = f_A(e)(x) \land g_B(e)(x) \) for all \( e \in E \), where \( C = A \cap B \). Here we write \( h_C = f_A \cap g_B \).

**Definition 2.10** [38]. The fuzzy soft set \( f_A \in FS(X,E) \) is called fuzzy soft point if \( A = \{e\} \subseteq E \) and \( f_A(e) \) is a fuzzy point in \( X \) i.e. there exists \( x \in X \) such that \( f_A(e)(x) = t \) \((0 < t \leq 1)\) and \( f_A(e)(y) = 0 \) for all \( y \in X \setminus \{x\} \). We denote this fuzzy soft point \( f_A = e_x^t = \{(e,x)\} \) and the set of all fuzzy soft point by \( SP^f_r(X,E) \).

**Definition 2.11** [38]. Let \( e_x^t,f_A \in FS(X,E) \). we say that \( e_x^t \in f_A \) read as \( e_x^t \) belongs to the fuzzy soft set \( f_A \) if for the element \( e \in A, t \leq f_A(e)(x) \).

**Definition 2.12** [5]. Let \((X,E)\) and \((Y,E^*)\) be classes of fuzzy soft sets over \( X \) and \( Y \) with attributes from \( E \) and \( E^* \) respectively. Let \( \rho : X \rightarrow Y \) and \( \psi : E \rightarrow E^* \) be mapping. Then a fuzzy soft mapping \( f = (\rho, \psi) : (X,E) \rightarrow (Y,E^*) \) would be defined as follows

For a fuzzy soft set \( F_A \) in \((X,E)\), \( f(F_A) \) is a fuzzy soft set in \((Y,E^*)\) obtained as follows: for \( \beta \in \psi(E) \subseteq E^* \) and \( y \in Y \),

\[
f(F_A)(\beta)(y) = \begin{cases} \lor_{x \in \rho^{-1}(y)} \lor_{\alpha \in \psi^{-1}(\beta)} F_A(\alpha)(x), & \text{if } \rho^{-1}(y) \neq \emptyset, \ \psi^{-1}(\beta) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}
\]

\( f(F_A) \) is called fuzzy soft image of the fuzzy soft set \( F_A \).

**Definition 2.13** [5]. Let \((X,E)\) and \((Y,E^*)\) be classes of fuzzy soft sets over \( X \) and \( Y \) with attributes from \( E \) and \( E^* \) respectively. Let \( \rho : X \rightarrow Y, \psi : E \rightarrow E^* \) be mappings and \( f = (\rho, \psi) : (X,E) \rightarrow (Y,E^*) \) be fuzzy soft mapping. Let \( A \subseteq E \) be a fuzzy soft set over \( X \) and \( B \subseteq E^* \) be fuzzy soft set over \( Y \), then \( f(F_A) \) is called \( f \)-fuzzy soft image of \( A \) in \( B \), denoted by \( f(F_A)(B) \).
\( (X, E) \to (Y, E^*) \) a fuzzy soft mapping. Then for a fuzzy soft set \( g_B \) in \((Y, E^*)\) \( f^{-1}(g_B) \) is a fuzzy soft set in \((X, E)\) obtained as follows: for \( \alpha \in \psi^{-1}(E^*) \subseteq E \) and \( x \in E \),

\[
f^{-1}(g_B)(\alpha)(x) = g_B(\psi(\alpha))(\rho(x)).
\]

\( f^{-1}(g_B) \) is called a fuzzy soft inverse image of the fuzzy soft set \( g_B \).

3. \((L, M)\)-fuzzy soft topological spaces

Let \( L \) be a completely distributive lattice with 0 and 1 elements and \( M \) be a strictly two-sided, commutative quantale lattice.

**Definition 3.1.** A map \( \mathcal{T} : L\text{-}FS(X, E) \to M \) is called an \((L, M)\)-fuzzy soft topology on \((X, E)\) if it satisfies the following conditions:

- (LSO1) \( \mathcal{T}(\emptyset) = \mathcal{T}(\overline{1}) = 1 \).
- (LSO2) \( \mathcal{T}(f_{A_1} \cap f_{A_2}) \geq \mathcal{T}(f_{A_1}) \odot \mathcal{T}(f_{A_2}), \) for all \( f_{A_1}, f_{A_2} \in L\text{-}FS(X, E) \).
- (LSO3) \( \mathcal{T}(\bigcup_{i \in A} f_{A_i} \geq \bigwedge_{i \in A} \mathcal{T}(f_{A_i})), \) for all \( f_{A_i} \in L\text{-}FS(X, E) \).

The triple \((X, E, \mathcal{T})\) is called \((L, M)\)-fuzzy soft topological space.

Let \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \) be \((L, M)\)-fuzzy soft topologies on \((X, E)\). We say that \( \mathcal{T}_1 \) is finer than \( \mathcal{T}_2 \) (\( \mathcal{T}_2 \) is coarser than \( \mathcal{T}_1 \)), denoted by \( \mathcal{T}_2 \subseteq \mathcal{T}_1 \), if \( \mathcal{T}_2(f_A) \leq \mathcal{T}_1(f_A) \), for all \( f_A \in L\text{-}FS(X, E) \).

Let \((X, E, \mathcal{T}_1)\) and \((Y, E^*, \mathcal{T}_2)\) be \((L, M)\)-fuzzy soft topological spaces. A soft map \( \phi : (X, E, \mathcal{T}_1) \to (Y, E^*, \mathcal{T}_2) \) is called LFS-continuous if and only if \( \mathcal{T}_2(f_A) \leq \mathcal{T}_1(\phi^*(f_A)) \), for all \( f_A \in L\text{-}FS(Y, E^*) \).

**Remark 3.2.** (1) If \((L = [0, 1], \wedge)\) and \( M = \{0, 1\} \), \((L, M)\)-fuzzy soft topological space is fuzzy soft topological space [37].

(2) If \((L = M = [0, 1], \odot = \wedge)\) then \((L, M)\)-fuzzy soft topological space is fuzzy soft topological space [4].
Definition 3.3. A map $\mathcal{F} : L\text{-}FS(X,E) \longrightarrow M$ is called an $(L,M)$-fuzzy soft filter on $(X,E)$ if it satisfies the following conditions:

(LSF1) $\mathcal{F}(\tilde{0}) = 0$ and $\mathcal{F}(\tilde{1}) = 1$.

(LSF2) $\mathcal{F}(f_{A_1} \cap f_{A_2}) \geq \mathcal{F}(f_{A_1}) \odot \mathcal{F}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

(LSF3) If $f_{A_1} \subseteq f_{A_2}$ we have $\mathcal{F}(f_{A_1}) \leq \mathcal{F}(f_{A_2})$.

The triple $(X,E,\mathcal{F})$ is called an $(L,M)$-fuzzy soft filter space.

Theorem 3.4. Let $(X,E,\mathcal{F})$ be an $(L,M)$-fuzzy soft filter space. We define a mapping $\mathcal{T}_{\mathcal{F}} : L\text{-}FS(X,E) \rightarrow M$ as follows:

$$\mathcal{T}_{\mathcal{F}}(f_A) = \begin{cases} \mathcal{F}(f_A), & \text{if } f_A \not\sim \tilde{0}, \\ 1, & \text{if } f_A \sim \tilde{0}. \end{cases}$$

Then $(X,E,\mathcal{T}_{\mathcal{F}})$ is an $(L,M)$-fuzzy soft topological space.

Proof. We show the condition (LSO3). For $f_{A_i} \in L\text{-}FS(X,E)$, since $f_{A_i} \subseteq \bigcup_{i \in \Gamma} f_{A_i}$ for all $i \in \Gamma$, we have $\mathcal{F}(f_{A_i}) \leq \mathcal{F}(\bigcup_{i \in \Gamma} f_{A_i})$, so

$$\bigwedge_{i \in \Gamma} \mathcal{T}_{\mathcal{F}}(f_{A_i}) \leq \mathcal{T}_{\mathcal{F}}(\bigcup_{i \in \Gamma} f_{A_i}).$$

Definition 3.5. A map $\mathcal{B} : L\text{-}FS(X,E) \rightarrow M$ is called an $(L,M)$-fuzzy soft base on $(X,E)$ if it satisfies the following conditions:

(LSB1) $\mathcal{B}(\tilde{0}) = \mathcal{B}(\tilde{1}) = 1$.

(LSB2) $\mathcal{B}(f_{A_1} \cap f_{A_2}) \geq \mathcal{B}(f_{A_1}) \odot \mathcal{B}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

Theorem 3.6. Let $\mathcal{B}$ be an $(L,M)$-fuzzy soft base on $(X,E)$. Define a map $\mathcal{T}_{\mathcal{B}} : L\text{-}FS(X,E) \rightarrow M$ as follows:

$$\mathcal{T}_{\mathcal{B}}(f_A) = \bigvee \{ \bigwedge_{i \in \Gamma} \mathcal{B}(f_{A_i}) : f_A = \bigcup_{i \in \Gamma} f_{A_i} \}.$$ 

Then $\mathcal{T}_{\mathcal{B}}$ is the coarsest $(L,M)$-fuzzy soft topology on $(X,E)$ such that $\mathcal{T}_{\mathcal{B}}(f_A) \geq \mathcal{B}(f_A)$ for all $f_A \in L\text{-}FS(X,E)$. 
Proof. (1) It is trivial from the definition of $\mathcal{T}_B$.

(2) For all families $\{f_A : f_A = \bigsqcup_{i \in \Delta} f_{A_i}\}$ and $\{g_B : g_B = \bigsqcup_{j \in \Gamma} g_{B_j}\}$ there exists a family $\{f_{A_i} \cap g_{B_j}\}$ such that:

$$f_A \cap g_B = \left( \bigsqcup_{i \in \Delta} f_{A_i} \right) \cap \left( \bigsqcup_{j \in \Gamma} g_{B_j} \right) = \bigsqcup_{i \in \Delta, j \in \Gamma} (f_{A_i} \cap g_{B_j}).$$

It implies

$$\mathcal{T}_B(f_A \cap g_B) \geq \bigwedge_{i \in \Delta, j \in \Gamma} \mathcal{B}(f_{A_i} \cap g_{B_j})$$

$$\geq \bigwedge_{i \in \Delta, j \in \Gamma} (\mathcal{B}(f_{A_i}) \circ \mathcal{B}(g_{B_j})) \quad \text{(by Definition 3.5 (LSB2))}$$

$$\geq (\bigwedge_{i \in \Delta} \mathcal{B}(f_{A_i})) \circ (\bigwedge_{j \in \Gamma} \mathcal{B}(g_{B_j})).$$

By definition 2.4 (L4) we have $\mathcal{T}_B(f_A \cap g_B) \geq \mathcal{T}_B(f_A) \circ \mathcal{T}_B(g_B)$.

(3) Let $\mathcal{J}_i$ be the collection of all index sets $K_i$ such that $\{f_{A_{i_k}} : f_{A_{i_k}} = \bigsqcup_{k \in K_i} f_{A_{i_k}}\}$ with $f_A = \bigsqcup_{i \in \Gamma} f_{A_i} = \bigsqcup_{i \in \Gamma} \bigsqcup_{k \in K_i} f_{A_{i_k}}$. For each $i \in \Gamma$ and each $\psi \in \Pi_{i \in \Gamma} \mathcal{J}_i$ with $\psi(i) = K_i$ we have

(1) $$\mathcal{T}_B(f_A) \geq \bigwedge_{i \in \Gamma} (\bigwedge_{k \in K_i} \mathcal{B}(f_{A_{i_k}})).$$

Put $a_{i, \psi(i)} = \bigwedge_{k \in K_i} \mathcal{B}(f_{A_{i_k}})$. From (3.1) we have

$$\mathcal{T}_B(f_A) \geq \bigvee_{\psi \in \Pi_{i \in \Gamma}} (\bigwedge_{i \in \Gamma} a_{i, \psi(i)}).$$

(Since $L$ is a completely distributive lattice,)

$$= \bigwedge_{i \in \Gamma} (\bigvee_{M_i \in \mathcal{J}_i} a_{i, M_i}) = \bigwedge_{i \in \Gamma} \bigwedge_{M_i \in \mathcal{J}_i, m \in M_i} \mathcal{B}(f_{A_{i_m}}))$$

$$= \bigwedge_{i \in \Gamma} \mathcal{T}_B(f_{A_i}).$$

Thus $\mathcal{T}_B$ is a $(L, M)$-fuzzy soft topology on $X$. 


If $T \geq B$ for every $f_A = \bigsqcup_{i \in \Delta} f_{A_i}$ we have

$$T(f_A) \geq \bigwedge_{i \in \Delta} T(f_{A_i}) \geq \bigwedge_{i \in \Delta} B(f_{A_i}).$$

Thus $T \supseteq T_B$.

From Theorem 3.6, we can easily prove the following lemma.

**Lemma 3.7.** Let $T$ be an $(L,M)$-fuzzy soft topology on $(X,E)$ and $B$ be an $(L,M)$-fuzzy soft base on $(Y,E^*)$. Then a map $\phi : (X,E,T) \to (Y,E^*,T_B)$ is LFS-continuous if and only if $T(\phi^{-}(f_A)) \geq B(f_A)$ for each $f_A \in L$-FS$(Y,E^*)$.

**Theorem 3.8.** Let $\{(X_i, E_i, T_i) : i \in \Gamma\}$ be a family of $(L,M)$-fuzzy soft topological spaces, $X$ be a set, $E$ be a set of parameters and for each $i \in \Gamma$, $\phi_i : (X,E) \to (X_i, E_i)$ a fuzzy soft map. Define a map $B : LFS(X,E) \to M$ on $(X,E)$ by:

$$B(f_A) = \bigvee \{ \cap_{j=1}^{n} T_{k_j}(g_{B_{k_j}}) : f_A = \cap_{j=1}^{n} \phi_{k_j}^{-}(g_{B_{k_j}}) \}$$

where $\bigvee$ is taken over all finite subsets $K = \{k_1, ..., k_n\} \subset \Gamma$.

Then: (1) $B$ is an $(L,M)$-fuzzy soft base on $(X,E)$.

(2) The $(L,M)$-fuzzy soft topology $T_B$ generated by $B$ is the coarsest $(L,M)$-fuzzy soft topology on $(X,E)$ for which all $\phi_i, i \in \Gamma$ are LFS-continuous maps.

**Proof.** (1)(LSB1) Since $f_A = \phi_i^{-}(f_{A_i})$ for each $f_A \in \{0,1\}$ we have $B(0) = B(1) = 1$.

(LSB2) For all finite subsets $K = \{k_1, ..., k_p\}$ and $J = \{j_1, ..., j_q\}$ of $\Gamma$ such that

$$f_A = \cap_{i=1}^{p} \phi_{k_i}^{-}(f_{A_{k_i}}), \quad g_B = \cap_{j=1}^{q} \phi_{j_i}^{-}(g_{B_{j_i}}),$$

we have

$$f_A \cap g_B = (\cap_{i=1}^{p} \phi_{k_i}^{-}(f_{A_{k_i}})) \cap (\cap_{j=1}^{q} \phi_{j_i}^{-}(g_{B_{j_i}})).$$

Furthermore, we have for each $k \in K \cap J$,

$$\phi_{k}^{-}(f_{A_{k}}) \cap \phi_{k}^{-}(g_{B_{k}}) = \phi_{k}^{-}(f_{A_{k}} \cap g_{B_{k}}).$$
Put \( f_A \cap g_B = \cap_{m_i \in K \cup J} \phi_{m_i}^{-1}(h_{C_{m_i}}) \) where

\[
h_{C_{m_i}} = \begin{cases} 
  f_{A_{m_i}}, & \text{if } m_i \in K - (K \cap J), \\
  g_{B_{m_i}}, & \text{if } m_i \in J - (K \cap J), \\
  f_{A_{m_i}} \cap g_{B_{m_i}}, & \text{if } m_i \in K \cap J.
\end{cases}
\]

We have

\[
\mathcal{B}(f_A \cap g_B) \geq \bigcirc_{j \in K \cup J} \mathcal{T}_j(h_{C_j}) \\
\geq (\bigcap_{m_i \in K - K \cap J} \mathcal{F}_m(f_{A_{m_i}})) \bigcirc (\bigcap_{i=1}^{q} \mathcal{I}_{m_i}(g_{B_{m_i}})) \\
\geq (\bigcap_{i=1}^{p} \mathcal{T}_{l_i}(f_{A_{m_i}})) \bigcirc (\bigcap_{i=1}^{q} \mathcal{I}_{l_i}(g_{B_{l_i}})).
\]

By Definition 2.4 (L4) we have \( \mathcal{B}(f_A \cap g_B) \geq \mathcal{B}(f_A) \circ \mathcal{B}(g_B) \).

(2) For each \( f_{A_i} \in LFS(X_i, E_i) \), one family \( \{\phi_i^{-1}(f_{A_i})\} \) and \( i \in \Gamma \) we have

\[
\mathcal{T}_\mathcal{B}(\phi_i^{-1}(f_{A_i})) \geq \mathcal{B}(\phi_i^{-1}(f_{A_i})) \geq \mathcal{I}_i(f_{A_i}).
\]

Thus, for each \( i \in \Gamma, \phi_i : (X, E, \mathcal{T}_\mathcal{B}) \rightarrow (X_i, E_i, \mathcal{R}_i) \) is LFS-continuous. Let \( \phi_i : (X, E, \mathcal{T}_0) \rightarrow (X_i, E_i, \mathcal{R}_i) \) is LFS-continuous, that is for each \( i \in \Gamma \) and \( f_{A_i} \in LFS(X_i, E_i) \), \( \mathcal{T}_0(\phi_i^{-1}(f_{A_i})) \geq \mathcal{R}_i(f_{A_i}) \). For all finite subsets \( K = \{k_1, ..., k_p\} \) of \( \Gamma \) such that \( f_A = \bigcirc_{i=1}^{p} \phi_{k_i}^{-1}(f_{A_{k_i}}) \) we have

\[
\mathcal{T}_0(f_A) \geq \bigcirc_{i=1}^{p} \mathcal{T}_0(\phi_{k_i}^{-1}(f_{A_{k_i}})) \geq \bigcirc_{i=1}^{p} \mathcal{R}_{k_i}(f_{A_{k_i}}).
\]

It implies \( \mathcal{T}_0(f_A) \geq \mathcal{B}(f_A) \) for each \( f_A \in LFS(X, E) \). By Theorem 3.6 \( \mathcal{T}_0 \geq \mathcal{T}_\mathcal{B} \).

**Example 3.9.** Let \( X = \{x, y\} \) be a set, \( E = \{e_1, e_2, e_3\} \) be a set of parameters and \( L = M = [0, 1] \) a completely distributive lattice. Define a binary operation \( \bigcirc \) on \( M = [0, 1] \) by \( x \bigcirc y = \max\{0, x + y - 1\} \). Then \( ([0, 1], \leq, \bigcirc) \) is a stsc-quantale. Let \( g_B, h_C \in LFS(X, E) \) be defined as follows:

\[
g_B = \{g(e_1) = \{(x, 0.6), (y, 0.3)\}, g(e_2) = \overline{0}, g(e_2) = \overline{0}\}
\]

\[
h_C = \{h(e_1) = \{(x, 0.5), (y, 0.7)\}, h(e_2) = \overline{0}, h(e_2) = \overline{0}\}.
\]
Then we have

\[ g_B \cap h_C = \{(g_B \cap h_C)(e_1) = \{(x, 0.5), (y, 0.3)\}, \]

\[ (g_B \cap h_C)(e_2) = \emptyset, (g_B \cap h_C)(e_2) = \emptyset \}\]

\[ g_B \cup h_C = \{(g_B \cup h_C)(e_1) = \{(x, 0.6), (y, 0.7)\}, \]

\[ (g_B \cup h_C)(e_2) = \emptyset, (g_B \cup h_C)(e_2) = \emptyset \}\].

We define an \((L, M)\)-fuzzy soft topology \(\mathcal{F}: L-FS(X, E) \to [0, 1]\) as follows:

\[
\mathcal{F}(f_A) = \begin{cases} 
1, & \text{if } f_A \cong \tilde{0} \text{ or } \tilde{1}, \\
0.8, & \text{if } f_A \cong g_B, \\
0.4, & \text{if } f_A \cong h_C, \\
0.6, & \text{if } f_A \cong g_B \cup h_C, \\
0.2, & \text{if } f_A \cong g_B \cap h_C, \\
0, & \text{otherwise.}
\end{cases}
\]

Conflict of Interests

The authors declare that there is no conflict of interests.

References


