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MONITORING PROCESS PERFORMANCE USING SELF-STARTING CUMULATIVE SUM CONTROL CHART

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Abstract: Statistical process control (SPC) charts are important tools for detecting process shifts. The control chart is an important statistical technique that is used to monitor the quality of a process. Shewhart control charts help to detect larger shifts in the process parameters, but Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) charts are expected for smaller and moderate changes. The CUSUM control chart is normally used in industry for the result of small and moderate shifts in process spot and disparity. It can be shown that if there are sharp, irregular changes to a process, these types of charts are highly effective. On the other hand, if one involved in a small, persistent shift in a process, other types of control charts may be chosen, for instance the CUSUM control chart, originally developed by Page (1954). In this article, we used CUSUM control chart for monitoring the moisture level of the paper sheet.

Keywords: Statistical Process Control; Shewhart Control Chart; Cumulative Sum Control Chart; Self-Starting Cumulative Sum Control Chart.

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1. INTRODUCTION

A most important purpose for a product or a process control is to constantly look up its quality. This aim, in statistical terms, may be expressed as variability reduction. SPC is a notorious collection of methods aiming at this purpose and the control charts are considered as the main tools to detect shifts in a process. The most accepted control charts are the Shewhart charts, CUSUM charts and the EWMA charts. Shewhart type charts are used to detect large shifts in a process whereas CUSUM and EWMA charts are known to be fast in detecting small to moderate shifts. CUSUM control chart is a time-weighted control chart that displays the CUSUM of the variation of each sample value from the target value. A minor drifting in the process mean will direct to steadily rising or declining cumulative deviation value, Owing to the factor that it is cumulative. It was developed by E. S. Page of the University of Cambridge. CUSUM control chart consider being more efficient in detecting small shift in the mean of a process. CUSUM control chart shows a better result than Shewhart control chart when it is required to detect smaller shift. Also, CUSUM control chart is comparatively slow to respond to large shift and firm to detect and analyse special trend patterns.

2. REVIEW OF LITERATURE

CUSUM control chart conceived by Page, E.S., 1954 and which have been developed by many authors; in particular, Ewan(1963), Page(1961), Gan (1991), Lucas(1976), Hawkins(1981, 1993a) and Woodall and Adams (1993).They have been proposed as a substitute to Shewhart control charts. Since they detect small shifts in the process level more swiftly, they provide an early signal of process change and they are more meaningful graphically as they point out areas needing attention.

The self-starting control chart has many other advantages that get better the engineer's ability to control a process. One such benefit is the ability to chart in real time and essentially with the first units of production. By using a self-starting chart the engineer can still determine if a shift has occurred from the situation obtained at process start-up without knowing the exact parameter values. The chart uses the past observations to calculate approximately the in-control process parameters. The most fundamental self-starting methodology that we will focus on is the self-starting Q chart (Quesenberry, 1991). Q charts are Shewhart-type self-starting chart techniques. Hawkins (1987) at first proposed a self-starting CUSUM scheme that utilizes two pairs of CUSUMs one for monitoring the location of the process and the other for monitoring the spread. Quesenberry (1991) proposes the self-starting Q chart for both the mean and variance which applies transformations to the quality statistic so it can be plotted on standard normal control charts. Thus so far we have seen the self-starting methodology in the univariate case. However, if we want to chart two related quality statistics such as inner and outer diameters of a type of parts from the process start-up, multivariate self-starting quality control charts are the best method. Hawkins et al (2007) develops a multivariate equivalent of univariate self-starting charts, when it is needed to monitor two or more related quality characteristics for location and scale. Sullivan et al., 2002; Maboudou-Tchao et al., 2011; Capizzi et al., 2010 were investigated Multivariate self-starting charts.

3. METHODOLOGY

$\overline{X}$ **and R Chart:** Let a random variable follows normal distribution with mean μ and standard deviation σ, where μ and σ are known. If $\boldsymbol{x_1, x_2, x_{3,\dots,} x_n}$ is a sample of size n, then the average of this sample is $\bar{\boldsymbol{x}} = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$. Let $\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_m$ be the average of each sample the grand average then $\bar{\bar{x}} = \frac{\bar{x}_1, +_2 + \bar{x}_3 + \cdots + \bar{x}_m}{m}$. Thus the centreline of the $\bar{\boldsymbol{x}}$ control chart is $\bar{\bar{\boldsymbol{x}}}$. Control limits for the $\bar{\boldsymbol{x}}$ chart are $\mathrm{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$, $\mathrm{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$ and $\mathrm{CL}_{\bar{X}} = \bar{\bar{x}}$. If x is a sample of size n, then the range of the sample is the difference between the largest and smallest observations, that is $R = x_{max} - x_{min}$ and let $R_1, R_2, \dots, R_m$ be the ranges of m samples. Then the average range is $\bar{R} = \frac{R_1 + R_{2+\dots+}R_m}{m}$. The Control Limits for the R chart are $UCL_R = D_4\bar{R}$, $LCL_R = D_3\bar{R}$ and $CL_R = \bar{R}$. Where $\bar{\boldsymbol{X}}_{\boldsymbol{i}}$= Average of subgroup, $\bar{\bar{\boldsymbol{X}}}$ = Average of subgroup averages, R_i = Range of subgroup, $\bar{R}$= Average range of subgroup, N = Number of subgroup, UCL= Upper control limit, CL = Central Line, LCL = Lower control limit, A_2, D_4, D_3, d_2 are constant obtained from quality control table.

CUSUM Control Chart: The cumulative sum (CUSUM) of observations is defined as

$$\begin{Bmatrix} C_i = \sum_{j=1}^{i}(x_j - \mu) = (x_i - \mu) + \sum_{j=1}^{i-1}(x_j - \mu) = (x_i - \mu) + C_{i-1}; & \text{when } i \geq 1 \\ C_i = C_0 = 0; & \text{when } i = 0 \end{Bmatrix}$$

When the process remains in control with mean at μ, the cumulative sum is a random walk with mean zero. When the mean shifts upward with a value μ_0 such that $\mu > \mu_0$ then an upward or positive drift will be developed in the cumulative sum. When the mean shifts downward with a value μ_0 such that $\mu < \mu_0$ then a downward or negative drift will be developed in the CUSUM. There are two basic ways to present CUSUM control chart, which are tabular or algorithmic CUSUM and v-mask.

Tabular or Algorithmic CUSUM for Monitoring the Mean of the Process: The tabular CUSUM limits are defined as $C_i^+ = \mathrm{Max}[0, \bar{X}_i - (\mu_0 + K) + C_{i-1}^+]$ and $C_i^- = \mathrm{Max}[0, (\mu_0 - K) + C_{i-1}^-]$, where $\boldsymbol{C_i^-}$ and $\boldsymbol{C_i^+}$ are called one sided lower and one sided upper CUSUM respectively; K is the reference value and μ_0 is the targeted mean. If either $\boldsymbol{C_i^-}$ or $\boldsymbol{C_i^+}$ value exceeds the decision interval H, which usually defined as H=5σ then the process is considered to be out of control.

First Initial Response (Fir): The FIR feature gives a simple procedure for detecting an out-of-control situation at start-up more quickly. However, if the process is initially in control state, the FIR feature has little effect whereas if it is in an out-of-control state, a signal is given much more quickly. The FIR essentially just sets the starting value equal to non zero value typically H/2.

Standardized CUSUM: A number of CUSUM prefers using standardized CUSUM control chart, which is defined as $Z_i = \frac{(\bar{X} - \mu_0)}{S_i}$ for i = 1,2,3,... Where $S_i = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\bar{x} - \bar{x}_i)^2}$ then the standardized CUSUM is calculated using $C_i^+ = \mathrm{Max}[0, y_i - k + C_{i-1}^+]$ and $C_i^- = \mathrm{Max}[0, -k - y_i + C_{i-1}^-]$. Then Z_i becomes iid standard normal. Start with $C_0^+ = 0 = C_0^-$.

The Self Starting CUSUM: The self starting CUSUM for the mean of a normally distributed random variables can be applied immediately without any need for a phase I sample to estimate the process parameters, the mean μ and the variance $\boldsymbol{\sigma^2}$. Let $\bar{x}_n$ The average of the first n observations and let $w_n = \sum_{i=1}^{n} (x_i - \bar{x}_n)^2$ be the sum of squared deviations from the average of those observations. Formulas to update these quantities after each new observation $\bar{x}_n = \bar{x}_{n-1} + \frac{(x_n - \bar{x}_{n-1})}{n}$ and $w_n = w_{n-1} + \frac{(n-1)(x_n - \bar{x}_{n-1})^2}{n}$. Let $S_n{}^2 = \frac{w_n}{(n-1)}$ be the sample variance of the first n observations. Standardise each successive new process observation using $T_n =$

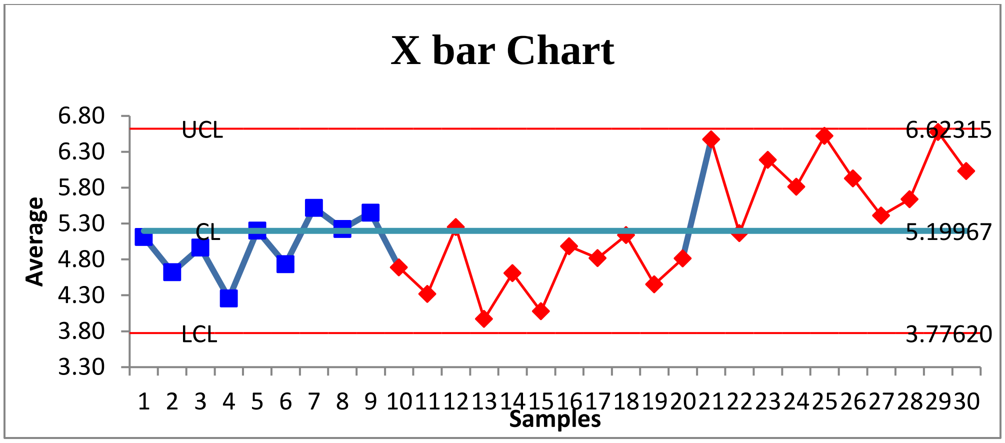
$\frac{x_n - \bar{x}_{n-1}}{s_{n-1}}$ for the case where n is greater than or equal to 3. If the observations are normally distributed, the distribution of $\sqrt{\frac{n-1}{n} T_n}$ is a t distribution with n-1 degrees of freedom. The cumulative distribution of T_n is $P(T_n \leq t) = F_{n-2}(t\sqrt{\frac{n-1}{n}})$, where F_n is the cumulative t distribution with n-1 degrees of freedom. If φ^{-1} is the inverse normal cumulative distribution, then the transformation $\mathbf{U_n} = \varphi^{-1}[F_{n-2}(a_n t_n)]$ where $a_n = \sqrt{\frac{n-1}{n}}$ converts the cusum quantity T_n into a standard normal random variable. So, we can plot all the values of $\mathbf{U_n}$ for n≥3 on N (0, 1) cusum. To estimate the process parameters for a conventional cusum this avoids the problem of using a large sample of phase I data.

4. RESULTS AND DISCUSSIONS

In this section, the moisture level in the paper sheet material is monitored using Self-Started CUSUM control chart. For monitoring the process, 30 samples were collected from the on-going process. The calculations of Shewhart control chart is given in the Table 1. From the Fig 1, it can be noted that Shewhart control chart did not show the out the control point. Because, this chart is used to find the larger shift in the process. So that CUSUM control chart is used to detect the smaller shift in the process if there is anything in the process.

TABLE 1: CALCULATIONS FOR SHEWHART $\overline{X}$ AND R CONTROL CHART

$\overline{X}$	UCL	+2 Sigma	+1 Sigma	CL	-1 Sigma	-2 Sigma	LCL	Range	UCL	+2 Sigma	+1 Sigma
5.1140	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.8727	5.2153	4.2992	3.3831
4.6227	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	3.5260	5.2153	4.2992	3.3831
4.9673	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.4730	5.2153	4.2992	3.3831
4.2599	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.9595	5.2153	4.2992	3.3831
5.2051	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.4059	5.2153	4.2992	3.3831
4.7376	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.2426	5.2153	4.2992	3.3831
5.5221	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.6134	5.2153	4.2992	3.3831
5.2287	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.0699	5.2153	4.2992	3.3831
5.4555	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.3203	5.2153	4.2992	3.3831
4.6899	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.4836	5.2153	4.2992	3.3831
. . . .											
6.4726	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.4808	5.2153	4.2992	3.3831
5.1661	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.4711	5.2153	4.2992	3.3831
6.1875	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.9130	5.2153	4.2992	3.3831
5.8140	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.9698	5.2153	4.2992	3.3831
6.5197	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	2.1998	5.2153	4.2992	3.3831
5.9289	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.7057	5.2153	4.2992	3.3831
5.4119	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	1.6153	5.2153	4.2992	3.3831
5.6407	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	3.2642	5.2153	4.2992	3.3831
6.5769	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	3.2869	5.2153	4.2992	3.3831
6.0317	6.6231	6.1487	5.6742	5.1997	4.7252	4.2507	3.7762	3.0660	5.2153	4.2992	3.3831



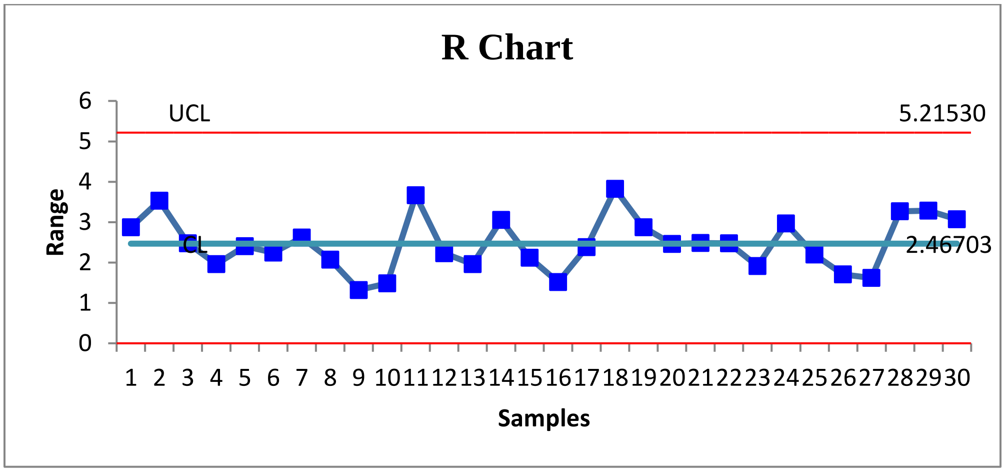


Figure 1: Shewhart Control Chart

The calculations values of the CUSUM control chart are given in the Table 2. From the Fig 2, it can be noted that the upward shift is started from the 21^{th} observation and the observations 29 and 30 are in out of control. So the mean value of the paper quality parameter changed (increased from the target mean value) from 21^{th} observation.

TABLE 2: CALCULATIONS FOR CUSUM CONTROL CHART

S. No	$\overline{X}$	Upper Cusum	C+	C-	Lower Cusum	N+	N-	Target
1	5.113973	2.8568	0.0000	0.0000	-2.8568	0	0	5.199673
2	4.622701	2.8568	0.0000	-0.2199	-2.8568	0	1	5.199673
3	4.967329	2.8568	0.0000	-0.0951	-2.8568	0	2	5.199673
4	4.259926	2.8568	0.0000	-0.6778	-2.8568	0	3	5.199673
5	5.205094	2.8568	0.0000	-0.3153	-2.8568	0	4	5.199673
6	4.737614	2.8568	0.0000	-0.4202	-2.8568	0	5	5.199673
7	5.522105	2.8568	0.0000	0.0000	-2.8568	0	0	5.199673
8	5.228674	2.8568	0.0000	0.0000	-2.8568	0	0	5.199673
9	5.455491	2.8568	0.0000	0.0000	-2.8568	0	0	5.199673
10	4.68992	2.8568	0.0000	-0.1527	-2.8568	0	1	5.199673
. . . .								
21	6.472604	2.8568	0.9158	-0.5111	-2.8568	1	12	5.199673
22	5.166059	2.8568	0.5251	-0.1876	-2.8568	2	13	5.199673
23	6.187529	2.8568	1.1559	0.0000	-2.8568	3	0	5.199673
24	5.814002	2.8568	1.4131	0.0000	-2.8568	4	0	5.199673
25	6.519731	2.8568	2.3761	0.0000	-2.8568	5	0	5.199673
26	5.928888	2.8568	2.7482	0.0000	-2.8568	6	0	5.199673
27	5.411949	2.8568	2.6034	0.0000	-2.8568	7	0	5.199673
28	5.640702	2.8568	2.6873	0.0000	-2.8568	8	0	5.199673
29	6.576862	2.8568	3.7074	0.0000	-2.8568	9	0	5.199673
30	6.031702	2.8568	4.1824	0.0000	-2.8568	10	0	5.199673

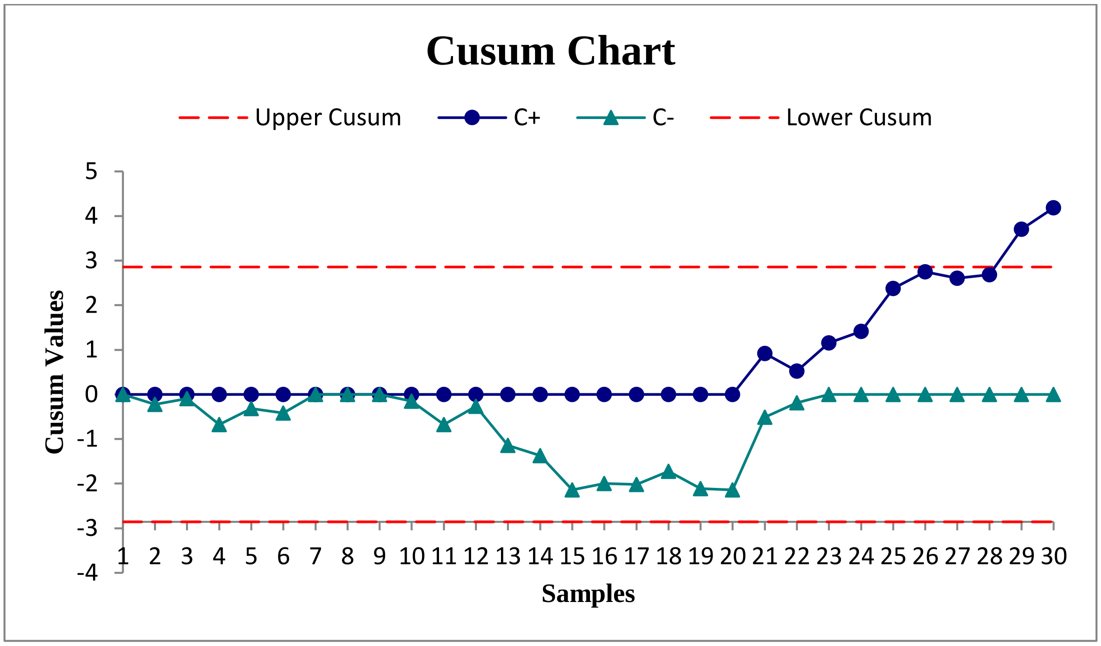


Figure 2. Cumulative Sum Control Chart

TABLE 3: CALCULATIONS FOR SELF-STARTING CUSUM

n	x_n	$\bar{x}_n$	w_n	S_n	T_n	a_nT_n	$F_{n-2}(a_nT_n)$	U_n
1	5.113973	5.113973	0	-	-	-	-	-
2	4.622701	4.868337	0.1207	0.347419	-	-	-	-
3	4.967329	4.901334	0.127233	0.252223	0.284936	0.232649	0.572757	0.1834
4	4.259926	4.740982	0.435786	0.381133	-2.54302	-2.20232	0.079276	-1.41
5	5.205094	4.833805	0.608106	0.389906	1.217717	1.089159	0.822122	0.92348
6	4.737614	4.817773	0.615817	0.350946	-0.2467	-0.22521	0.416431	-0.21103
7	5.522105	4.918392	1.041031	0.41654	2.006951	1.858075	0.938863	1.5453
8	5.228674	4.957177	1.125272	0.40094	0.744904	0.696794	0.743994	0.65571
9	5.455491	5.012545	1.345998	0.410183	1.242863	1.171782	0.86019	1.0812
10	4.68992	4.980283	1.439676	0.399955	-0.78654	-0.74618	0.238478	-0.71121
	. . . . . .							
21	6.472604	4.891083	6.335004	0.562806	3.758616	3.668034	0.999183	3.1497
22	5.166059	4.903582	6.407178	0.552362	0.48858	0.477346	0.680853	0.47008
23	6.187529	4.959406	7.984023	0.60242	2.324467	2.273373	0.983189	2.1246
24	5.814002	4.995014	8.683926	0.614461	1.418604	1.388735	0.910592	1.3444
25	6.51973	5.056003	10.9157	0.674404	2.481389	2.431255	0.988376	2.2693
26	5.928888	5.089575	11.64832	0.682593	1.294306	1.269172	0.891724	1.2358
27	5.411949	5.101515	11.74839	0.672206	0.472278	0.463449	0.676471	0.45785
28	5.640702	5.120772	12.02873	0.667464	0.802115	0.787661	0.780991	0.77554
29	6.576862	5.170982	14.07582	0.709019	2.181524	2.143582	0.979385	2.0412
30	6.031702	5.199672	14.79197	0.71419	1.213959	1.193555	0.878666	1.1683

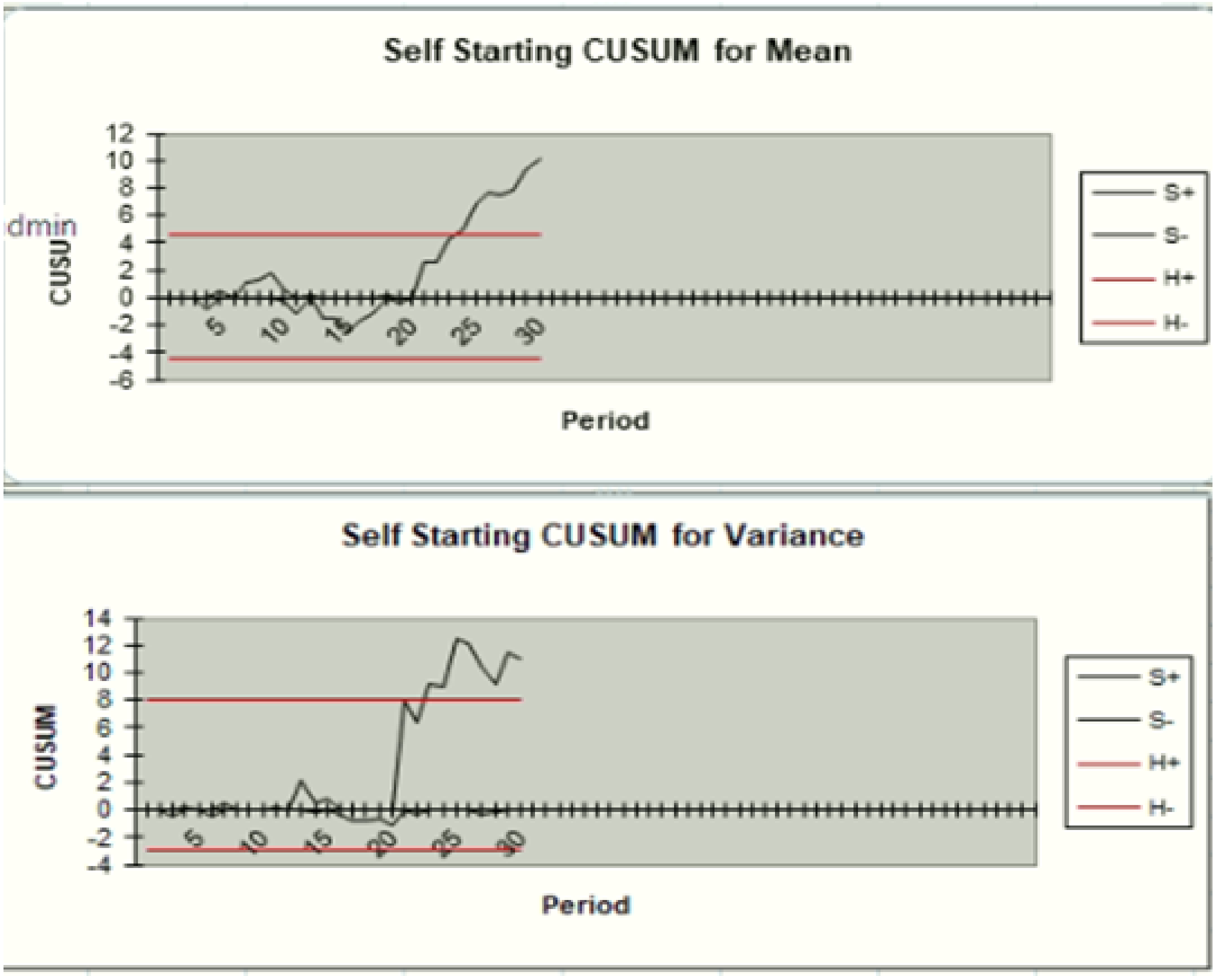


Figure 3: Self-Starting Cumulative Sum Control Chart

The calculation of the Self – Starting CUSUM control chart is given in the Table 3. From the Fig 3, it can be noted that the increasing trend started from the 21st observation and from the 24th observations onwards all the points are in out of control whereas in the CUSUM control chart the 29th and 30th observations are in out of control. From the Fig 2 and 3, it can be concluded that the self-starting CUSUM is more sensitive than CUSUM control chart. Because, CUSUM control chart showed the out of control at the last two observations. But in Self-Starting CUSUM, 24th observation onwards the points are in out of control. The Self-Staring CUSUM shows an alarm at the earliest compared with CUSUM control chart.

5. CONCLUSION

The CUSUM control chart is very authoritative chart for identifying changes in the process average. This paper discussed the problem based on monitoring the moisture level in the paper sheet material using the traditional and CUSUM control chart. From the standing theory, it is clear that CUSUM charts are valuable for monitoring the process performance, as they tend to detect process shifts on the 21^{th} observation itself, where the traditional charts showed no change. This will help the decision maker to proactively respond to the output and correct the problems in time. CUSUM chart is particularly adapted to detect the processes characterized by small shifts, whereas traditional chart is effective in detecting variation of the process response. Also Shewhart chart is better in detecting an immediate abrupt (transient) change, the CUSUM chart is more effective in detecting more sustained changes. Hence it is clear that the CUSUM chart is far superior to the traditional control chart in detecting the small (1 σ) shift. The Self Starting – CUSUM chart is very easy to use and it quickly detects both small and large shifts in the process mean and /or standard deviation. This Self Starting- control chart has then additional advantage over other charts because it swiftly detects decreases in the process SD as well. It also shows graphically the parameters and the sample that triggers an alarm at the earliest, thus indicating that the process is out of control.

Conflict of Interests

The authors declare that there is no conflict of interests.

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