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ON LEXICOGRAPHIC PRODUCTS OF TWO INTUITIONISTIC FUZZY GRAPHS

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Abstract. In this paper, lexicographic products of two intuitionistic fuzzy graphs, namely, lexicographic min product and lexicographic max product which are analogous to the concept lexicographic product in crisp graph theory are defined. It is illustrated that the operations lexicographic products are not commutative. The connected effective and complete properties of the operations lexicographic products are studied. The degree of a vertex in the lexicographic products of two intuitionistic fuzzy graph is obtained. A relationship between the lexicographic min product and lexicographic max–product is also obtained.

Key words: connected intuitionistic fuzzy graph; effective and regular intuitionistic fuzzy graph; lexicographic min-product and lexicographic max - product.

2010 AMS Subject Classification: 05C72.

1. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Later on Bhattacharya [1] gave some remarks on fuzzy graphs Operations on fuzzy graphs were introduced by Mordeson and Peng [3]. We defined the direct sum of two fuzzy graphs and studied its properties [8].

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In this paper we introduce the concept of lexicographic products of two intuitionistic fuzzy graphs namely, lexicographic min product and lexicographic max product which are analogous to the concept lexicographic product in crisp graph theory. We have illustrated that these operations are not commutative and studied the connected effective and complete properties of these operations. We have obtained the degree of a vertex in the lexicographic products of two intuitionistic fuzzy graphs and obtained a relationship between the lexicographic min-product and lexicographic max-product. First let us recall some preliminary definitions that can be found in [1]-[9].

2. PRELIMINARIES

Definition 2.1

An intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$ where,

i) $V = \{v_1, v_2 \dots \dots . v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and$0 \leq \mu_1(v_1) + \gamma_1(v_1) \leq 1 \ \forall \ v_i \in V(i, = 1,2 \dots . n) \to (1)$

ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that

$\mu_2(v_1, v_2) \leq \min\, [\mu_1(v_1), \mu_1(v_2)] \to (2)$

$\gamma_2(v_1, v_2) \leq \max\, [\gamma_1(v_1), \gamma_1(v_2)] \to (3)$

and $0 \leq \mu_2(v_1, v_2) + \gamma_2(v_1, v_2) \leq 1 \ \forall \ (v_1, v_2) \in E(1,2 \dots \dots n) \to (4)$.

Definition 2.2

Let $G =< V, E >$ be an intuitionistic fuzzy graph. The degree of a vertex u is defined by $d(u) = (d_\mu(u), d_\gamma(u))$ where $d_\mu(u) = \sum_{u \neq v} \mu_2(u, v)$ and

$d_\gamma(u) = \sum_{u \neq v} \gamma_2(u, v)$.

Definition 2.3

An edge $e = (u, v)$ of an intuitionistic fuzzy graph $G = (V, E)$ is called an effective edge if $\mu_2(u, v) = \min\, (\mu_1(u), \mu_1(v))$ and $\gamma_2(u, v) = \max\, (\gamma_1(u), \gamma_1(v))$.

Definition 2.4

An intuitionistic fuzzy graph is complete if $\mu_2(u, v) = \min\, \{(\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) = \max\, \{\gamma_1(u), \gamma_1(v)\}$.

Definition 2.5

An intuitionistic fuzzy graph $G = (V, E)$ is said to be regular, if every vertex is adjacent to a vertex with same degree.

Definition 2.6

The lexicographic product of $G_1: (V_1, E_1)$ with $G_2: (V_2, E_2)$ is defined as $G_1[G_2]: (V, E)$ where $V = V_1 \times V_2$ and $E = \{(u_1, v_1)(u_2, v_2)/u_1u_2 \in E_1$ or $u_1 = u_2$ and $v_1v_2 \in E_2\}$

3. LEXICOGRAPHIC MIN PRODUCT

Definition: 3.1

Let $G_1: (\mu_1, \mu_2), (\gamma_1, \gamma_2)$ and $G_2: (\mu_1', \mu_2'), (\gamma_1', \gamma_2')$ denote two intuitionistic fuzzy graphs. Define $G: (\mu_1, \mu_2)$, and (γ_1, γ_2) with underlying crisp graph $G^*: (V, E)$ Where $V = V_1 \mathrm{x} \mathrm{V}_2, \mathrm{E} = \{(u_1, v_1)\ (u_2, v_2)/u_1u_2 \in E,\ or$

$u_1 = u_2\ and v_1v_2 \in E_2\}$ by $\mu_1(u_1, v_1) = \mu_1(u_1) \vee \mu_1'(v_1)$

$$\gamma_1(u_1v_1), = \gamma_1(u_1) \vee \gamma_1'(v_1)\ \forall\ (u_1v_1) \in V_1 \mathrm{x}\, V_2 \text{ and}$$

$$\mu_2^*((u_1, v_1)\ (u_2, v_2)) = \begin{cases} \mu_2\ (u_1u_2) & if\ u_1u_2 \in E_1 \\ \mu_1(u_1) \wedge \mu_2'(v_1v_2) & if\ \ u_1 = u_2, V_1\, V_2\ \in E_2 \end{cases}$$

$$\gamma_2^*((u_1, v_1)\ (u_2, v_2)) = \begin{cases} \gamma_2\ (u_1u_2) & if\ u_1u_2 \in E_1 \\ \gamma_1(u_1) \wedge \gamma_2'(v_1v_2) & if\ \ u_1 = u_2, V_1\, V_2\ \in E_2 \end{cases}$$

$if\ u_1, u_2 \in E_1, \mu_2(u_1u_2) = \mu_1(u_1) \wedge \mu_1(u_2)$

$$\leq [\mu_1(u_1) \vee \mu_1'(v_1)] \wedge [\mu_1(u_2) \vee \mu_1'(v_2)]$$

$$= \mu_1^*\ (u_1, v_1\ ) \wedge\ \ \mu_1^*\ (u_2, v_2\ )$$

$if\ u_1, u_2 \in E_1, \gamma_2(u_1u_2) = \gamma_1(u_1) \wedge \gamma_1(u_2)$

$$\leq [\gamma_1(u_1) \vee \gamma_1'(v_1)] \wedge [\gamma_1(u_2) \vee \gamma_1'(v_2)]$$

$$= \gamma_1^*(u_1v_1\ ) \wedge\ \ \gamma_1^*\ (u_2v_2\ )$$

$if\ u_1 = u_2, v_1v_2 \in E_2\ \mu_1'\ (u_1) \wedge \mu_2'\ (v_1\ v_2) \leq \mu_1(u_1) \wedge [\mu_1'(v_1) \wedge \mu_1'(v_2)]$

$$= [\mu_1(u_1) \wedge \mu_1'(v_1)] \wedge [\mu_1(u_2) \wedge \mu_1'(v_2)]$$

$$\leq [\mu_1(u_1) \vee \mu_1'(v_1)] \wedge [\mu_1(u_2) \vee \mu_1'(v_2)]$$

$$= [\mu_1^*(\ u_1, v_1) \wedge \mu_1^*(\ u_2, v_2)]$$

$if u_1 = u_2, v_1v_2 \in E_2\ \gamma_1'(u_1) \wedge \gamma_2'(v_1v_2) \leq \gamma_1(u_1) \wedge [\gamma_1'(v_1) \wedge \gamma_1'(v_2)]$

$$= [\gamma_1(\ u_1) \wedge \gamma_1'(v)] \wedge [\gamma_1(\ u_2) \wedge \gamma_1'(\ v_2)]$$

$$\leq [\gamma_1(\ u_1) \vee \gamma_1'(v_1)] \wedge [\gamma_1(\ u_2) \vee \gamma_1'(\ v_2)]$$

$= [\gamma_1{}^{*}(u_1, v_1) \wedge \gamma_1{}^{*}(u_2, v_2)]$

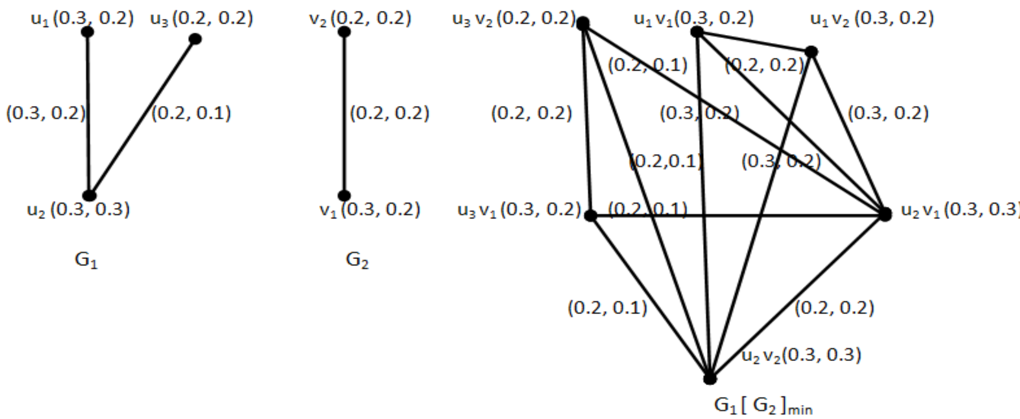
Hence $\mu_2{}^{*}((u_1, v_1)(u_2, v_2)) \leq \mu_1{}^{*}(u_1, v_1) \wedge \mu_1{}^{*}(u_2, v_2)$ and

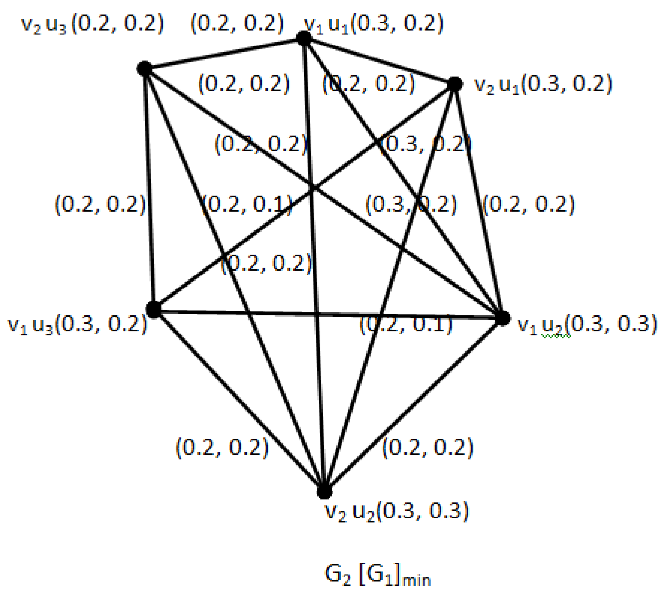
$\gamma_2{}^{*}((u_1, v_1)(u_2, v_2)) \leq \gamma_1{}^{*}(u_1, v_1) \wedge \gamma_1{}^{*}(u_2, v_2)$

$\therefore G: (\mu_1, \gamma_2)$ is an intuitionistic fuzzy graph. This is called the Lexicographic min product of G_1 with G_2 and is denoted by $G_1[G_2]_{min}: (\mu_1, \mu_2)(\gamma_1, \gamma_2)$

Remark 3.2

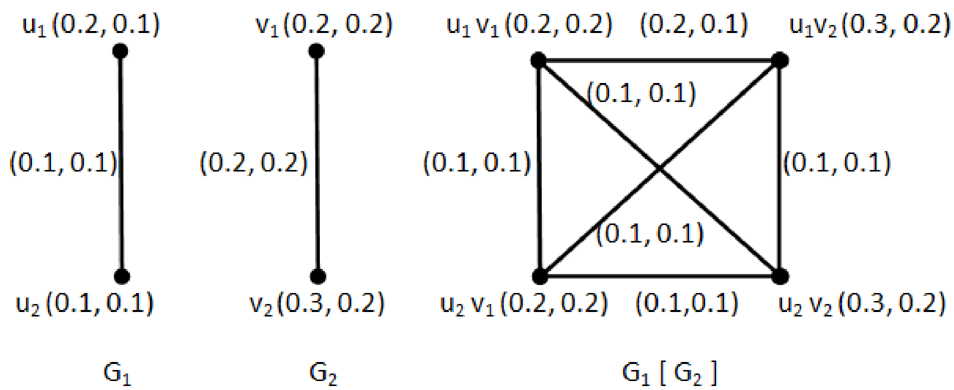
The operation Lexicographic min product of two intuitionistic fuzzy graphs is not commutative. That is $G_1[G_2]_{min}: (\mu_1, \mu_2), (\gamma_1, \gamma_2)$ is different from $G_2[G_1]_{min}: (\mu_1', \mu_2'), (\gamma_1', \gamma_2')$. This is illustrated through the following figure.





Remark 3.3

The lexicographic min - product of two effective intuitionistic fuzzy graphs need not be effective. Also the lexicographic min - product of two complete intuitionistic fuzzy graphs need not be complete.



Theorem 3.4

If G_1 and G_2 are two effective intuitionistic fuzzy graphs with underlying crisp graph ${G_1}^*$ and ${G_2}^*$ respectively such that $\mu_1 \geq {\mu_1}^*, \gamma_1 \geq {\gamma_1}^*$ and $(\mu_2, {\mu_2}^*)(\gamma_2\,, {\gamma_2}^*)$ are constant functions of same value, Then the lexicographic min product of G_1 with G_2 is an effective intuitionistic fuzzy graph.

Proof:

$If\ u_1u_2 \in E_1, \mu^*\big((u_1v_1)(u_2v_2)\big) = \mu_2\ (u_1u_2) = \mu_1(u_1) \wedge \mu_1(u_2) = {\mu_1}^*(u_1v_1) \wedge {\mu_1}^*(u_2v_2)$

$\gamma^*\ ((u_1, v_1)(u_2, v_2)) = \gamma_2\ (u_1, u_2) = \gamma_1(u_1) \wedge \gamma_1(u_2) = {\gamma_1}^*(u_1, v_1) \wedge {\gamma_1}^*(u_2, v_2)$

$If\ u_1 = u_2\ and\ v_1v_2 \in E_2\ \mu^*\ ((u_1v_1)(u_2v_2)) = {\mu_2}^*\ (\ v_1v_2) = \mu_2(u_1, u_2) = \mu_1(u_1) \wedge \mu_1(u_2)$

$= {\mu_1}^*(u_1, v_1) \wedge {\mu_1}^*(u_2, v_2)$

$\gamma^*\ ((u_1, v_1)(u_2, v_2)) = {\gamma_2}^*(v_1, v_2) = \gamma_2(u_1, u_2) = \gamma_1(u_1) \wedge \gamma_1(u_2)\ = {\gamma_1}^*(u_1, v_1) \wedge \gamma_1(u_2, v_2)$

Thus $\mu^*\big((u_1, v_1)(u_2, v_2)\big) = {\mu_1}^*\ (u_1, v_1) \wedge {\mu_1}^*\ (u_2, v_2) and$

$\gamma^*\big((u_1, v_1), (u_2, v_2)\big) = {\gamma_1}^*\ (u_1, v_1) \wedge {\gamma_1}^*(u_2, v_2)\ \big((u_1, v_1)(u_2, v_2)\big) \in E$ Hence $G_1[G_2]_{min}$ is an effective intuitionistic fuzzy graph.

Theorem 3.5

The lexicographic min product $G_1[G_2]_{min}$ of two connected intuitionistic fuzzy graphs G_1 and G_2 is connected intuitionistic fuzzy graph iff G is connected.

Proof:

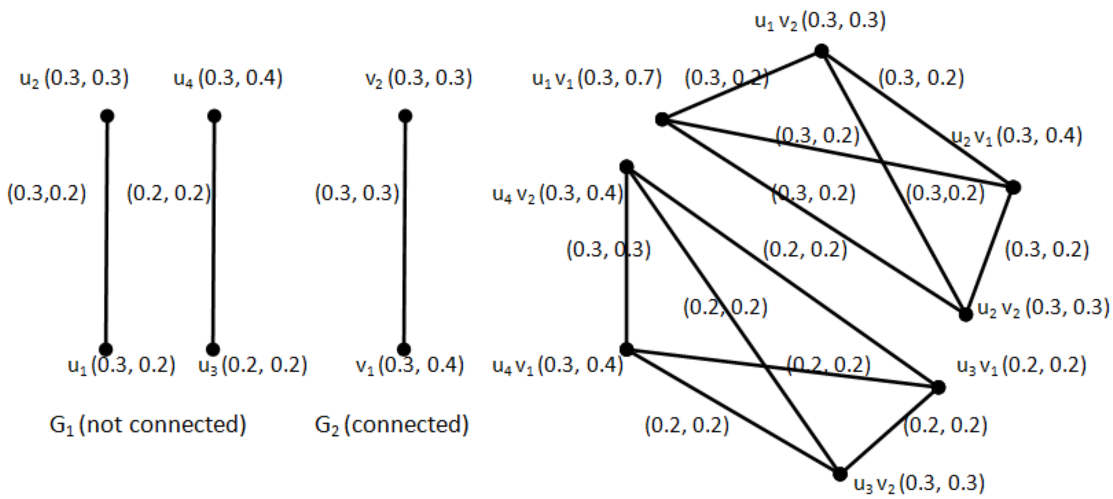
From the definition $G_1[G_2]_{min}$ has $|V_2|$ copies of G_1 That is for each vertex in G_2 there is a copy of G_1 in $G_1[G_2]_{min}$. Also G_1 is connected. Hence $G_1[G_2]_{min}$ is connected.

Conversely, assume that G_1 and G_2 be two intuitionistic fuzzy graphs such that $G_1[G_2]_{min}$ is connected.

To prove: G_1 is connected. Suppose that G_1 is not connected. Then ∃ at least two different vertices u_1, u_2 in V_1 such that there is no path between them. But since $G_1[G_2]_{min}$ is connected, for any two vertices of the form (u_1, v_i) and $(u_2, v_j) \in V_1 \text{x}\, V_2$ There is at least one path between them. This implies that there must be at least one path between the vertices (u_1, u_2). This is a contradiction. Hence G_1 is connected.

Example 3.6

Consider two intuitionistic fuzzy graphs G_1 and G_2 where G_1 is not a connected intuitionistic fuzzy graph and G_2 is a connected intuitionistic fuzzy graph. The lexicographic min product of G_1 with $G_2$is not connected intuitionistic fuzzy graph.



G₁ (not connected)   G₂ (connected)   G₁ [G₂]min (not connected)

Theorem 3.7

The number of connected components in the lexicographic min- product $G_1[G_2]_{min}$ of the intuitionistic fuzzy graph G_1 with G_2 is equal to that of the intuitionistic fuzzy graph G_1.

Proof:

Let G_1 be a connected intuitionistic fuzzy graph and G_2 be an intuitionistic fuzzy graph. Then the lexicographic min product $G_1[G_2]_{min}$ is connected. This implies that both G_1 and $G_1[G_2]_{min}$ are connected and hence the theorem. Suppose that the intuitionistic fuzzy graph G_1 is not connected and has 'm' disjoint connected components (say). Then we can rename the vertices of G_1 in such a way that.

$\{u_1, u_2 \dots \dots u_{k_1}\}, \{u_{k+1}, u_{k+2} \dots . u_{k_2}\}, \dots \dots .., \{u_{k_{m+1}}, u_{k_{m+2}} \dots . u_{k_{m+n}}\}$ are the vertex sets of the m disjoint connected components of $G_1. If$ $\{v_1, v_2 \dots \dots .. v_n\}$ is the vertex set of G_2 then for each vertex v_i in G_2, there is a copy of each connected component of G_1 in the lexicographic min product $G_1[G_2]_{min}$. There is no edge between these components. For if there is an edge between $u, v_i, u_{k_{1+1}} v_i$ Then there must be an edge between u_1, u_{k+1} in G_1 which is a contradiction. Thus each connected component in the Lexicographic min product $G_1[G_2]_{min}$ is disjoint from every other component and hence the theorem.

4. Degree of a vertex in the Lexicographic min product:

Definition 4.1

The degree of any vertex in the Lexicographic min- product $G_1[G_2]_{min}$ of the intuitionistic fuzzy graph G_1 with G_2 is given by

$d_{G_1[G_2]_{min}}(u_{i,} v_j) = \sum_{u_i u_k \in E, v_l \in V_2} \mu_2(u_i u_k) + \sum_{u_i = u_k, v_j v_l \in E_2} \mu_1(u_i) \wedge \mu_2'(v_j v_l)$ and

$d_{G_1[G_2]_{min}}(u_{i,} v_j) = \sum_{u_i u_k \in E, v_l \in V_2} \gamma_2(u_i u_k) + \sum_{u_i = u_k, v_j v_l \in E_2} \gamma_1(u_i) \wedge \gamma_2'(v_j v_l)$

Theorem 4.2

If G_1 and G_2 are two intuitionistic fuzzy graphs such that $(\mu_1 \geq \mu_2')$, $(\gamma_1 \geq \gamma_2')$ Then the degree of a vertex in the Lexicographic min product $G_1[G_2]_{min}$ of the intuitionistic fuzzy graph G_1 with G_2 is given by,

$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$

Proof:

Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $\mu_1 \geq \mu_2'$ and $\gamma_2 \geq \gamma_2'$. This implies that $\mu_1 \wedge \mu_2' = \mu_2'$ and $\gamma_1 \wedge \gamma_1' = \gamma_1'$. Then the degree of any vertex $(u_i, v_j) \in V_1 \text{x}\, V_2$

is given by

$$d_{G_1[G_2]_{min}}(u_i,v_j) = \sum_{u_iu_k \in E_1, v_l \in V_2} \mu_2(u_iu_k) + \sum_{u_i = u_k, v_jv_l \in E_2} \mu_1(u_i) \wedge {\mu_2}'(v_jv_l)$$

$$= |V_2| \sum_{u_iu_k \in E_1} \mu_2(u_i\, u_k) + \sum_{u_i = u_k, v_jv_l \in E_2} {\mu_2}'(v_jv_k)$$

$$= |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)$$

$$d_{G_1[G_2]_{min}}(u_i,v_j) = \sum_{u_iu_k \in E_1, v_l \in V_2} \gamma_2(u_iu_k) + \sum_{u_i = u_k, v_jv_l \in E_2} \gamma_1(u_i) \wedge {\gamma_2}'(v_jv_l)$$

$$= |V_2| \sum_{u_iu_k \in E_1} \gamma_2(u_iu_k) + \sum_{u_i = u_k, u_jv_l \in E_2} {\gamma_2}'(v_jv_l)$$

$$= |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)$$

Corollary 4.3

If G_1 and G_2 are two intuitionistic fuzzy graphs such that $\mu_1 \geq {\mu_2}'$ and $\gamma_1 \geq {\gamma_2}'$ and ${\mu_2}', {\gamma_2}'$ is a constant function of value ‘c’ then the degree of vertex in the lexicographic min – product $G_1[G_2]_{min}$ of the intuitionistic fuzzy graph $G_1$with G_2 is given by

$$d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)\, C$$

Theorem 4.4

If G_1 and G_2 are two intuitionistic fuzzy graphs such that $\mu_1 \leq {\mu_2}'$ and $\gamma_1 \leq {\gamma_2}'$ Then the degree of a vertex in the lexicographic min product $G_1[G_2]_{min}$ of the intuitionistic fuzzy graph $G_1$with G_2 is given by

$d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)\, \mu_1(u_1)$ and

$d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)\, \gamma_1(u_i)$

Proof:

Let G_1 and G_2 be two intuitionitic fuzzy graphs such that $\mu_1 \leq {\mu_2}'$ and $\gamma_1 \leq {\gamma_2}'$. This implies that $\mu_1 \wedge {\mu_2}' = \mu_1$ and $\gamma_1 \wedge {\gamma_2}' = \gamma_1$. Then the degree of any vertex $(u_i, v_j) \in V_1 \text{ x } V_2$is given by

$$d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_jv_l \in E_2} \mu_1(u_i) \wedge {\mu_2}'(v_jv_l)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i=u_k, v_jv_l \in E_2} \mu_1(u_i)$$

$$= |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)\, \mu_1(u_i)$$

and

$$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k,\, v_j v_l \in E_2} \gamma_1(u_i) \wedge \gamma_2'(v_j v_l)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k,\, v_j v_l \in E_2} \gamma_1(u_i)$$

$$= |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)\, \gamma_1(u_i)$$

5. Lexicographic Max – Product

Definition: 5.1

Define G with underlying crisp graph G^* Where $V = V_1 \text{ x } V_2$, $E = \{(u_1v_1)(u_2v_2)/ u_1u_2 \in E_1$ or $u_1 = u_2$ and $v_1v_2 \in E_2\}$ by $\mu_1(u_1v_1), = \mu_1(u_1) \vee \mu_1'(u_1)$, $\gamma_1(u_1v_1) = \gamma_1(u_1) \vee \gamma_1'(u_1)$ for all $(u_1v_1) \in V_1 \text{ x } V_2$ and

$$\mu_2^*((u_1, v_1)\,(u_2, v_2)) = \begin{cases} \mu_2(u_1, u_2) & if\ u_1u_2 \in E_1 \\ \mu_1(u_1) \vee (\mu_2'(v_1, v_2) & if u_1 = u_2, v_1v_2 \in E_2 \end{cases}$$

$$\gamma_2^*((u_1, v_1)\,(u_2, v_2)) = \begin{cases} \gamma_2(u_1, u_2) & if\ u_1u_2 \in E_1 \\ \gamma_1(u_1) \vee (\gamma_2'(v_1, v_2) & if u_1 = u_2, v_1v_2 \in E_2 \end{cases}$$

$if\ u_1u_2 \in E_1, \mu_2(u_1, u_2) = \mu_1(u_1) \wedge \mu_1(u_2) \quad \leq [\mu_1(u_1) \wedge \mu_1'(v_1)] \vee [\mu_1(u_2) \wedge \mu_1'(v_2)]$

$= \mu_1^*(u_1v_1) \wedge \mu_1^*(u_2v_2)$

$if\ u_1u_2 \in E_1, \gamma_2(u_1, u_2) = \gamma_1(u_1) \wedge \gamma_1'(u_2) \quad \leq [\gamma_1(u_1) \wedge \gamma_1'(v_1)] \vee [\gamma_1(u_2) \wedge \gamma_1'(v_2)]$

$= \gamma_1^*(u_1v_1) \wedge \gamma_1^*(u_2v_2)$

$if\ u_1 = u_2, v_1v_2 \in E_2\ \mu_1'(u) \vee \mu_2'(v_1v_2) \leq \mu_1(u_1) \vee [\mu_1'(v_1) \wedge \mu_1'(v_2)]$

$= [\mu_1(u_1) \vee \mu_1'(v_1)] \wedge [\mu_1(u_2) \vee \mu_1'(v_2)]$

$\leq [\mu_1(u_1) \vee \mu_1'(v_1)] \wedge [\mu_1(u_2) \vee \mu_1'(v_2)]$

$= \mu_1^*(u_1v_1) \wedge \mu_1^*(u_2v_2)]$

$if\ u_1 = u_2, v_1v_2 \in E_2\ \gamma_1'(u) \wedge \gamma_2'(v_1v_2) \leq \gamma_1(u_1) \wedge [\gamma_1'(v_1) \wedge \gamma_1'(v_2)]$

$= [\gamma_1(u_1) \vee \gamma_1'(v_1)] \wedge [\gamma_1(u_2) \wedge \gamma_1'(v_2)]$

$\leq [\gamma_1(u_1) \wedge \gamma_1'(v_1)] \wedge [\gamma_1(u_2) \wedge \gamma_1'(v_2)]$

$= \gamma_1^*(u_1v_1) \wedge \gamma_1^*(u_2v_2)]$

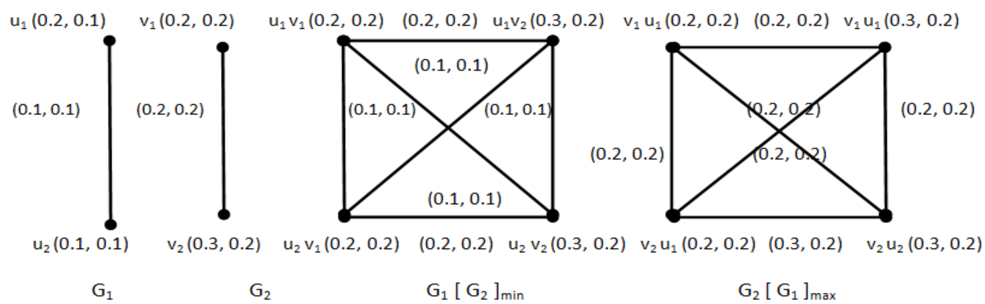
Hence $(u_1v_1)\,(u_2v_2) \leq \mu_1^*(u_1v_1) \wedge \mu_1^*(u_2v_2)$ and

$\gamma_2^*\,(u_1v_1)(u_2v_2) \leq \gamma_1^*(u_1v_1) \wedge \gamma_1^*(u_2v_2)$

$\therefore G$ is a intuitionistic fuzzy graph. This is called the Lexicographic max product of $G_1$with G_2 and is denoted by $G_1[G_2]_{max}$

Remark 5.2

The operation Lexicographic max – product of two intuitionistic fuzzy graphs is not commutative. That is, $G_1[G_2]_{max} \neq G_2[G_1]_{max}$. Also the Lexicographic max product of two effective intuitionistic fuzzy graphs need not be effective and the lexicographic max- product of two complete intuitionistic fuzzy graph not be a complete



Theorem 5.3

The Lexicographic max-product $G_1[G_2]_{max}$of two connected intuitionistic fuzzy graphs G_1 and G_2 is a connected intuitionistic fuzzy graph if and only if $G_1$is connected.

(Proof of this theorem is similar to the proof of the theorem 5.2)

Theorem 5.4

The number of connected components in the Lexicographic max-product $G_1[G_2]$ of the intuitionistic fuzzy graph G_1 with G_2 is equal to that of the intuitionistic fuzzy graph G_1

(Proof this theorem is similar to the proof of the theorem 5.3)

6. DEGREE OF A VERTEX IN LEXICOGRAPHIC MAX-PRODUCT

Definition 6.1

The degree of any vertex in the lexicographic max-product $G_1[G_2]$ of the intuitionistic fuzzy graph $G_1$with G_2 is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_l \in V_2} \mu_2(u_i, u_k) + \sum_{u_i = u_k, v_jv_l \in E_2} \mu_1(u_i) \vee \mu_2{}'(v_jv_l) \text{ and}$$

$$= \sum_{u_iu_k \in E_1, v_l \in V_2} \gamma_2(u_i, u_k) + \sum_{u_i = u_k, v_jv_l \in E_2} \gamma_1(u_i) \wedge \gamma_2{}'(v_jv_l)$$

Theorem 6.2

If G_1 and G_2be two intuitionistic fuzzy graph such that $\mu_1 \leq \mu_2'$. Then the degree of any vertex in $G_1[G_2]_{max}$ is given by

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$$

Proof:

Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $\mu_1 \leq \mu_2'$. This implies that $\mu_1 \vee \mu_2' = \mu_2', \gamma_1 \vee \gamma_2' = \gamma_2'$,

Then the degree of any vertex$(u_i, v_j) \in V_1 \text{x}\, V_2$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = \sum_{u_i u_k \in E_1, v_l \in V_2} \mu_2(u_i u_k) + \sum_{u_i = u_k, v_j v_l \in E_2} \mu_1(u_i) \vee \mu_2'(v_j v_l)$$

$$= |V_2| \sum_{u_i u_k \in E_1} \mu_2(u_i u_k) + \sum_{u_i = u_{k,} v_j v_l \in E_2} \mu_2'(v_j v_l)$$

$$= |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)$$

and

$$d_{G_1[G_2]_{max}}(u_i, v_j) = \sum_{u_i u_k \in E_1, v_l \in V_2} \gamma_2(u_i u_k) + \sum_{u_i = u_k, v_j v_l \in E_2} \gamma_1(u_i) \vee \gamma_2'(v_j v_l)$$

$$= |V_2| \sum_{u_i u_k \in E_1} \gamma_2(u_i u_k) + \sum_{u_i = u_{k,} v_j v_l \in E_2} \gamma_2'(v_j v_l)$$

$$= |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)$$

Corollary 6.3

If G_1 and G_2 are two intuitionistic fuzzy graphs such that $\mu_1 \leq \mu_2'$ and $\gamma_1 \leq \gamma_2', \mu_2', \gamma_2'$, is a constant function of value c, then for any vertex (u_i, v_j) in $G_1[G_2]$ is given by

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_i)\, C$$

Theorem 6.4

If G_1 and G_2 are two intuitionistic fuzzy graphs such that $\mu_1 \geq \mu_2', \gamma_1 \geq \gamma_2'$ then the degree of any vertex in $G_1[G_2]_{max}$ is given by

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)\mu_1(u_i) \quad \text{and}$$

$$= |V_2|\, d_{G_2}(u_i) + d_{G_2}(v_j)\gamma_1(u_i)$$

Proof:

Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $\mu_1 \geq \mu_2{}'$. and $\gamma_1 \geq \gamma_2{}'$. This implies that $\mu_1 \vee \mu_2{}' = \mu_1$ and $\gamma_1 \vee \gamma_2{}' = \gamma_1$ then the degree of any vertex $(u_i, v_j) \in V_1 \text{x} V_2$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_j v_l \in E_2} \mu_1(u_i) \vee \mu_2{}'(v_j v_l)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_j v_l \in E_2} \mu_1(u_i) = |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)\mu_1(u_i)$$

and

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_j v_l \in E_2} \gamma_1(u_i) \vee \gamma_2{}'(v_j v_l)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_j v_l \in E_2} \gamma_1(u_i)$$

$$= |V_2| d_{G_1}(u_i) + d_{G_2}(v_j)\gamma_1(u_i)$$

7. RELATIONSHIP BETWEEN THE LEXICOGRAPHIC PRODUCTS

Theorem 7.1

The lexicographic min-product $G_1[G_2]_{\,min}$ of the intuitionistic fuzzy graph $G_1$with G_2 is a spanning intuitionistic fuzzy subgraph of the lexicographic max-product $G_1[G_2]_{\,max}$.

Proof:

Consider the lexicographic products $G_1[G_2]_{\,max}$ and $G_1[G_2]_{min}$ of G_1 with G_2 defined on G^* where $V = V_1 \text{x} V_2$ $E = \{(u_1 v_1)\,(u_2 v_2)/u_1 u_2 \in E_1 or \quad u_1 = u_2 and\ v_1 v_2 \in E_2\}$ from the definitions of the lexicographic max-product and the lexicographic min product it is clear that

$\mu_1^*(u_1 v_1) = \mu_1(u_1 v_1) \forall\, (u_1 v_1) \in V$

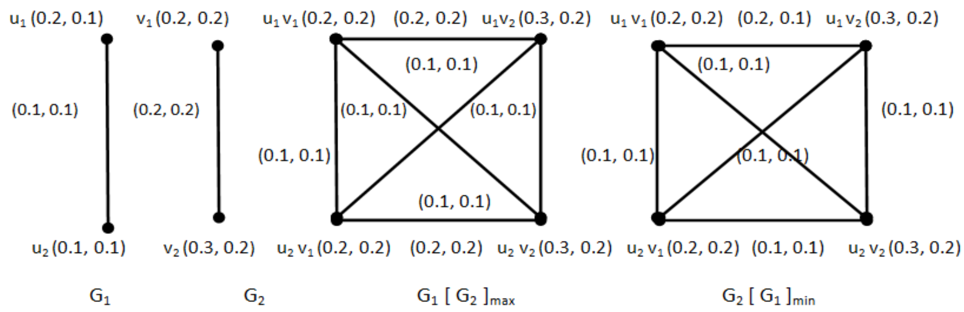
$\gamma_1^*(u_1 v_1) = \gamma_1(u_1 v_1) \forall\, (u_1 v_1) \in V$ and

$\mu_2{}^*((u_1 v_1)(u_2 v_2)) \geq \mu_2((u_1 v_1)(u_2 v_2)) \forall\, (u_1 v_1)(u_2 v_2) \in E$

Thus $\mu_1^*= \mu_2$ and $\mu_2 \subseteq \mu_2{}^*$ Hence the lexicographic min-product is a spanning intuitionistic subgraph of the lexicographic max-product

Example 7.2

Consider the following example. Here the lexicographic min–product $G_1[G_2]$ is a spanning intuitionistic fuzzy subgraph of the lexicographic max-product $G_1[G_2]$



8. CONCLUSION

In this paper, we have introduced the concept of lexicographic products of two intuitionistic fuzzy graphs namely, lexicographic min – product and lexicographic max – product which are analogous to the concept of lexicographic products in crisp graph theory. We have illustrated that the operations lexicographic products are not commutative and studied the connected, effective and complete properties of these operations. We have obtained the degree of a vertex in the lexicographic product of two intuitionistic fuzzy graph. Also we have obtained a relationship between the lexicographic min – product and lexicographic max – product. In addition to the existing operations there operations and properties will also be helpful to study large intuitionistic fuzzy graph as a combination of small intuitionistic fuzzy graph and to derive its properties from those of the small ones.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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