PERFORMANCE ANALYSIS OF A COMPLEX REPAIRABLE SYSTEM WITH TWO SUBSYSTEMS IN SERIES CONFIGURATION WITH AN IMPERFECT SWITCH

VIJAY VIR SINGH[1,*], PRAVEEN KUMAR POONIA[2] AND AMEER HASSAN ADBULLAHI[3]

[1]Department of Mathematics, Yusuf Maitama Sule University, Kano State, Nigeria

[2]Department of General Requirement, Sur College of Applied Sciences, Oman

[3]Department of Mathematics, Kano University of Science & Technology Wudil, Kano State, Nigeria

Abstract: This paper presents the study of reliability measures of a complex system consisting of two subsystems, subsystem-1, and subsystem-2, in a series configuration with switching device. The subsystem-1 has five units that are working under 2-out-of-5: G policy and the subsystem-2 has two units that are working under 1-out-of-2: G policy. Moreover, the switching device in the system is unreliable, and as long as the switch fails, the whole system fails immediately. Failure rates of units of subsystems are constant and assumed to follow the exponential distribution. Still, their repair supports two types of distribution, namely general distribution and Gumbel-Hougaard family copula distribution. Using the supplementary variable technique, Laplace transformations, and copula approach differential equations developed. Important reliability characteristics such as availability of the system, reliability of the system, MTTF, profit analysis, and sensitivity analysis for MTTF have computed for fixed values of failure and repair rates. Particular cases corresponding to the switching device have also considered. Graphs demonstrate results, and consequently, conclusions have done.

Keywords: k-out-of-n: G system; availability; reliability; MTTF; cost analysis; Gumbel-Hougaard family-copula distribution.

2010 AMS Subject Classification: 90B25.

---

*Corresponding author

E-mail address: singh_vijayvir@yahoo.com

1. INTRODUCTION

Complicated systems such as computers, automobiles industry, telephone networks, and various electronic networks are becoming a prevalent feature and essential requirements of our society. The systems are built with multiple components/ parts to perform specified tasks adequately. It is often difficult to assure that the systems will perform particular tasks efficiently for which they designed. Due to various causes, it is difficult to anticipate the failure of a component and sometimes impossible to prevent the failure of the entire system. Reliability is a vital need for proper uses and repair of any engineering system. Achieving a high or required level of reliability and availability of the system is often an essential requisite based on system designed structure. The importance and utility of a system depend on its successful performance, and its performance depends on its design. The availability and reliability of an industrial system may be enhancing using a highly reliable structural design of the system or subsystem of higher reliability. The best way to improve system reliability is to add redundant components in the design. A constructive and common form of redundancy is a k-out-of-n configuration. Many researchers have brought their attention to the study of k-out-of-n: G systems and k-out-of-n: F systems. The k-out-of-n: G system is good if and only if at least k of its n components is good, while k-out-of-n: F system fails if and only if at least k of its n components fails. For example, an airplane with four engines can be modeled as a 3-out-of-4: G system. Furthermore, consider a large truck with ten tires is an example of 6-out-of-10: G system. Although the system performance may be degraded if less than ten tires are operational, rearrangement of the tire configuration will result in adequate performance as long as at least six tires are operational. In nuclear power plant system 2-out-of-4: G; system can perform adequate power supply. Conclusively a k-out-of-n system plays a very crucial role in system reliability theory to the proper operation of the system.

In the past decade, many researchers have focused on k-out-of-n-type systems mainly because such systems are more general than pure parallel or pure series systems, and they frequently come across in practice. There is an extensive literature available for reliability analysis of k-out-of-n-type systems under various situation such as [10], repairable systems with different failure modes [25], three-unit series system under warm standby [15], consecutive k-out-of-n system using standby with multiple working vacations [19], generalized block replacement policy with respect to a threshold number of failed components and risk costs [13], non-identical components subject to repair priorities [1] and non-identical components considering shut-off rules using quasi-birth-death process [18] among others. There are some real systems such as

satellites, transmission systems, or computer systems where some new equipment groups need to add because of the requirement for better output of the system. Realizing this fact, authors like Alka and Singh [3] analyzed reliability analysis of a complex repairable system composed of two 2-out-of-3: G subsystems connected in a parallel configuration. They analyzed the system by using the supplementary variable technique and obtain various measures such as mean time to failure, steady-state probability, availability, and cost analysis. Yusuf et al. [7] focus on the comparative study of 2-out-of-3: G system for the different situations under the concept of general repair analyzed using Kolmogorov's forward equations method. The objective of this study is to see the effect of preventive maintenance and system design of 2-out-of-3. In addition, Yusuf et al. [8, 9] developed an explicit expression for mean time to system failure for a 3-out-of-5 warm standby system involving common cause failure and ensured the maximum overall MTSF of the system.

Considering one type of repair/general repair to a totally failed system may cause a massive loss due to the non-operation of the system, and the industry/organization may drop its market reputation. Several authors, including El-Said and EL-Sherbeny [11], Bulama et al. [12], Gupta et al. [16, 17] and Malik et al. [20] examined the reliability characteristics under the presumption that the failed unit can be repaired by employing only one type of repair. There are many situations in real life where more than one repair is possible between two adjacent transition states for quick repair of the failed system. When such type of possibility exists, the system is repaired using the Gumbel-Hougaard family copula; it couples the two distributions, namely general distribution and exponential distribution. Therefore, in contrast to this, authors have considered models in which they tried to address a problem where two different repair facilities are available between adjacent states, i.e., the initial state and totally failed state. Ram and Singh [14] have studied availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using the Gumbel-Hougaard family copula in repair. Singh et al. [21] have studied cost analysis of an engineering system involving two subsystems in a series configuration with controllers and human failure under the concept of k-out-of-n: G policy using Gumbel-Hougaard family copula distribution. Also, in [22, 23], Singh et al. have studied the performance analysis of the complex system in the series configuration under different failure and repair disciplines using copula and controllers. Bona et al. [5] have discussed the reliability allocation based integrated factor method (IFM) approach to the aerospace system. The consequence of the study of reliability allocation method Di, Bona, Forcina, A, and Silvestri, A, [6] have proposed a new reliability allocation method as a critical flow method (CFM) for the

thermonuclear system. Recently Lado et al. [2] analyzed two subsystems connected in a series configuration and operated by a human operator. In this study, they concluded that copula repair is more reliable compared to general repair. Also, Babu et al. [4] studied a δ-shock maintenance model for a deteriorating system with an imperfect delayed repair under partial process. In addition, Singh and Poonia [24] studied two units parallel system with correlated lifetime under inspection using regenerative point technique.

Authors who studied k-out-of-n systems have put attention toward the operation of units in parallel/series configuration or in a circular arrangement with catastrophic failure and preventive maintenance but did not consider transfer switch and its failure. Therefore, realizing the fact and necessity of such type of configuration, we in the present analyzing a complex system having two subsystems viz. subsystem-1 and subsystem-2 under k-out-of-n: G configuration. Both subsystems connected in series, and each linked with a switching device for the proper functioning of the system, which may be perfect or imperfect at the time of need. The subsystem-1 follows 2-out-of-5: good configuration, and subsystem-2 follows 1-out-of-2: good configuration. All the units in both the subsystems are in a parallel configuration. The system has three possible transition states: Good, partially failed and complete failed. The system may move to the failed state as per the following options:

(i) More than three units of subsystem-1 fail, but both units of subsystem-2 are in good working condition.

(ii) Both of the units of the subsystem-2 fail.

(iii) The switching device of the subsystem-1 / subsystem-2 fails.

In addition to this, the system will be in a partially failed state in the following situations:

(i) At least one and maximum up to 3 units of subsystem-1 failed, and all the units of subsystem-2 are good.

(ii) All units of subsystem-1 are good, and anyone unit of subsystem-2 fails.

To carry out performance analysis, the authors have evaluated the expressions for the availability of the system, reliability of the system, MTTF, profit analysis, and sensitivity analysis corresponding to MTTF using the supplementary variable technique. All the results analyzed in the model using *mapple17*. This paper is planned in various sections. Section 1 describes the brief introduction of the paper, which focuses on the relevant literature reviewed for the study of

the proposed design. Section 2 to 6 covers the state description, assumptions, nomenclature of notation used for the study of a mathematical model, and transition diagram. Section 7 and 8 cover the analytical part of the paper in which some particular cases are taken for discussion and elaboration. Section 8 describes the conclusion of the study with results.

2. STATE DESCRIPTION

The description of the various possible state of the model after failing the units in both the subsystems, including transfer switch failure, is given in Table 1. The states $\{S_0, S_1, S_2, S_3, and S_5\}$ are operative states, and $\{S_4, S_6, S_7, and S_8\}$ are inoperative states of the system.

Table 1 State Description of the model

State	State description
S_0	This is a perfect state, and all units of subsystem-1 and subsystem-2 are in good working condition.
S_1	The indicated state represents that the system is degraded but is in operational mode after the failure of any one unit in subsystem-1, but both units of subsystem-2 are in a good operational state. The system is under repair.
S_2	The indicated state represents that the system is degraded but is in operational mode after the failure of any two units in subsystem-1, but both units of subsystem-2 are in a good operational state. The system is under repair.
S_3	The indicated state represents that the system is degraded but is in operational mode after the failure of any three units in subsystem-1. Still, both units of subsystem-2 are in a good operational state. The system is under repair.
S_5	The indicated state represents that the system is degraded but is in operational mode after the failure of anyone unit in subsystem-2, but all the units of subsystem-1 are in a good operational state. The system is under repair.
S_4	The states represent that the system is in totally failed mode after failing more than three units in the subsystem 1. The system is under repair using the Gumbel-Hougaard family copula distribution.
S_6	The states represent that the system is in a complete failed state after failing both units in subsystem 2. The system is under repair using employing copula distribution.
S_7	It is a complete failed state due to switch failure in the subsystem-1.
S_8	The state S_8 represents a complete failed state by failing switch device in subsystem-2.

3. ASSUMPTIONS

The following assumptions have been made throughout the study of the model:

1. Initially, the system is in the state S_0, and all the units of subsystem-1 and subsystem-2 are in good working conditions.

2. The subsystem-1 works successfully until three or more than three units are in good working condition, i.e., 2-out-of-5:G policy.

3. The subsystem-2 works successfully if one or both units are in good working condition, i.e., 1-out-of-2:G policy.

4. Both the subsystems having switching devices, which may be unreliable at the time as long as the switch fails, the whole system fails immediately.

5. The units in both the subsystems are in parallel mode and hot standby and ready to start within a negligible time after the failure of any unit in the subsystems.

6. Repairperson is available to full time with the system and maybe called as soon as the system reaches to partially or totally failed state.

7. All failure rates are constant and follow the exponential distribution.

8. The complete failed system needs repair immediately. For this, copula can be employed to restore the system.

9. No damage reported due to the repair of the system.

10. As soon as the failed unit repaired, it is ready to perform the task as good as new.

4. NOTATIONS

t, s	Time scale and Laplace transform variable
λ_1 / λ_2	The failure rate of each unit in the subsystem-1/subsystem-2.
$\lambda_{s_1} / \lambda_{s_2}$	The failure rate of the switching devices between the units for subsystem-1/subsystem-2.
$\phi_1(x)/\phi_1(y)$	The Repair rate of each unit in the subsystem-1/subsystem-2.
$\mu_0(z)$	Repair rate of the switching device for both the subsystems.
$P_0(t)$	The state transition probability that the system is in state S_i at an instant $i = 0$.
$\bar{P}(s)$	Laplace transformation of the state transition probability $P(t)$.
$P_i(x,t)$	The probability that the system is in the state S_i for $i = 1$ to 8 and the system is under repair with elapsed repair time is x,t. x is repaired variable and t is time variable.
$E_p(t)$	Expected profit in the interval. [0, t)
K_1, K_2	Revenue generated and service cost per unit time, respectively.
$\mu_0(x)$	An expression of the joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula, is given $\mu_0(x) = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}$ where $u_1(x) = \phi(x)$ and $u_2(x) = e^x$. Here θ is the parameter $1 < \theta < \infty$.

5. SYSTEM CONFIGURATION AND STATE TRANSITION DIAGRAM

System configuration is shown in Fig 1 (a) while the transition diagram in Fig 1 (b). In transition diagram, S_0 is perfect state, S_1, S_2, S_3, and S_5 partial failed/degraded and S_4, S_6, S_7, and S_8 are complete failed states. Due to failure in any unit in the subsystem-1 and in subsystem-2, the transitions approach to partially failed states S_1, S_2, S_3, and S_5, respectively. The state S_4 and S_6 are complete failed states due to the failure of units in both the subsystems. The states S_7 and S_8 are complete failed states due to transfer switch failure.
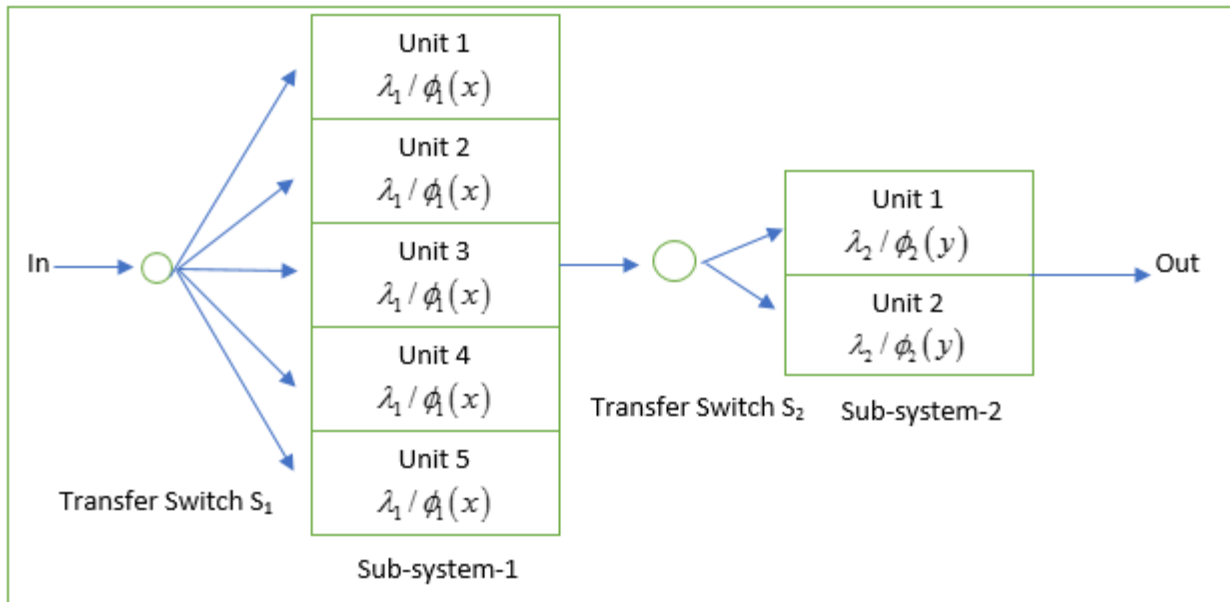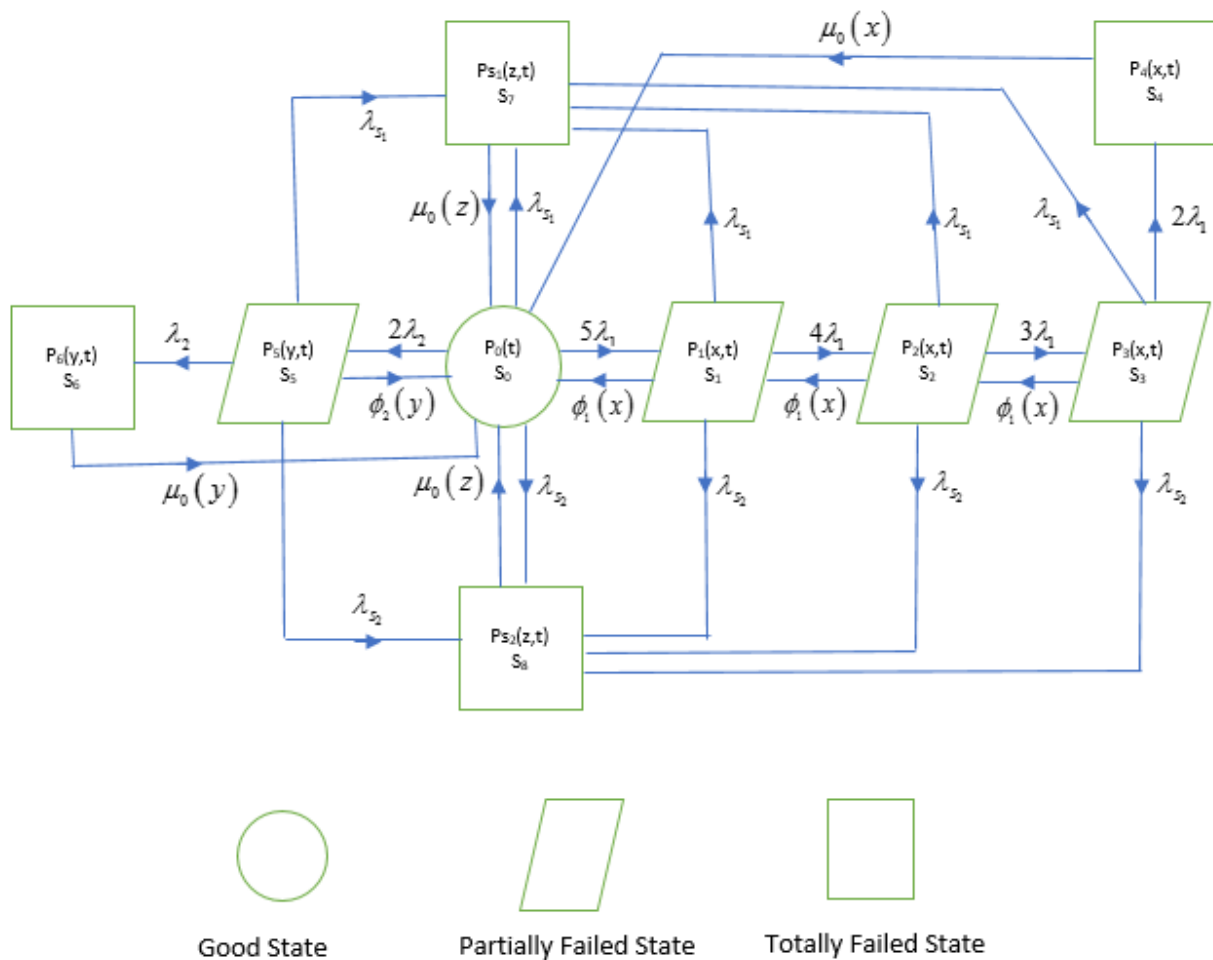
Figure 1 (a) System configuration

Figure 1 (b) State transition diagram of the model

6. FORMULATION OF THE MODEL

By a probability of considerations and continuity arguments, we can obtain the following set of difference-differential equations associated with the present mathematical model:

$$\left[\frac{\partial}{\partial t}+5\lambda_1+2\lambda_2+\lambda_{s_1}+\lambda_{s_2}\right]P_0(t)=\left[\int_0^\infty\phi_1(x)P_1(x,t)dx+\int_0^\infty\phi_2(y)P_5(y,t)dy\right.$$

$$+\int_0^\infty\mu_0(x)P_4(x,t)dx+\int_0^\infty\mu_0(y)P_6(y,t)dy$$

$$\left.+\int_0^\infty\mu_0(z)P_{s_1}(z,t)dz+\int_0^\infty\mu_0(z)P_{s_2}(z,t)dz\right] \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+4\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\phi_1(x)\right]P_1(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+3\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\phi_1(x)\right]P_2(x,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\phi_1(x)\right]P_3(x,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\right]P_4(x,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\lambda_{s_1}+\lambda_{s_2}+\phi_2(y)\right]P_5(y,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\exp\left[y^\theta+\{\log\phi(y)\}^\theta\right]^{1/\theta}\right]P_6(y,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\exp\left[z^\theta+\{\log\phi(z)\}^\theta\right]^{1/\theta}\right]P_{s_1}(z,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\exp\left[z^\theta+\{\log\phi(z)\}^\theta\right]^{1/\theta}\right]P_{s_2}(z,t)=0 \tag{9}$$

Boundary conditions

$$P_1(0,t)=5\lambda_1 P_0(t) \tag{10}$$

VIJAY VIR SINGH, PRAVEEN KUMAR POONIA AND AMEER HASSAN ADBULLAHI

$$P_2(0,t) = 4\lambda_1 P_1(0,t) = 20\lambda_1^2 P_0(t) \tag{11}$$

$$P_3(0,t) = 3\lambda_1 P_2(0,t) = 60\lambda_1^3 P_0(t) \tag{12}$$

$$P_4(0,t) = 2\lambda_1 P_3(0,t) = 120\lambda_1^4 P_0(t) \tag{13}$$

$$P_5(0,t) = 2\lambda_2 P_0(t) \tag{14}$$

$$P_6(0,t) = \lambda_2 P_5(0,t) = 2\lambda_2^2 P_0(t) \tag{15}$$

$$P_{s_1}(0,t) = \lambda_{s_1} \left[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) \right] \tag{16}$$

$$P_{s_2}(0,t) = \lambda_{s_2} \left[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) \right] \tag{17}$$

Initials conditions

$$P_0(0) = 1, \text{ and other state probabilities are zero at } t = 0 \tag{18}$$

Solution of the model

Taking Laplace transformation of equations (1) to (17) and using equation (18), we obtain

$$\left[s + 5\lambda_1 + 2\lambda_2 + \lambda_{s_1} + \lambda_{s_2} \right] \bar{P}_0(s) = 1 + \int_0^\infty \phi_1(x) \bar{P}_1(x,s) dx + \int_0^\infty \phi_2(y) \bar{P}_5(y,s) dy$$

$$+ \int_0^\infty \mu_0(x) \bar{P}_4(x,s) dx + \int_0^\infty \mu_0(y) \bar{P}_6(y,s) dy$$

$$+ \int_0^\infty \mu_0(z) \bar{P}_{s_1}(z,s) dz + \int_0^\infty \mu_0(z) \bar{P}_{s_2}(z,s) dz] \tag{19}$$

where $\quad \bar{P}_i(x,s) = \int_0^\infty e^{-st} P_i(x,t) dt$

$$\left[s + \frac{\partial}{\partial x} + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1(x) \right] \bar{P}_1(x,s) = 0 \tag{20}$$

$$\left[s + \frac{\partial}{\partial x} + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1(x) \right] \bar{P}_2(x,s) = 0 \tag{21}$$

$$\left[s + \frac{\partial}{\partial x} + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1(x) \right] \bar{P}_3(x,s) = 0 \tag{22}$$

$$\left[s + \frac{\partial}{\partial x} + \exp\left[x^\theta + \left\{ \log\phi(x) \right\}^\theta \right]^{1/\theta} \right] \bar{P}_4(x,s) = 0 \tag{23}$$

$$\left[s + \frac{\partial}{\partial y} + \lambda_2 + \lambda_{s_1} + \lambda_{s_2} + \phi_2(y) \right] \bar{P}_5(y,s) = 0 \tag{24}$$

$$\left[s + \frac{\partial}{\partial y} + \exp\left[y^\theta + \left\{ \log\phi(y) \right\}^\theta \right]^{1/\theta} \right] \bar{P}_6(y,s) = 0 \tag{25}$$

$$\left[s + \frac{\partial}{\partial z} + \exp\left[z^\theta + \left\{ \log\phi(z) \right\}^\theta \right]^{1/\theta} \right] \bar{P}_{s_1}(z,s) = 0 \tag{26}$$

$$\left[s + \frac{\partial}{\partial z} + \exp\left[z^\theta + \left\{ \log\phi(z) \right\}^\theta \right]^{1/\theta} \right] \bar{P}_{s_2}(z,s) = 0 \tag{27}$$

$$\bar{P}_1(0,s) = 5\lambda_1 \bar{P}_0(s) \tag{28}$$

$$\bar{P}_2(0,s) = 4\lambda_1 \bar{P}_1(0,s) = 20\lambda_1^2 \bar{P}_0(s) \tag{29}$$

$$\bar{P}_3(0,s) = 3\lambda_1 \bar{P}_2(0,s) = 60\lambda_1^3 \bar{P}_0(s) \tag{30}$$

$$\bar{P}_4(0,s) = 2\lambda_1 \bar{P}_3(0,s) = 120\lambda_1^4 \bar{P}_0(s) \tag{31}$$

$$\bar{P}_5(0,s) = 2\lambda_2 \bar{P}_0(s) \tag{32}$$

$$\bar{P}_6(0,s) = \lambda_2 \bar{P}_5(0,s) = 2\lambda_2^2 \bar{P}_0(s) \tag{33}$$

$$\bar{P}_{s_1}(0,s) = \lambda_{s_1} \left[\bar{P}_0(s) + \bar{P}_1(0,s) + \bar{P}_2(0,s) + \bar{P}_3(0,s) + \bar{P}_5(0,s) \right]$$

$$= \lambda_{s_1} \left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2 \right) \bar{P}_0(s) \tag{34}$$

$$\bar{P}_{s_2}(0,s) = \lambda_{s_2} \left[\bar{P}_0(s) + \bar{P}_1(0,s) + \bar{P}_2(0,s) + \bar{P}_3(0,s) + \bar{P}_5(0,s) \right]$$

$$= \lambda_{s_2} \left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2 \right) \bar{P}_0(s) \tag{35}$$

Laplace transformation of boundary conditions after repair

$$\bar{P}_1(0,s) = 5\lambda_1 \bar{P}_0(s) + \int_0^\infty \phi_1(x) \bar{P}_2(x,s) dx = 5\lambda_1 \bar{P}_0(s) + \int_0^\infty \phi_1(x) \left(e^{-\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2} \right)x - \int_0^x \phi_1(x) dx} \right) \bar{P}_2(0,s) dx$$

$$= 5\lambda_1 \bar{P}_0(s) + \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\bar{P}_2(0,s)$$

$$= 5\lambda_1 \bar{P}_0(s) + \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)4\lambda_1 \bar{P}_1(0,s)$$

$$\bar{P}_1(0,s) = \frac{5\lambda_1}{1 - 4\lambda_1 \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)}\bar{P}_0(s) \tag{36}$$

Similarly

$$\bar{P}_2(0,s) = \frac{20\lambda_1^2}{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)}\bar{P}_0(s) \tag{37}$$

$$\bar{P}_3(0,s) = 3\lambda_1 \bar{P}_2(0,s) = \frac{60\lambda_1^3}{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)}\bar{P}_0(s) \tag{38}$$

$$\bar{P}_4(0,s) = 2\lambda_1 \bar{P}_3(0,s) = \frac{120\lambda_1^4}{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)}\bar{P}_0(s) \tag{39}$$

$$\bar{P}_5(0,s) = 2\lambda_2 \bar{P}_0(s) \tag{40}$$

$$\bar{P}_6(0,s) = \lambda_2 \bar{P}_5(0,s) = 2\lambda_2^2 \bar{P}_0(s) \tag{41}$$

$$\bar{P}_{s_1}(0,s) = \lambda_{s_1}\left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2\right)\bar{P}_0(s) \tag{42}$$

$$\bar{P}_{s_2}(0,s) = \lambda_{s_2}\left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2\right)\bar{P}_0(s) \tag{43}$$

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s) = \frac{1}{D(s)} \tag{44}$$

$$\bar{P}_1(s) = \frac{5\lambda_1}{D(s)}\frac{\left\{1 - \bar{S}_{\phi_1}\left(s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 4\lambda_1 \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \tag{45}$$

$$\bar{P}_2(s) = \frac{20\lambda_1^2}{D(s)}\frac{\left\{1 - \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \tag{46}$$

$$\bar{P}_3(s) = \frac{60\lambda_1^3}{D(s)}\frac{\left\{1 - \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \tag{47}$$

$$\bar{P}_4(s) = \frac{120\lambda_1^4}{D(s)} \frac{\left\{1 - \bar{S}_{\mu_0}(s)\right\}}{s\left\{1 - 3\lambda_1 \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \tag{48}$$

$$\bar{P}_5(s) = \frac{2\lambda_2}{D(s)} \frac{\left\{1 - \bar{S}_{\phi_2}\left(s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2}\right)} \tag{49}$$

$$\bar{P}_6(s) = \frac{2\lambda_2^2}{D(s)} \frac{\left\{1 - \bar{S}_{\mu_0}(s)\right\}}{s} \tag{50}$$

$$\bar{P}_{s_1}(s) = \frac{\lambda_{s_1}\left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2\right)}{D(s)} \frac{\left\{1 - \bar{S}_{\mu_0}(s)\right\}}{s} \tag{51}$$

$$\bar{P}_{s_2}(s) = \frac{\lambda_{s_2}\left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2\right)}{D(s)} \frac{\left\{1 - \bar{S}_{\mu_0}(s)\right\}}{s} \tag{52}$$

where $D(s) = s + 5\lambda_1 + 2\lambda_2 + \lambda_{s_1} + \lambda_{s_2} - \dfrac{5\lambda_1 P}{1 - 4\lambda_1 Q} - 2\lambda_2 S - \dfrac{120\lambda_1^4 T}{1 - 3\lambda_1 R} - 2\lambda_2^2 T$

$$-\left(\lambda_{s_1} + \lambda_{s_2}\right)\left(1 + 5\lambda_1 + 20\lambda_1^2 + 60\lambda_1^3 + 2\lambda_2\right)T$$

and $P = \bar{S}_{\phi_1}\left(s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right) = \dfrac{\phi_1}{s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1}$

$$Q = \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right) = \frac{\phi_1}{s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1}$$

$$R = \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right) = \frac{\phi_1}{s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \phi_1}$$

$$S = \bar{S}_{\phi_2}\left(s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2}\right) = \frac{\phi_2}{s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2} + \phi_2}$$

$$T = \bar{S}_{\mu_0}(s) = \frac{\mu_0}{s + \mu_0}$$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows.

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_5(s)$$

$$= \frac{1}{D(s)} \left[\begin{array}{c} 1 + \dfrac{5\lambda_1\left\{1 - \bar{S}_{\phi_1}\left(s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 4\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 4\lambda_1\bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \\[4mm] + \dfrac{20\lambda_1^2\left\{1 - \bar{S}_{\phi_1}\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 3\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 3\lambda_1\bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \\[4mm] + \dfrac{60\lambda_1^3\left\{1 - \bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\left\{1 - 3\lambda_1\bar{S}_{\phi_1}\left(s + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}} \\[4mm] + \dfrac{2\lambda_2\left\{1 - \bar{S}_{\phi_2}\left(s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2}\right)\right\}}{\left(s + \lambda_2 + \lambda_{s_1} + \lambda_{s_2}\right)} \end{array} \right] \tag{53}$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \tag{54}$$

7. ANALYTICAL STUDY

7.1 Availability Analysis

When repair follows general and Gumbel-Hougaard family copula distribution, we have

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp\left[x^\theta + \left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}}(s) = \frac{\exp\left[x^\theta + \left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}}{s + \exp\left[x^\theta + \left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}}$$

setting $\quad \bar{S}_{\alpha_i}(s) = \dfrac{\alpha_i}{s + \alpha_i}, i = 1, 2$ and $\bar{S}_\phi(s) = \dfrac{\phi}{s + \phi}$

Here we have considered the following three cases on switching device for the availability of the system:

Case I: When both the subsystems have switching device, the availability of the system by taking the values of different parameters as

$\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0.025, \lambda_{s_2} = 0.022, \phi = 1, \theta = 1, \ x = 1, y = 1, z = 1, \ \alpha_i = 1(i = 1, 2),$ in (53),

then taking the inverse Laplace to transform, we obtain,

$$P_{up}(t) = -0.003240e^{-1.1370t} + 0.022538e^{-2.7829t} - 0.035199e^{-1.3118t} + 0.098452e^{-1.0638t}$$
$$+ 1.012130e^{-0.01070t} - 0.000578e^{-1.1070t} - 0.094103e^{-1.0670t} \tag{55}$$

Case II: When subsystem-2 does not have switching device, then the availability of the system by taking the values of different parameters as $\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0.025, \lambda_{s_2} = 0, \phi = 1, \theta = 1,$

$x = 1, y = 1, z = 1, \alpha_i = 1 (i = 1, 2)$, in (53), then taking the inverse Laplace to transform, we obtain,

$$P_{up}(t) = -0.003268e^{-1.1150t} - 0.0005880e^{-1.0850t} + 0.012383e^{-2.7532t} - 0.031792e^{-1.2960t}$$
$$+ 0.109886e^{-1.0421t} + 1.018479e^{-0.0119t} - 0.105102e^{-1.0450t} \tag{56}$$

Case III: When subsystem-1 and two both do not have switching device, the availability of the system by taking the values of different parameters as

$\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0, \lambda_{s_2} = 0, \phi = 1, \theta = 1, x = 1, y = 1, z = 1, \alpha_i = 1 (i = 1, 2)$, in (53), then taking

the inverse Laplace to transform, we obtain,

$$P_{up}(t) = 0.000365e^{-2.7193t} - 0.027264e^{-1.2785t} + 0.125983e^{-1.0175t} + 1.025501e^{-0.0131t}$$
$$- 0.003298e^{-1.0900t} - 0.1206870e^{-1.0200t} - 0.000598e^{-1.0600t} \tag{57}$$

For $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values

$P_{up}(t)$ with the help of (55-57), as shown in table-2 and figure-2.

Table 2 Variation of availability with respect to time in various cases

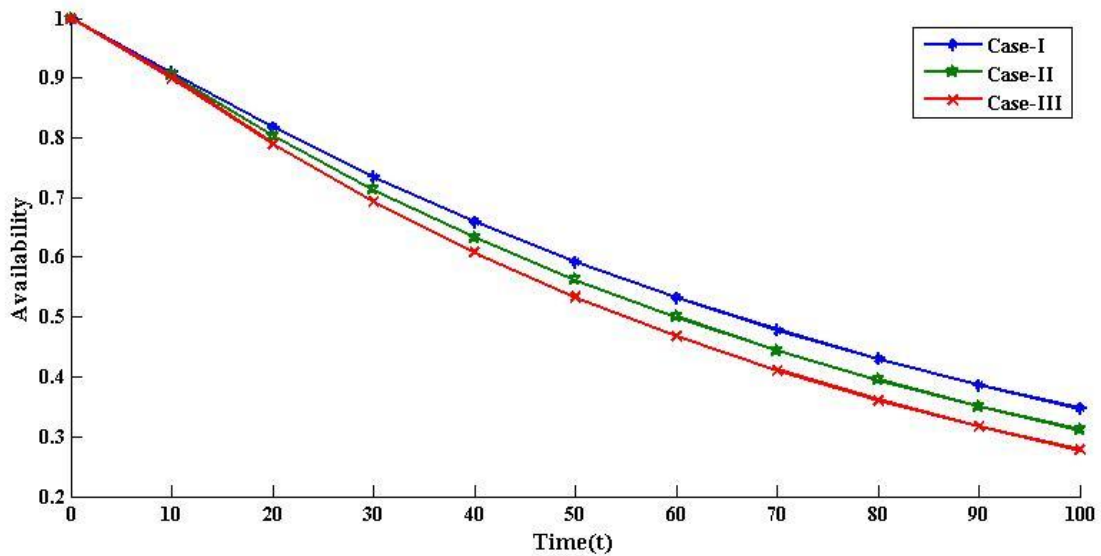| Time (t)	Case-I	Case-II	Case-III
0	1.0000	1.0000	1.0000
10	0.9094	0.9045	0.8999
20	0.8171	0.8032	0.7897
30	0.7342	0.7133	0.6930
40	0.6597	0.6335	0.6082
50	0.5927	0.5626	0.5337
60	0.5326	0.4996	0.4683
70	0.4785	0.4437	0.4110
80	0.4300	0.3941	0.3607
90	0.3863	0.3500	0.3165
100	0.3471	0.3108	0.2777

Figure 2 Availability as a function of time

7.2 Reliability Analysis

In order to obtain system reliability, consider repair rates equal to zero. Like availability, the same three cases are discussed here.

Case I: When both the subsystems have a switching device, the reliability of the system by taking the values of different parameters as $\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0.025, \lambda_{s_2} = 0.022$ in (53), we obtain,

$$R(t) = 0.012465e^{-0.1070t} - 1.570613e^{-0.2370t} + 0.180000e^{-0.1370t} + 2.142857e^{-0.1670t} + 0.235294e^{-0.0670t} \tag{58}$$

Case II: When subsystem-2 does not have a switching device, the availability of the system by taking the values of different parameters as $\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0.025, \lambda_{s_2} = 0$ in (53), we obtain,

$$R(t) = -1.570613e^{-0.2150t} + 0.180000e^{-0.1150t} + 2.142857e^{-0.1450t} + 0.235294e^{-0.0450t} + 0.012462e^{-0.0850t} \tag{59}$$

Case III: When subsystem-1, as well as the subsystem-2, do not have a switching device, the availability of the system by taking the values of different parameters as

$\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0, \lambda_{s_2} = 0$ in (53), we obtain,

PERFORMANCE ANALYSIS OF A COMPLEX REPAIRABLE SYSTEM

$$R(t) = -1.570613e^{-0.1900t} + 2.142857e^{-0.1200t} + 0.180000e^{-0.0900t} + 0.012462e^{-0.0600t}$$
$$+ 0.235294e^{-0.0200t}$$
(60)

For $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values $R(t)$

with the help of (58-60), as shown in table-3 and figure-3.

Table 3 Computed values of reliability corresponding to the different cases

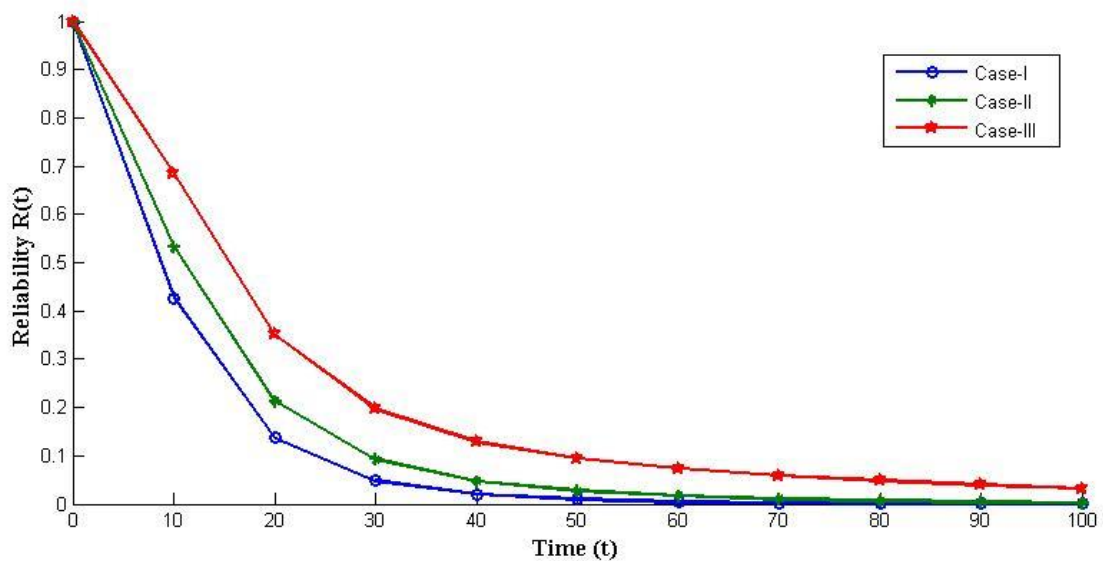| Time (t)	Case-I	Case-II	Case-III
0	1.0000	1.0000	1.0000
10	0.4269	0.5320	0.6831
20	0.1369	0.2125	0.3504
30	0.0479	0.0928	0.1965
40	0.0196	0.0473	0.1286
50	0.0090	0.0270	0.0943
60	0.0044	0.0164	0.0736
70	0.0022	0.0102	0.0590
80	0.0011	0.0065	0.0478
90	0.0005	0.0041	0.0390
100	0.0002	0.0026	0.0319

Figure 3 Reliability as a function of time

7.3 Mean Time to Failure (MTTF)

Taking all repair rate to zero and the limit as s tends to zero in (53) for the exponential distribution; we can obtain the MTTF as:

$$MTTF = \frac{1}{\lambda}\left[1 + \frac{5\lambda_1}{4\lambda_1 + \mu} + \frac{20\lambda_1^2}{3\lambda_1 + \mu} + \frac{60\lambda_1^3}{2\lambda_1 + \mu} + \frac{2\lambda_2}{\lambda_2 + \mu}\right] \tag{61}$$

where $\lambda = 5\lambda_1 + 2\lambda_2 + \lambda_{s_1} + \lambda_{s_2}$ and $\mu = \lambda_{s_1} + \lambda_{s_2}$

Now taking the values of different parameters as $\lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_{s_1} = 0.025$ and $\lambda_{s_2} = 0.022$ and varying $\lambda_1, \lambda_2, \lambda_{s_1}$ and λ_{s_2} one by one respectively as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07,$ $0.08, 0.09, 0.10$ in (61), the variation of MTTF, with respect to failure rates, can be obtained in table-4 and figure-4.

Table 4 Computation of MTTF corresponding to the failure rates

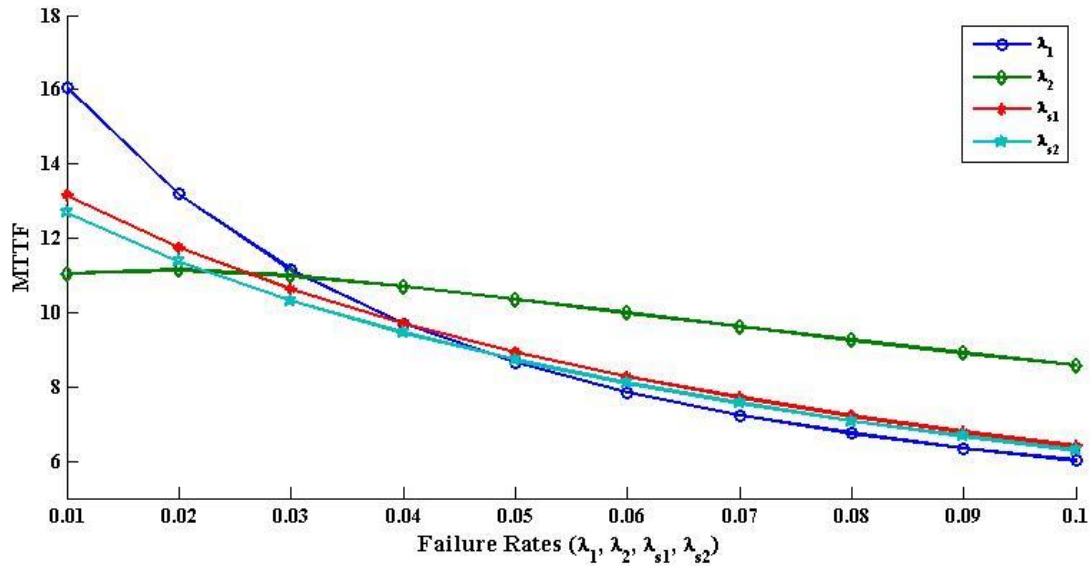| Failure Rate	MTTF λ_1	MTTF λ_2	MTTF λ_{s_1}	MTTF λ_{s_2}
0.01	16.0481	11.0396	13.1586	12.6936
0.02	13.1802	11.1466	11.7385	11.3754
0.03	11.1466	10.9881	10.6151	10.3212
0.04	9.7040	10.7013	9.6971	9.4528
0.05	8.6468	10.3557	8.9290	8.7221
0.06	7.8477	9.9880	8.2749	8.0970
0.07	7.2282	9.6181	7.7101	7.5552
0.08	6.7378	9.2565	7.2167	7.0806
0.09	6.3433	8.9087	6.7818	6.6611
0.10	6.0219	8.5775	6.3952	6.2875

Figure 4 MTTF as a function of failure rates

7.4 Cost Analysis

Let the service facility be always available, then expected profit during $[0,t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t \tag{62}$$

For the same set of parameters defined in (53), one can obtain (63). Therefore,

$$
\begin{aligned}
E_p(t) = K_1 \{ & 0.002850 e^{-1.1370t} - 0.008099 e^{-2.7829t} + 0.026831 e^{-1.3119t} - 0.092545 e^{-1.0638t} \\
& - 94.595676 e^{0.0107t} + 0.000523 e^{-1.1070t} + 0.088194 e^{-1.0670t} + 94.57792 \} - K_2 t
\end{aligned}
\tag{63}
$$

Setting $K_1 = 1$ $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively, and varying $t = 0, 10, 20, 30, 40,$

$50, 60, 70, 80, 90$ and 100 units of time, the results for expected profit can be obtained as per table-5 and figure-5.

VIJAY VIR SINGH, PRAVEEN KUMAR POONIA AND AMEER HASSAN ADBULLAHI

Table 5 Profit computation for different values of time

Time (t)	$K_2 = 0.6$	$K_2 = 0.5$	$K_2 = 0.4$	$K_2 = 0.3$	$K_2 = 0.2$	$K_2 = 0.1$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	3.5808	4.5808	5.5808	6.5809	7.5809	8.5809
20	6.2056	8.2056	10.2056	12.2056	14.2056	16.2051
30	7.9551	10.9551	13.9551	16.9551	19.9551	22.9551
40	8.9182	12.9182	16.9182	20.9182	24.9182	28.9182
50	9.1748	14.1748	19.1748	24.1748	29.1748	34.1748
60	8.7966	14.7966	20.7966	26.7966	32.7986	38.7966
70	7.8479	14.8479	21.8479	28.8479	35.8479	42.8479
80	6.3867	14.3867	22.3867	30.3867	38.3867	46.3867
90	4.4649	13.4649	22.4649	31.4649	40.4649	49.4649
100	2.1293	12.1293	22.1293	32.1293	42.1293	52.1293
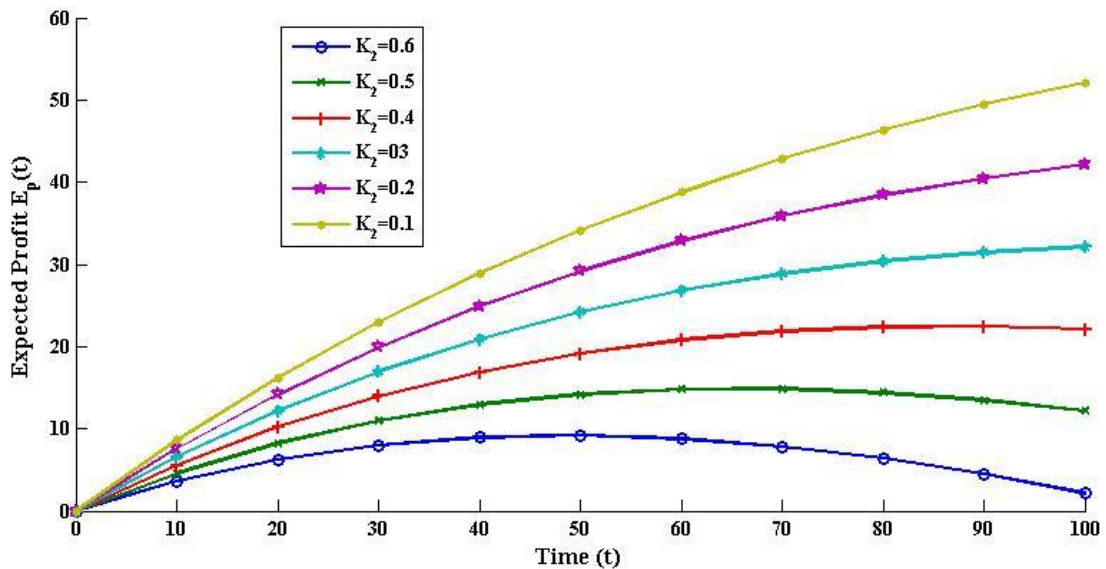
Figure 5 Expected profit as a function of time

7.5 Sensitivity Analysis corresponding to MTTF

The sensitivity in MTTF of the system can be studied through the partial differentiation of MTTF with respect to the failure rates of the system. By applying the set of parameters as

$\lambda_1 = 0.3, \lambda_2 = 0.2, \lambda_{s_1} = 0.25$ and $\lambda_{s_2} = 0.22$ in the partial differentiation of MTTF, one can calculate MTTF sensitivity, as shown in table-6 and figure-6.

Table 6 MTTF sensitivity as a function of failure rates

Failure Rate	$\dfrac{\partial(MTTF)}{\partial\lambda_1}$	$\dfrac{\partial(MTTF)}{\partial\lambda_2}$	$\dfrac{\partial(MTTF)}{\partial\lambda_{s1}}$	$\dfrac{\partial(MTTF)}{\partial\lambda_{s2}}$
0.01	-326.7760	31.5790	-161.4625	-148.8523
0.02	-241.9781	-5.7094	-125.2098	-116.9666
0.03	-169.5936	-23.8210	-100.9229	-95.1384
0.04	-122.3188	-32.4317	-83.5807	-79.3132
0.05	-91.1812	-36.0978	-70.6214	-67.3551
0.06	-69.9076	-37.1163	-60.6048	-58.0337
0.07	-54.8285	-36.7045	-52.6592	-50.5901
0.08	-43.7890	-35.5322	-46.2253	-44.5304
0.09	-35.4802	-33.9767	-40.9280	-39.5191
0.10	-29.0776	-32.2544	-36.5055	-35.3198

Figure 6 MTTF sensitivity as a function of failure rates

VIJAY VIR SINGH, PRAVEEN KUMAR POONIA AND AMEER HASSAN ADBULLAHI
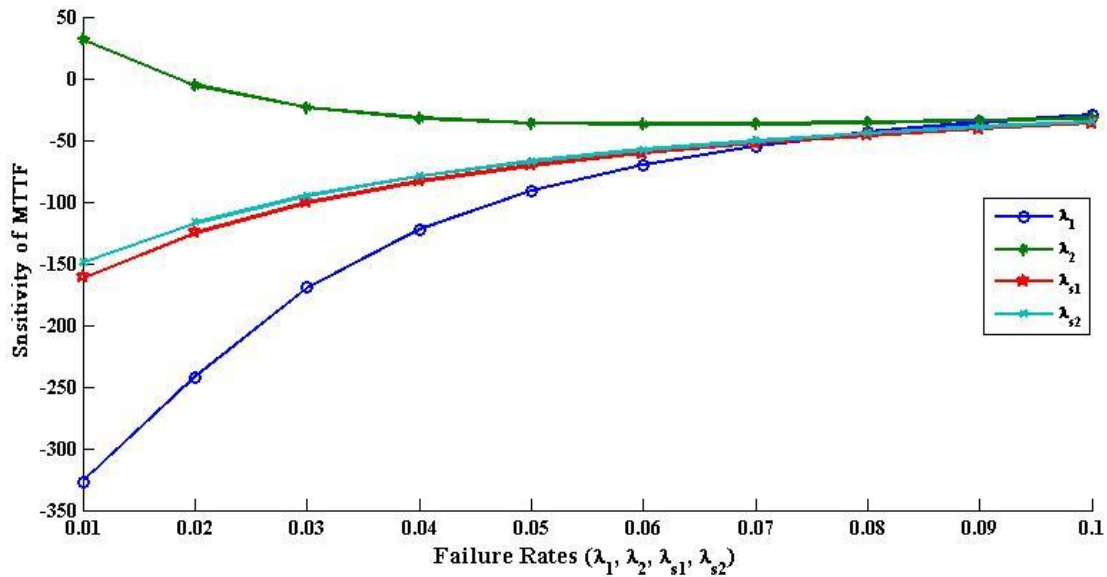
8. CONCLUSION

In this paper, the reliability analysis of a complex system consisting of two subsystems, subsystem-1 and subsystem-2 in a series configuration with the switching device, is studied. The subsystems have five and two units, respectively. Furthermore, the switching device in the system is unreliable, and the function of the switch is: "as long as the switch fails, the whole system fails immediately." Using the supplementary variable technique and the Laplace transform various measures like availability of the system, reliability of the system, MTTF, profit analysis, and sensitivity analysis for MTTF are derived in this model.

Table-2 and corresponding figure-2 give the analysis of availability in three different cases on the switching device. In case I, $\lambda_{s_1} = 0.025, \lambda_{s_2} = 0.022$ i.e., both subsystems have a switching device, in case II, $\lambda_{s_1} = 0.025, \lambda_{s_2} = 0$ i.e., only the first subsystem has switching device, while in case III, $\lambda_{s_1} = 0, \lambda_{s_2} = 0$ i.e., no subsystem have switching device. It reveals from the graph that availability constantly decreases as time increases in all the three cases. The reliability of the system is evaluated in three different cases, like availability and shown in table-3 and figure-3. It concludes that reliability decreases significantly in the beginning, and thereafter it decreases approximately in a constant manner. Thus, both the availability and reliability decrease with an increase in time. Investigation through figure 2 and figure 3 concludes that availability values are greater than reliability for the same values of failure rates. Thus, one can understand the need for repair for repairable systems for better performance.

Table-4 and figure-4 yield the MTTF of the system with respect to variation in failure rate $\lambda_1, \lambda_2, \lambda_{s_1}$ and λ_{s_2}, respectively, when other parameters have been kept constant. MTTF of the system is decreasing concerning different failure rates. MTTF of the system is highest for the failure rate of subsystem-1 and is lowest concerning the failure rate of subsystem-2. The MTTF of switching devices and subsystem-1 are almost the same on failure rate variation value after 0.03.

When revenue cost per unit time fixed at $K_1 = 1$ and service costs at $K_2 = 0.6, 0.5, 0.4, 0.3,$ 0.2 and 0.1, the expected profit has been calculated (Table 5), and the results are demonstrated by the graph (Figure 5). It reveals that expected profit increased as time increased for lower values of K_2 while it is decreased for higher values of service cost. Thus, for low service costs,

the profit is higher as compared to the high service cost. The sensitivities of the system MTTF concerning the system parameter $\lambda_1, \lambda_2, \lambda_{s_1}$ and λ_{s_2} s shown in table-6 and figure-6. The sensitivity of MTTF for system parameters becomes constant for higher values. Thus, in general, with the study, the behavior of such systems can be analyzed and prognosticate in advance. This paper may be important to engineers, maintenance managers, and plant management for proper maintenance analysis, decision, and for the safety of the system as a whole.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] A. Khattab, N. Nahas, and M. Nourelfath. Availability of k-out-of-n: G systems with non-identical components subject to repair priorities, Reliab. Eng. Syst. Saf. 94(2)(2009), 142–151.

[2] A. Lado and V.V. Singh. Cost assessment of complex repairable system consisting of two subsystems in the series configuration using Gumbel Hougaard family copula, Int. J. Quality Reliab. Manage. 36(10)(2019), 1683-1698.

[3] A. Munjal and S. B. Singh. Reliability analysis of a complex repairable system composed of two 2-out-of-3: G subsystems connected in parallel, J. Reliab. Stat. Stud. 7(S)(2014), 89-111.

[4] D. Babu, P. Govindaraju, and U. Rizwan. A δ-Shock maintenance model for a deteriorating system with an imperfect delayed repair under partial product process, J. Math. Comput. Sci. 9(5)(2019), 571-581.

[5] Di Bona., G., Forcina, A., Petrillo, A., De Felice, F. A- IFM reliability allocation model based on multicriteria approach. Int. J. Quality Reliab. Manage. 33(5) (2016), 676-698.

[6] Di Bona., G., Forcina, A., Silvestri, A. Critical flow method: A new reliability allocation approach for a thermonuclear system. Quality Reliab. Eng. Int. 32(5)(2016), 1677-1691.

[7] I. Yusuf, and N. Hussaini. A comparative analysis of three redundant unit systems with three types of failures, Arab. J. Sci. Eng. 39(4)(2014), 3337-3349.

[8] I. Yusuf, A, S. Halilu, and N.G. Khalil. Reliability modeling and analysis of a non-repairable series-parallel system, J. Math. Comput. Sci. 6(2) (2016), 247-253.

[9] I. Yusuf, B. Yusuf, and S.I. Bala. Meantime to system failure analysis of a linear consecutive 3-out-of-5 warm standby system in the presence of common cause failure, J. Math. Comput. Sci. 4(1) (2014), 58-66.

[10] K. G. Ramamurthy. Reliability of consecutive -k-out-of-n systems for general case, Sankhya B - Indian J. Stat. 59(3)(1997), 396–420.

[11] K.M. El-Said and M. S. EL-Sherbeny. Evaluation of reliability and availability characteristics of two different systems by using linear first-order differential equations, J. Math. Stat. 1(2)(2005), 119–123.

[12] L. Bulama, I. Yusuf, and S.I. Bala. Stochastic modeling and analysis of some reliability characteristics of a repairable warm standby system, Appl. Math. Sci. 7(2013), 5847-5862.

[13] M. Park and H. Pham. A generalized block replacement policy for a k-out-of-n system with respect to a threshold number of failed components and risk costs, IEEE Trans. Syst. Man Cybern. Part A: Syst. Humans, 42(2)(2012), 453–463.

[14] M. Ram and S. B. Singh. Availability and Cost Analysis of a parallel redundant complex system with two types of failure under preemptive- resume repair discipline using Gumbel-Hougaard family copula in repair, Int. J. Reliab. Quality Safety Eng. 15(4)(2008), 341–365.

[15] N. Goyal, M. Ram M, S. Amoli, and A. Suyal A. Sensitivity analysis of a three-unit series system under k-out-of-n redundancy, Int. J. Quality Reliab. Manage. 34(6)(2017), 770-784.

[16] R. Gupta and A. Tyagi. Stochastic analysis of a two-unit warm standby system with two-phase repair and geometric distributions of the events, J. Math. Comput. Sci. 3(6)(2013), 1586-1600.

[17] R. Gupta, A. Chaudhary, and S. Jaiswal. Stochastic analysis of a three-unit complex system with active and passive redundancies and correlated failures and repair times, J. Math. Comput. Sci. 5 (5)(2015), 694-707.

[18] R. Moghaddass, M.J. Zuo, and W. Wang. Availability of a general k-out-of-n: G system with non-identical components considering shut-off rules using quasi-birth-death process, Reliab. Eng. Syst. Safety, 96(4) (2011), 489–496.

[19] R. Sharma and G. Kumar. Availability improvement for the successive k-out-of-n machining system using standby with multiple working vacations, Int. J. Reliab. Safety, 11(3)(2017), 256-267.

[20] S. C. Malik and S. Deswal. Reliability modeling and profit analysis of a repairable system of non-identical units with no operation and repair in abnormal weather, Int. J. Computer Appl. 51(11)(2012), 43–49.

[21] V. V. Singh, M. Ram, and D. K. Rawal. Cost Analysis of an Engineering System involving subsystems in Series Configuration, IEEE Trans. Autom. Sci. Eng. 10(1)(2013), 1124-1130.

[22] V. V. Singh, S. B. Singh, M. Ram, and C. K. Goel. Availability, MTTF, and cost analysis of a system having two units in a series configuration with the controller, Int. J. Syst. Assur. Manage. 4(4)(2013), 341–352.

[23] V. V. Singh, J. Gulati, D. K. Rawal, and C. K. Goel. Performance analysis of the complex system in the series configuration under different failure and repair disciplines using a copula, Int. J. Reliab. Quality Safety Eng. 23(2)(2016), 1-21.

[24] V. V. Singh and P. K. Poonia. Probabilistic assessment of two units parallel system with correlated lifetime under inspection using regenerative point technique, Int. J. Reliab. Risk Safety, Theory Appl. 2(1)(2019), 5-14.

[25] X. Liang, Y. Xiong, and Z. Li. Exact reliability formula for consecutive k-out-of-n repairable systems, IEEE Trans. Reliab. 59(2)(2010), 313–318.