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INVESTIGATION OF A PRODUCTION INVENTORY MODEL WITH TWO SERVERS HAVING MULTIPLE VACATIONS

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Abstract. In this paper, a production inventory system with two servers involving multiple vacations is considered. Customers' demand is in accordance with the Poisson process. The time for production and addition of each item to the inventory is exponentially distributed. The reloading of the inventory is done as per the (s, S) policy. When no customer waits for service in the system or no inventory is available to satisfy their demands or both, multiple vacations are taken by the servers. The period taken by servers 1 and 2 for their vacation is also exponentially distributed. The system works with the assumption that both the servers are heterogeneous. It is also presumed that their service rates are exponentially distributed with parameters μ_1 and μ_2. The minimum service rates of both the servers are taken as μ and such a case is also considered with two homogeneous servers. The final algorithmic solution to the problem is obtained by Matrix Analytic Method (MAM). We could also derive some significant measures of performance of the model in the steady state. Finally, we could also construct and analyze cost function numerically.

Keywords: cost analysis; matrix analytic method; heterogeneous servers; homogeneous servers.

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1. INTRODUCTION

The study of production inventory models with multiple vacations has received remarkable attention of researchers in recent decades. To redress the problem of the long queue with customers waiting for service, the decision maker provides another server. Multiple servers are more flexible and practically applicable compared to single server models. Another advantage of a multi-server queueing system is that the servers can engage in secondary jobs when they are idle. The credit for developing a production inventory system having service time and vacation to the server goes to Krishnamoorthy and Viswanath C. Narayanan [3]. The system had a MAP/PH/1 queue attached to it. Items were produced in the inventory as per the time that follows a ”Markovian Production Scheme”. They calculated system stability and performance. Another M/M/2 queueing inventory system was discussed by Krishnamoorthy and Sreenivasan [4]. They modeled the system with two heterogeneous servers, in which one was a vacationing server. Customers enjoyed a different quality of service with this heterogeneous service mechanism. Two production inventory systems with varying production rates were devised by Jose and Salini [1]. The system had both retrials and buffer. To find the numerical solution they used the MAM method. Referring to Neuts [9] can provide more details of the MAM.

Yet another (s, S) production inventory system was studied by Anoop and Jacob [8]. It was a queueing system that had controlled self service. It provided service only to one customer at a time though it was a multi sever Markovian queueing model. Using the MAM method one could easily check ergodicity and steady state solutions. Vijayashree and Janani [12] experimented with a multi server queueing system that had a single exponential vacation.The customers arrive as per the Poisson process and services take place according to an exponential distribution. All the servers in the system go for a vacation when the system is empty and returns after the fixed time gap. The model helps us to explicitly obtain the stationary and transient probabilities for the number of customers during the ideal and functional state of the servers. For an in-depth study of a Markovian queueing system having two heterogeneous servers that take working vacations, Vishwanath Maurya [7] demonstrated a mathematical model. At first, a service policy where both the heterogeneous servers take a vacation was considered. In the model, they worked out a busy period analysis.

A new model was developed by Vijayalaxmi and Jyothsna [6] who observed the performance of a working vacation queue with heterogeneous servers and had a renewal input. Supplementary variable and recursive techniques were used to find the different performance measures and the steady state probabilities of the model. A Markovian (s, Q) inventory queueing system was developed by Jeganathan [2] that had two heterogeneous servers with server interruptions. This was to compare the efficiency of heterogeneous and homogeneous systems; by deriving their performance measures. Palanivel and Uthayakumar [10] devised an economic production quantity model with varying production costs and probabilistic deterioration. For a Quasi Birth Death Process Latouche and Ramaswami [5] derived a logarithmic reduction algorithm. The non-ergodicity and mean drifts of the Markov chain were derived by Sennott et al. [11]. The following section of this paper discusses ; 1. A detailed description of the model 2. Analysis of the system after modeling it mathematically 3. Derivation of its stability condition 4. Discussion of Steady state probability vector and its explicit expressions 5. Derivation of various relevant performance measures and their explicit form 6. Numerical experiments.

2. DESCRIPTION OF THE MODEL

This paper considers a production facility with two servers; server 1 and server 2. The servers go for a vacation when the inventory level reaches zero or no demand is made or both. θ_1 and θ_2 are the parameters with which the vacation times of the servers are exponentially distributed.The servers take independent and identically distributed time gaps for their vacations.The model works with the assumption that the service rates of the heterogeneous servers are exponentially distributed with parameters μ_1 and μ_2. Then, a minimum service rate of μ_1 and μ_2 named μ with homogeneous servers is also considered. In these two instances, only when there is a positive inventory level with at least one customer waiting for service does the service get started at the end of the vacation. In all the other cases both the servers go for another vacation until there is a positive inventory level and customer level. A customer who comes and finds a free server with a positive inventory level can get the service immediately. The (s, S) policy renews the inventory. Customer demands are in accordance with the Poisson process with rate

λ. Production has to be started when there is a decline in inventory level say s. It gets stopped when the level of inventory reaches S. Two successive items are added to the inventory and their time gap follows an exponential distribution with rate β.

3. Mathematical Modeling and Analysis

The following are the assumptions and notations used in this model.

Assumptions

(i) customer demands are in accordance with the Poisson process with rate λ.

(ii) the service rates of the heterogeneous servers are exponentially distributed with parameter μ_1 and μ_2.

(iii) the time gap of the addition of two successive items (by production) to the inventory is exponential with rate β.

(iv) θ_1 and θ_2 are the parameters with which vacation durations of server 1 and server 2 are exponentially distributed.

Notations

At time t, N(t) be the number of customers in the system

At time t, the inventory level is I(t)

$$C(t): \begin{cases} 0 \text{ if both servers are on vacation} \\ 1 \text{ if server 1 is busy and server 2 is on vacation} \\ 2 \text{ if server 1 is on vacation and server 2 is busy} \\ 3 \text{ if server 1 and server 2 are busy} \end{cases}$$

$$J(t): \begin{cases} 0 \text{ if the production process is switched off} \\ 1 \text{ if the production process gets switched on} \end{cases}$$

$$\mathbf{e} : (1,1,1,...,1)'$$

Then the quadruplet $\{(N(t),C(t),J(t),I(t)),t \geq 0\}$ is a continuous time Markov chain on the state space $\{(0,0,0,k), s+1 \leq k \leq S\} \cup \{(0,0,1,k), 0 \leq k \leq S-1\} \cup \{(1,j,0,k), j = 1,2, s+$

$1 \leq k \leq S\} \cup \{(1,j,1,k), j=1,2, 1 \leq k \leq S-1\} \cup \{(i,0,1,0), i \geq 1\} \cup \{(i,j,0,k), i \geq 2, j = 1,2,3, s+1 \leq k \leq S\} \cup \{(i,j,1,k), i \geq 2, j=1,2, 1 \leq k \leq S-1\} \cup \{(i,3,1,k), i \geq 2, 2 \leq k \leq S-1\}$.

Now the infinitesimal generator of the process is

$$Q = \begin{bmatrix} B_{00} & B_{01} & 0 & 0 & 0 & \dots \\ B_{10} & B_{11} & B_{12} & 0 & 0 & \dots \\ 0 & B_{21} & A_1 & A_0 & 0 & \dots \\ 0 & 0 & A_2 & A_1 & A_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where A_0, A_1, A_2 are square matrices of order $6S-3s-3$.

$$B_{00} = \begin{bmatrix} -\lambda I_{S-s} & 0 \\ \Psi_1 & D_1 \end{bmatrix}_{(2S-s)} ; [D_1]_(uv) = \begin{cases} \text{-}\beta, if u = v, u = 1 \\ \beta, \text{ if } 1 \leq u \leq S-1, v = u+1 \\ \text{-}(\lambda+\beta), \text{ if } 2 \leq u \leq S, v = u \end{cases}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_0 & \Delta_1 & 0 & 0 & 0 & 0 \\ \Delta_2 & 0 & \Delta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & \Delta_5 & 0 & 0 \\ \Delta_6 & 0 & 0 & 0 & \Delta_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_8 & \Delta_9 \\ 0 & 0 & \Delta_6 & 0 & \Delta_2 & 0 & \Delta_{10} \end{bmatrix}$$

$$[\Delta_0]_{uv} = \begin{cases} \mu_1, \text{ if } 2 \leq u \leq S-s, v = u-1 \\ 0, \text{ otherwise} \end{cases} ; [\Delta_1]_{uv} = \begin{cases} \mu_1, \text{ if } u = 1, v = s \\ 0, \text{ otherwise} \end{cases}$$

$$[\Delta_2]_{uv} = \begin{cases} \mu_1, \text{ if } u = v, u = 1 \\ 0, \text{ otherwise} \end{cases} ; [\Delta_3]_{uv} = \begin{cases} \mu_1, \text{ if } 2 \leq u \leq S-1, v = u-1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Delta_4]_{uv} = \begin{cases} \mu_2, \text{ if } 2 \leq u \leq S-s, v = u-1 \\ 0, \text{ otherwise} \end{cases} ; [\Delta_5]_{uv} = \begin{cases} \mu_2, \text{ if} u = 1, v = s \\ 0, \text{ otherwise} \end{cases}$$

$$[\Delta_6]_{uv} = \begin{cases} \mu_2, \text{ if } u=1, v=u \\ 0, \text{ otherwise} \end{cases} ; [\Delta_7]_{uv} = \begin{cases} \mu_2, \text{ if } 2 \leq u \leq S-1, v=u-1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Delta_8]_{uv} = \begin{cases} (\mu_1+\mu_2), \text{ if } 2 \leq u \leq S-s, v=u-1 \\ 0, \text{ otherwise} \end{cases} ; [\Delta_9]_{uv} = \begin{cases} (\mu_1+\mu_2,) \text{ if } u=1, v=s-1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Delta_{10}]_{uv} = \begin{cases} (\mu_1+\mu_2), \text{ if } 2 \leq u \leq S-2, v=u-1 \\ 0, \text{ otherwise} \end{cases}$$

$$B_{10} = \begin{bmatrix} 0 & 0 \\ \Delta_0 & \Delta_1 \\ 0 & \nabla_1 \\ \Delta_4 & \Delta_5 \\ 0 & \nabla_2 \end{bmatrix}_{(4S-2s-1)\times(2S-s)} ; \nabla_1 = \begin{bmatrix} \mu_1 I_{S-1} & 0 \end{bmatrix}_{(4S-2s)\times(2S-s)}$$

$$\nabla_2 = \begin{bmatrix} \mu_2 I_{S-1} & 0 \end{bmatrix}_{(4S-2s)\times(2S-s)}$$

$$A_1 = \begin{bmatrix} -\beta & 0 & \Psi_0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\mu_1+\theta_2)I_{S-s} & 0 & 0 & 0 & \theta_2 I_{S-s} & 0 \\ 0 & \Psi_1 & \Psi_2 & 0 & 0 & 0 & \Psi_3 \\ 0 & 0 & 0 & -(\lambda+\mu_2+\theta_1)I_{S-s} & 0 & \theta_1 I_{S-s} & 0 \\ 0 & 0 & 0 & \Psi_1 & \Psi_4 & 0 & \Psi_5 \\ 0 & 0 & 0 & 0 & 0 & -(\lambda+\mu_1+\mu_2)I_{S-s} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Psi_6 & \Psi_7 \end{bmatrix}$$

$$[\Psi_0]_{uv} = \begin{cases} \beta \text{ if u= 1 ,v=u} \\ 0 \text{ otherwise} \end{cases} ; [\Psi_1]_{uv} = \begin{cases} \beta, \text{ if } u=S-1, v=S-s \\ 0, \text{ otherwise} \end{cases}$$

$$[\Psi_2]_{uv} = \begin{cases} \text{-}(\lambda+\mu_1+\beta), \text{ if } u=v, u=1 \\ \text{-}(\lambda+\beta+\mu_1+\theta_2), \text{ if } 2 \leq u \leq S-1, v=u \\ \beta, \text{ if } 1 \leq u \leq S-2, v=u+1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Psi_3]_{uv} = \begin{cases} \theta_2, \text{ if } 2 \le u \le S-1, v = u-1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Psi_4]_{uv} = \begin{cases} \text{-}(\lambda+\mu_2+\beta), \text{ if } u = v, u = 1 \\ \text{-}(\lambda+\beta+\mu_2+\theta_1), \text{ if } 2 \le u \le S-1, v = u \\ \beta, \text{ if } 1 \le u \le S-2, v = u+1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\Psi_5]_{uv} = \begin{cases} \theta_1, \text{ if } 2 \le u \le S-1, v = u-1 \\ 0, \text{ otherwise} \end{cases} \quad ; [\Psi_6]_{uv} = \begin{cases} \beta, \text{ if } u = S-3, v = S-s \\ 0, \text{ otherwise} \end{cases}$$

$$[\Psi_7]_{uv} = \begin{cases} \text{-}(\lambda+\mu_1+\mu_2+\beta), \text{ if } 1 \le u \le S-2, v = u \\ \beta, \text{ if } 1 \le p \le S-3, v = u+1 \\ 0, \text{ otherwise} \end{cases}$$

$$B_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_0 & \Delta_1 & 0 & 0 \\ \Delta_2 & 0 & \Delta_3 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & \Delta_5 \\ \Delta_6 & 0 & 0 & 0 & \Delta_7 \\ 0 & \Delta_4 & \Delta_5 & \Delta_0 & \Delta_1 \\ 0 & 0 & K_1 & 0 & \nabla_1 \end{bmatrix}_{(6S-3s-3)\times(4S-2s-1)} \quad ; K_1 = \begin{bmatrix} \mu_2 I_{S-1} & 0 \end{bmatrix}_{(4S-2s)\times(2S-s)}$$

$$B_{11} = \begin{bmatrix} -\beta & 0 & \Psi_0 & 0 & 0 \\ 0 & -(\lambda+\mu_1)I_{S-s} & 0 & 0 & 0 \\ 0 & \Psi_1 & \chi_1 & 0 & 0 \\ 0 & 0 & 0 & -(\lambda+\mu_2)I_{S-s} & 0 \\ 0 & 0 & 0 & \Psi_1 & \chi_2 \end{bmatrix}_{(4S-2s-1)}$$

$$[\chi_1]_{uv} = \begin{cases} \text{-}(\lambda+\beta+\mu_1), \text{ if } 1 \le u \le S-1, v=u \\ \beta, \text{ if } 1 \le u \le S-2, v=u+1 \\ 0, \text{ otherwise} \end{cases}$$

$$[\chi_2]_{uv} = \begin{cases} \text{-}(\lambda+\beta+\mu_2), \text{ if } 1 \le u \le S-1, v=u \\ \beta, \text{ if } 1 \le u \le S-2, v=u+1 \\ 0, \text{ otherwise} \end{cases}$$

$$A_0 = \begin{bmatrix} 0 & & & & & & \\ & I_{S-s} & & & & & \\ & & I_{S-1} & & & & \\ & & & I_{S-s} & & & \\ & & & & I_{S-1} & & \\ & & & & & I_{S-s} & \\ & & & & & & I_{S-2} \end{bmatrix}$$

$$B_{12} = \begin{bmatrix} 0 & & & & & 0 \\ & \lambda I_{S-s} & & & & 0 \\ & & \lambda I_{S-1} & & & 0 \\ & & & \lambda I_{S-s} & & 0 \\ & & & & \lambda I_{S-1} & 0 \end{bmatrix}_{(4S-2s-1)\times(6S-3s-3)}$$

$$B_{01} = \begin{bmatrix} 0 & \lambda I_{S-s} & 0 & 0 \\ 0 & 0 & Z_1 & 0 \end{bmatrix}_{(4S-2s-1)} ; Z_1 = \begin{bmatrix} 0 & \lambda I_{S-1} \end{bmatrix}_{(S)}$$

4. Algorithmic Analysis

4.1. Stability Condition.

Theorem 1. $\frac{\lambda}{\mu_1+\mu_2} < 1$ *is the necessary and sufficient condition for the system to be stable.*

Proof. To prove the stability condition of the system, we use Pake's Lemma. According to Pake's Lemma,"if there exists an $\varepsilon > 0$ such that the mean drift $\psi_j = E[(T_{i+1} - T_i)/T_i = j]$ is finite for all j$\in N$ and $\psi_j \le -\varepsilon$ for all $j \in N$ except perhaps for a finite number then the irreducible aperiodic Markov chain is ergodic". Let T_i be the number of customers in the system immediately after the service completion of the i^{th} customer, then $\{T_i : i \in N\}$ satisfies

$$T_i = \begin{cases} T_{i-1} - 1 + V_i, & \text{if } T_{i-1} \geq 1 \\ V_i, & \text{if } T_{i-1} = 0 \end{cases}$$

where V_i denotes the number of arrivals into the system during the service of the i^{th} customer. Here the Mean drift is

$$\psi_j = \begin{cases} -1 + \rho, & \text{if } j \geq 1 \\ \rho, & \text{if } j = 0 \end{cases}$$

The Markov chain $\{T_i : i \in N\}$ is ergodic if $\rho < 1$. For proving the necessary condition, we assume that $\rho \geq 1$. Then using the theorem in Sennot, if $\{T_i : i \in N\}$ satisfies Kaplan's condition then the Markov chain is non-ergodic. Here $\psi_j < \infty$ for $j \geq 0$ and there exists a j_0 such that $\psi_j \geq 0$, for $j \geq j_0$. Hence the Markov chain is not ergodic when $\rho \geq 1$

□

5. Steady State Probability Vector

Let $P = (P_0, P_1, \ldots)$ be the steady state probability vector of Q. Our objective is to compute the stationary probability vector P from the system of equations $PQ = 0$. Under the conditions of stability P_i's are given by $P_i = P_{i-1} * R$ for (i = 3,4,5...) where R is the minimal nonnegative solution of the matrix quadratic equation $R^2 A_2 + RA_1 + A_0 = 0$, with spectral radius of R is less than one. The rate matrix R is computed from $R = -A_0(A_1)^{-1} - R^2 A_2 A_1^{-1}$. R is approximated by the successive substitution method developed by Neuts namely $R_0 = 0$,
$R_{n+1} = -A_0(A_1)^{-1} - R_n^2 A_2 (A_1)^{-1}, n = 0, 1, 2, \ldots$. The elements of R will increase after each iteration. The process is continued until the successive difference in the value of R is less than a specified tolerance criterion. The sub vectors P_0, P_1, and P_2 can be calculated using

$$\left.\begin{aligned} P_0 B_{00} + P_1 B_{10} &= 0 \\ P_0 B_{01} + P_1 B_{11} + P_2 B_{21} &= 0 \\ P_1 B_{12} + P_2 [A_1 + RA_2] &= 0 \end{aligned}\right\} \quad (1)$$

The normalizing equation is

$$P_0 e + P_1 e + P_2 (I-R)^{-1} e = 1 \Big\} \tag{2}$$

The boundary probabilities P_0, P_1, and P_2 and the probabilities P_i for $i \geq 3$ can be obtained from using equations 1, 2 and R.

6. System Performance Measures

We partition the components of P_i as

$$P_i = (y_{i,0,1,0}, y_{i,1,0,s+1}, \dots, y_{i,1,0,S}, y_{i,1,1,1} \cdots, y_{i,1,1,S-1}, y_{i,2,0,s+1}, \dots, y_{i,2,0,S},$$

$$y_{i,2,1,1}, \dots, y_{i,2,1,S-1}, y_{i,3,0,s+1}, \dots, y_{i,3,0,S}, y_{i,3,1,2}, \dots, y_{i,3,1,S-1}), (i \geq 2)$$

Also,

$$P_0 = (y_{0,0,0,s+1}, \dots, y_{0,0,0,S}, y_{0,0,1,0} \cdots, y_{0,0,1,S-1}),$$

and

$$P_1 = (y_{1,1,0,s+1}, \dots, y_{1,1,0,S}, y_{1,1,1,1} \cdots, y_{1,1,1,S-1}, y_{1,2,0,s+1}, \dots, y_{1,2,0,S}, y_{1,2,1,1}, \dots, y_{1,2,1,S-1})$$

Now we derive some performance measures of the system under steady state

(i) Expected inventory level, $\vee_I$, is given by

$$\vee_I = \sum_{k=s+1}^{S} k y_{0,0,0,k} + \sum_{k=1}^{S-1} k y_{0,0,1,k} + + \sum_{j=1}^{2} \sum_{k=s+1}^{S} k y_{1,j,0,k} + \sum_{j=1}^{2} \sum_{k=1}^{S-1} k y_{1,j,1,k} +$$

$$\sum_{j=1}^{3} \sum_{k=s+1}^{S} \sum_{i=2}^{\infty} k y_{i,j,0,k} + \sum_{j=1}^{2} \sum_{k=1}^{S-1} \sum_{i=2}^{\infty} k y_{i,j,1,k} + \sum_{k=2}^{S-1} \sum_{i=2}^{\infty} k y_{i,3,1,k}$$

(ii) Expected number of customers in the system, $\vee_C$, is given by

$$\vee_C = (\sum_{i=1}^{\infty} i P_i) e = P_1 e + P_2 \{(I-R)^{-1} + (I-R)^{-2}\} e$$

(iii) Expected reorder rate, $\vee_R$, is given by

$$\vee_R = \mu_1 \sum_{i=1}^{\infty} y_{i,1,0,s+1} + \mu_2 \sum_{i=1}^{\infty} y_{i,2,0,s+1} + (\mu_1 + \mu_2) \sum_{i=2}^{\infty} y_{i,3,0,s+1}$$

(iv) The fraction of time production process is on $\vee_{ON}$ is given by

$$\vee_{ON} = \sum_{j=1}^{2} \sum_{k=1}^{S-1} y_{1,j,1,k} + \sum_{k=0}^{S-1} y_{0,0,1,k} + \sum_{i=1}^{\infty} y_{i,0,1,0} + \sum_{i=1}^{\infty} \sum_{k=1}^{S-1} \sum_{j=1}^{2} y_{i,j,1,k} + \sum_{i=2}^{\infty} \sum_{k=2}^{S-1} y_{i,3,1,k}$$

(v) Expected number of departures $\vee_D$ after completing service is given by

$$\vee_D = \mu_1\{\sum_{i=1}^{\infty}\sum_{k=s+1}^{S} y_{i,1,0,k} + \sum_{i=1}^{\infty}\sum_{k=1}^{S-1} y_{i,1,1,k}\} + \mu_2\{\sum_{i=1}^{\infty}\sum_{k=s+1}^{S} y_{i,2,0,k} + \sum_{i=1}^{\infty}\sum_{k=1}^{S-1} y_{i,2,1,k}\}$$
$$+(\mu_1+\mu_2)\{\sum_{i=2}^{\infty}\sum_{k=s+1}^{S} y_{i,3,0,k} + \sum_{i=2}^{\infty}\sum_{k=2}^{S-1} y_{i,3,1,k}\}$$

(vi) The probability that server 2 is on vacation

$$\vee_{vac2} = P_0 + \sum_{i=1}^{\infty}\sum_{k=s+1}^{S} y_{i,1,0,k} + \sum_{i=1}^{\infty} y_{i,0,1,0} + \sum_{i=1}^{\infty}\sum_{k=1}^{S-1} y_{i,1,1,k}$$

where P_0 is the probability that no customer is in the system when both servers are on vacation.

(vii) The probability that server 1 is on vacation

$$\vee_{vac1} = P_0 + \sum_{i=1}^{\infty}\sum_{k=s+1}^{S} y_{i,2,0,k} + \sum_{i=1}^{\infty} y_{i,0,1,0} + \sum_{i=1}^{\infty}\sum_{k=1}^{S-1} y_{i,2,1,k}$$

7. Cost Analysis

Define the expected total cost of the system per unit per unit time is given by

$$T_{cost} = (C + (S-s)c_1)\vee_R + c_2 \vee_I + c_3 \vee_C + c_4 \vee_D, \text{ where}$$

C : Fixed cost/unit/unit time

c_1 : The procurement cost per unit per unit time

c_2 : The holding cost of inventory per unit per unit time

c_3 : The holding cost of customers per unit per unit time

c_4 : The cost due to service per unit per unit time

7.1. Numerical Experiments.

The effect of the arrival rate of the customers on different system performance measures and T_{cost} are illustrated in table 1. In tables 2,3,4 we represent the effect of $\theta_1, \theta_2, \beta$ on some of the performance measures and T_{cost} of the system for heterogeneous and homogeneous servers, keeping the other values of the parameters fixed.

TABLE 1. Effect of variation of λ with heterogeneous and homogeneous servers

λ	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HT)$	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HM)$
0.8	13.416	0.40938	0.53499	0.019096	**2100.5**	13.395	0.49710	0.54396	0.019256	**2103.3**
0.9	13.362	0.45026	0.59150	0.020838	**2100.0**	13.331	0.57571	0.60817	0.022162	**2101.8**
1.0	13.306	0.49318	0.64966	0.022891	**2099.6**	13.265	0.66851	0.67689	0.025572	**2103.1**
1.1	13.249	0.53910	0.64966	0.022891	**2099.5**	13.198	0.77966	0.75108	0.029451	**2105.5**
1.2	13.190	0.58893	0.77335	0.027856	**2099.6**	13.131	0.91433	0.83151	0.033755	**2109.5**
1.3	13.130	0.64361	0.83984	0.030714	**2100.0**	13.064	1.07920	0.91880	0.038433	**2115.4**
1.4	13.070	0.70407	0.90991	0.033779	**2101.1**	12.997	1.28310	1.01340	0.043429	**2123.5**

$C = 20, c_1 = 15, c_2 = 151, c_3 = 35, c_4 = 104, \mu_1 = 1.2, \mu_2 = 2.5, \beta = 5, \theta_1 = 3, \theta_2 = 4, S = 20, s = 5,$

TABLE 2. Effect of variation of θ_1 with heterogeneous and homogeneous servers

θ_1	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HT)$	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HM)$
3.1	13.553	0.28760	0.36653	0.015959	**325.7501**	13.564	0.31265	0.36584	0.013661	**327.4075**
3.2	13.554	0.28824	0.36661	0.015768	**325.7183**	13.564	0.31262	0.36583	0.013651	**327.4024**
3.3	13.554	0.28825	0.36660	0.015768	**325.7193**	13.564	0.31259	0.36582	0.013657	**327.3973**
3.4	13.554	0.28826	0.36660	0.015768	**325.7205**	13.563	0.31256	0.36582	0.013655	**327.3724**
3.5	13.554	0.28826	0.36659	0.015768	**325.7202**	13.563	0.31253	0.36581	0.013654	**327.3679**
3.6	13.554	0.28827	0.36659	0.015767	**325.7207**	13.563	0.31250	0.36580	0.013652	**327.3628**
3.7	13.554	0.28827	0.36658	0.015767	**325.7205**	13.563	0.31248	0.36580	0.013651	**327.3598**

$C = 20, c_1 = 42.2, c_2 = 20.1, c_3 = 118.1, c_4 = 24.4, \mu_1 = 1.2, \mu_2 = 2.5, S = 20, s = 5, \beta = 5, \lambda = .5, \theta_2 = 4$

TABLE 3. Effect of variation of θ_2 with heterogeneous and homogeneous servers

θ_2	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HT)$	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HM)$
4.1	13.554	0.28806	0.36658	0.015762	**197.7897**	13.563	0.31255	0.36581	0.013658	**331.6369**
4.2	13.553	0.28789	0.36655	0.015755	**197.7674**	13.563	0.31242	0.36577	0.013654	**331.6241**
4.3	13.553	0.28774	0.36652	0.015749	**197.7584**	13.563	0.31230	0.36574	0.013649	**331.6088**
4.4	13.553	0.28759	0.36650	0.015743	**197.7498**	13.562	0.31218	0.36570	0.013645	**331.5751**
4.5	13.553	0.28745	0.36647	0.015737	**197.7409**	13.562	0.31206	0.36567	0.013641	**331.5626**
4.6	13.552	0.28731	0.36644	0.015732	**197.7210**	13.562	0.31196	0.36564	0.013638	**331.5530**
4.7	13.552	0.28718	0.36642	0.015726	**197.7125**	13.561	0.31186	0.36561	0.013634	**331.5196**

$C = 20, c_1 = 184.2, c_2 = 21, c_3 = 7.1, c_4 = 18, S = 20, s = 5, \beta = 5, \lambda = .5, \theta_1 = 3, \mu_1 = 1.2, \mu_2 = 2.5$

TABLE 4. Effect of variation of β with heterogeneous and homogeneous servers

β	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HT)$	$\vee_I$	$\vee_C$	$\vee_D$	$\vee_R$	$T_{cost}(HM)$
5.1	13.556	0.28823	0.36662	0.015770	**37.3104**	13.565	0.31269	0.36585	0.013665	**32.7315**
5.2	13.557	0.28823	0.36662	0.015771	**37.3119**	13.567	0.31269	0.36585	0.013666	**32.7340**
5.3	13.559	0.28823	0.36662	0.015771	**37.3139**	13.568	0.31269	0.36585	0.013668	**32.7359**
5.4	13.560	0.28823	0.36662	0.015772	**37.3154**	13.570	0.31269	0.36585	0.013671	**32.7379**
5.5	13.561	0.28823	0.36662	0.015773	**37.3169**	13.572	0.31269	0.36585	0.013673	**32.7424**
5.6	13.563	0.28823	0.36662	0.015774	**37.3193**	13.573	0.31269	0.36585	0.013674	**32.7438**
5.7	13.564	0.28823	0.36662	0.015775	**37.3208**	13.574	0.31269	0.36585	0.013676	**32.7458**

$C = 20, c_1 = 31, c_2 = 1, c_3 = 5, c_4 = 40, S = 20, s = 5, \lambda = .5, \theta_1 = 3, \theta_2 = 4, \mu_1 = 1.2, \mu_2 = 2.5$

7.2. Numerical Inferences.

Here for both the heterogeneous and homogeneous systems, one of the parameters is varied at a time to analyze the total expected cost per unit per unit time. Table 1 indicates that when λ becomes 1.1 the T_{cost} of the heterogeneous servers becomes minimum with a value of 2099.50. On the contrary, the minimum value of the T_{cost} for the homogeneous servers is 2105.50. Table 2 indicates that for a heterogeneous system T_{cost} shows a minimum value of 325.72 when $\theta_1 = 3.2$. In the case of homogeneous servers, the minimum value of 327.40 is shown at θ_1. The heterogeneous and homogeneous systems show 197.79 and 331.64 respectively the value of T_{cost} at $\theta_2 = 4.1$(from table 3). The homogeneous servers show 32.73 as the value of T_{cost} at $\beta = 5.1$. In the same case, the value of the T_{cost} of the heterogeneous servers is 37.31. In all cases, heterogeneous servers show minimum T_{cost} though it can vary for replenishment rates.

8. CONCLUDING REMARKS

The paper considered a production inventory system with two servers taking multiple vacations. The model was examined using MAM. One could easily calculate the stability condition and system performance measures. Based on the performance measures of the model, a cost function was constructed and was analyzed numerically. This work gives way to new researchers by considering a multi server production inventory system with Markovian Arrival Process and Phase type service time.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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