Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 4, 863-880
https://doi.org/10.28919/jmcs/4408
ISSN: 1927-5307

RELIABILITY ANALYSIS OF COMMUNICATION NETWORK SYSTEM WITH REDUNDANT RELAY STATION UNDER PARTIAL AND COMPLETE FAILURE

IBRAHIM YUSUF[1,*], ABDULKAREEM LADO ISMAIL[2] AND U.A. ALI[3]

[1]Department of Mathematical Sciences, Bayero University, Kano, Nigeria

[2]Department of Mathematics. Kano State College of Arts, Science and Remedial Studies, Kano, Nigeria

[3]Department of Mathematics, Federal University, Dutse, Nigeria

Abstract: The purpose of this paper is to study the performance of a communication network system consisting of a transmitter, two relay stations and a receiver arranged in series parallel. Through the transition diagram, the partial differential equations are derived, and Laplace transforms are then taken on these equations to derive system reliability, availability, the mean time to system failure (*MTTF*) and cost function. It is assumed that failure rates are constant and follows exponential distribution, repair rates of partial failure state are assumed to follow general distribution and complete failure states are repaired through Gumbel-Hougaard family copula. The system is analyzed through supplementary variable technique and Laplace transform. Different measures of testing system effectiveness which include reliability, availability, mean time to failure (MTTF) and profit function have been calculated for particular values of time, failure and repair rates. From the study, it is clear that time and failure rates of both transmitter, relay stations and receiver influence the reliability, availability, MTTF and profit function. Mathematical models developed in this paper can aid plant management for proper maintenance and system safety, avoiding incorrect reliability, availability and profit assessment and leading to inadequate maintenance decision making, which may result in unnecessary expenditures and reduction of safety standards.

Keywords: communication network; reliability; availability; mean time to failure.

2010 AMS Subject Classification: 90B25.

---

[*]Corresponding author

E-mail address: ibrahimyusuffagge@gmail.com

Received December 2, 2019

1. INTRODUCTION

System reliability and availability are vital towards system performance, quality, production output, expected revenue as well as industrial growth. To this end, reliability and availability of a system may be enhanced by maintenance, fault tolerance components as well as adequate system design. Many researchers have studied comparison and performance evaluation problem of different systems. Munial and Singh [12] analyzed the reliability of a complex system with subsystems connected in parallel. Negi and Singh [13] analyzed non repairable system reliability with serial subsystems. Niwas et al [14] presents probabilistic analysis models of a single system with preventive maintenance. Niwas and Kadyan [15] modelled the reliability of a system with warranty. Kakkar et al [4] studied reliability of two unit parallel industrial system. Kakkar et al [5] presents reliability of two dissimilar parallel unit in the present of preventive maintenance. Kumar and Malik [6] studied reliability modelling of a computer system where hardware repair has priority over replacement. Kumar and Malik [7] presents reliability measures of a computer system where preventive maintenance has priority over hardware repair. Kumar et al [8] presents reliability optimization of complex system through cuckoos search algorithm. Kumar et al [9] studied availability and cost analysis of a system with subsystems in series. Gahlot et al [2] analyzed performance measure for a serial system under different types of failure. Gulati et al [3] investigated the performance modelling and assessment of two unit system with different types of failure. Lado et al [11], Singh et al [20] and Singh et al [21]. Large volumes of literature exist on the issue relating to prediction of performance communication network. Cetinkaya and Sterbenz [1] present network challenges through past and potential and the correlation with taxonomy events. Ioan et al [10] studied the damage to the network leading to its malfunction leading to the network failure. Neumayer and Modiano [16] present network reliability with geographically correlated failures. Rafiee and Shabgani [17] presents fault tree model in addressing reliability evaluation of communication network. Sterbenz et al [22] presents three resilience and mechanism to redundancy, multilevel resilience, and diversity for disruption tolerance. Rawal et al [18] studied modeling of internet data center with various maintenance policies. Sadeghi and Roghanian [19] analyzed reliability of warm standby repairable system with two cases of imperfect switching mechanism.

From the literature study above, little or no attention is paid on the reliability analysis of communication network system with standby relay stations. The present paper is aim at reliability modelling and analysis of a communication network system with transmitter, two relay stations in parallel and a receiver. Explicit expressions for reliability, availability, mean time to failure and coat function have been obtained. The objectives of this paper are twofold: First is to derive the explicit expressions for the reliability, availability, mean time to failure and profit function. Second is to capture the effect of both passage time and failure rates on reliability, availability, MTTF and profit.

The organization of the paper is as follows. Section 2 presents the notations of the study. Section 3 contains a description of the system under study. Section 4 presents formulations of the models. The results of our analytical comparison between the systems are presented in section 5. Numerical examples are presented in section 5. Finally, we make some concluding remarks in Section 6.

2. NOTATIONS

t : Time variable

s : Laplace transform variable

λ_A : Failure rate of transmitter (subsystem A)

λ_1 : Failure rate of the first relay station (Subsystem B)

λ_2 : Failure rate of the second relay station (subsystem C)

λ_D : Failure rate of the receiver (subsystem D)

$\phi_1(x)$: Repair rate of the first relay station (Subsystem B)

$\phi_2(x)$: Repair rate of the second relay station (Subsystem C)

$\mu_0(x)$: Repair rate of complete failure state

$p_i(t)$: Probability that the system is in state S_i for $i = 0,1,2,...,8$

$\overline{p(s)}$:Laplace transformation of transition probability $p(t)$

$p_i(x,t)$: Probability that the system is in state S_i for $i = 0,1,2,...,8$, the system is under repair with elapse repair time (x,t) with variables x and t for repair and time respectively.

$E_p(t)$: Revenue generated in the interval $[0,t)$

k_1, k_2: Revenue and service cost

$S_f(x) = f(x)e^{-\int_0^s f(x)(dx)}$ Notation function with repair distribution $f(x)$

$\overline{S}_f(s) = \int_0^\infty e^{-sx} f(x)e^{-\int_0^s f(x)dx} dx$ Laplace transform of $S_f(x)$

3. Description and States of the System

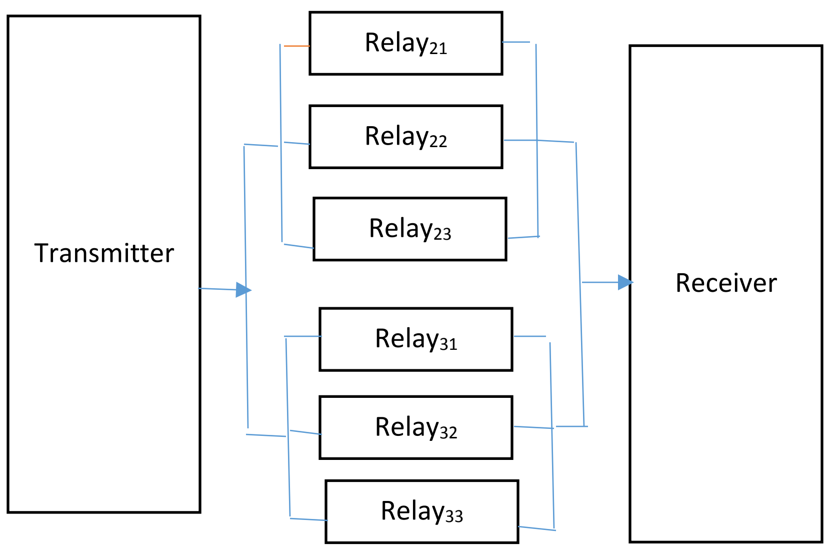


Figure 1: Reliability Block Diagram of the System

The system consisting of four subsystems A, B, C and D arranged in series-parallel. Subsystems A and D are single unit while subsystem B and C are parallel to each other consisting of 2-out-of-3 units each. Signal is received from the transmitter by relay stations in subsystems B and C, is distributed to the receiver (subsystem D) for consumption as shown in Figure 1. Subsystem A is the transmitter, subsystems B and C are relay stations, while subsystem D is the receiver. However the two paths are parallel to each other. The system failed when any of the subsystem A or D failed or B and C.

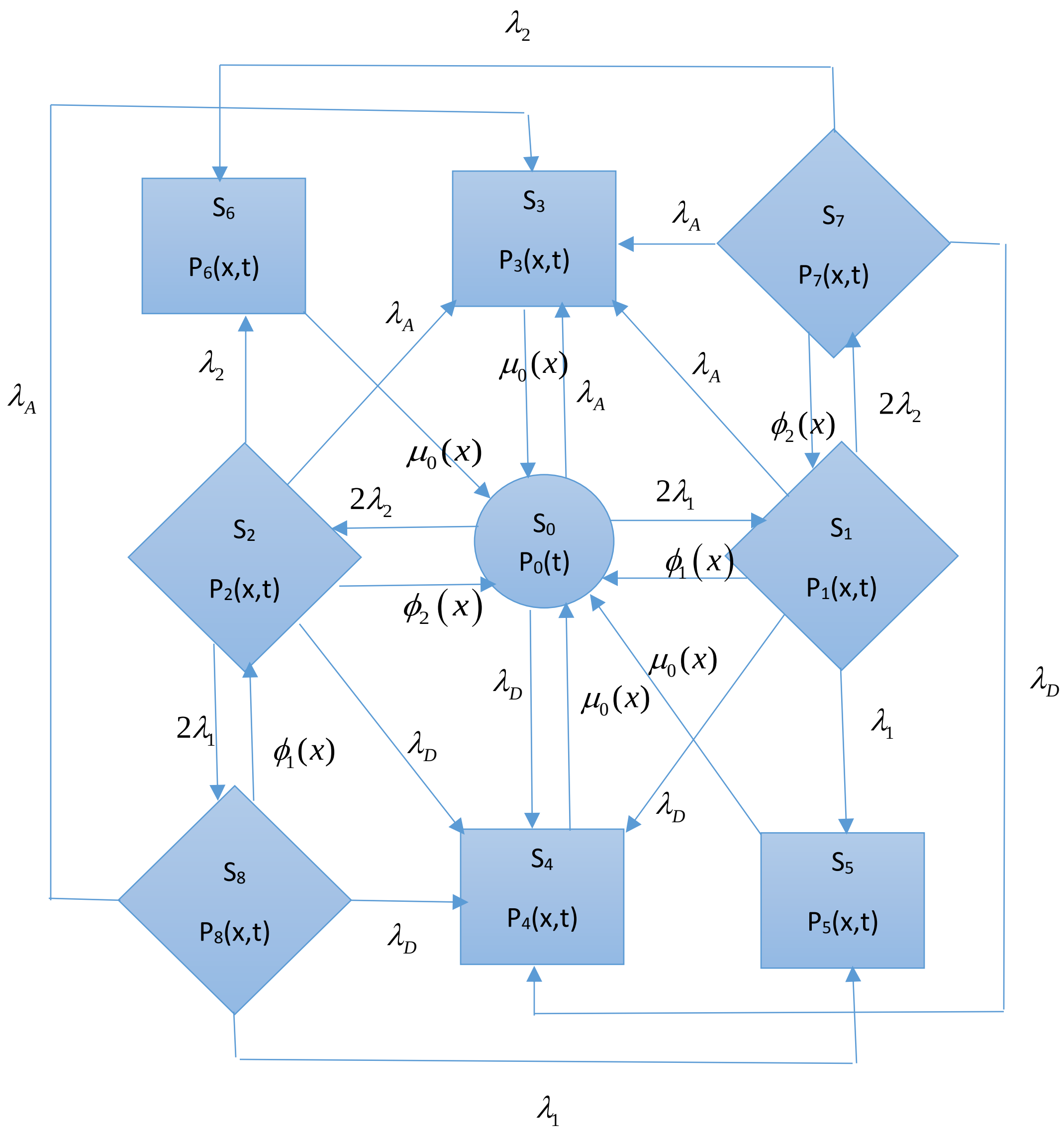


Figure 2: State Transition Diagram

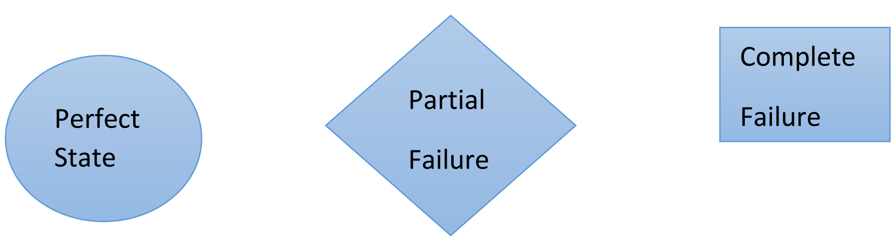


Table 1: States of the System

State	Description
S_0	Initial state, transmitter, two consecutive relays from subsystems 2 and 3, receiver are working, one relay each from subsystems B and C are on standby. The system is working.
S_1	Transmitter, two consecutive relays from subsystems B and C, receiver are working, one relay has failed in subsystems B and one relay is on standby in subsystem C. The system is working
S_2	Transmitter, two consecutive relays from subsystems B and C, receiver are working, one relay has failed in subsystems C and one relay is on standby in subsystem B. The system is working
S_3	Transmitter and one relay each from subsystems B and C have failed, receiver is idle. The system is down.
S_4	Receiver has failed, transmitter and relay stations are ide. The system is down
S_5	Transmitter, two consecutive relays from subsystems C, receiver are working, two relays have failed in subsystems B and one relay is on standby in subsystem C. The system is working
S_6	Relay stations in both subsystems B and C have failed. The system is down.
S_7	Two relays one each from subsystems B and C have failed, transmitter, relay stations from subsystems B and C, and receiver are working. The system is working.
S_8	Two relays one each from subsystems B and C and receiver have failed. The system is down.

4. MATHEMATICAL MODELS FORMULATION

By probability of considerations and continuity arguments, the following set of difference differential equations are associated with the present mathematical model.

$$\left(\frac{\delta}{\delta t}+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D\right)p_0(t)=\int_0^\infty \phi_1(x)\,p_1(x,t)\,dx+\int_0^\infty \phi_2(x)\,p_2(x,t)\,dx+\int_0^\infty \mu_0(x)\,p_3(x,t)\,dx+$$

$$\int_0^\infty \mu_0(x)\,p_3(x,t)\,dx+\int_0^\infty \mu_0(x)\,p_4(x,t)\,dx+\int_0^\infty \mu_0(x)\,p_5(x,t)\,dx+\int_0^\infty \mu_0(x)\,p_6(x,t)\,dx \qquad (1)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+2\lambda_2+\lambda_1+\lambda_A+\lambda_D+\phi_1(x)\right)p_1(x,t)=0 \qquad (2)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+2\lambda_1+\lambda_2+\lambda_A+\lambda_D+\phi_2(x)\right)p_2(x,t)=0 \qquad (3)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)p_3(x,t)=0 \qquad (4)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)p_4(x,t)=0 \tag{5}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)p_5(x,t)=0 \tag{6}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)p_6(x,t)=0 \tag{7}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\lambda_2+\lambda_A+\lambda_D+\phi_2(x)\right)p_7(x,t)=0 \tag{8}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\lambda_1+\lambda_A+\lambda_D+\phi_1(x)\right)p_8(x,t)=0 \tag{9}$$

BOUNDARY CONDITIONS**:**

$$p_1(0,t)=2\lambda_1 p_0(t) \tag{10}$$

$$p_2(0,t)=2\lambda_2 p_0(t) \tag{11}$$

$$p_3(0,t)=\lambda_A\left(p_0(t)+p_1(0,t)+p_2(0,t)+p_7(0,t)+p_8(0,t)\right) \tag{12}$$

$$p_4(0,t)=\lambda_D\left(p_0(t)+p_1(0,t)+p_2(0,t)+p_7(0,t)+p_8(0,t)\right) \tag{13}$$

$$p_5(0,t)=\lambda_1\left(p_1(0,t)+p_8(0,t)\right) \tag{14}$$

$$p_6(0,t)=\lambda_2\left(p_2(0,t)+p_7(0,t)\right) \tag{15}$$

$$p_7(0,t)=2\lambda_2 p_1(0,t) \tag{16}$$

$$p_8(0,t)=2\lambda_1 p_2(0,t) \tag{17}$$

INITIALS CONDITIONS:

$$p_k(t)=\begin{cases}1, & k=0\\ 0, & k=1,2,\dots,8\end{cases} \quad \text{at } t=0 \tag{18}$$

SOLUTION OF THE MODEL

Taking Laplace transform of (1) to (17) using the initial condition above, to obtained

$$\left(s+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D\right)p_0(s)=1+\int_0^\infty \phi_1(x)\overline{p_1}(x,s)\,dx+\int_0^\infty \phi_2(x)\overline{p_2}(x,s)\,dx+\int_0^\infty \mu_0(x)\overline{p_3}(x,s)\,dx+$$

$$\int_0^\infty \mu_0(x)\overline{p_4}(x,s)dx + \int_0^\infty \mu_0(x)\overline{p_5}(x,s)dx + \int_0^\infty \mu_0(x)\overline{p_6}(x,s)dx \quad (19)$$

$$\left(s+\frac{\delta}{\delta x}+2\lambda_2+\lambda_1+\lambda_A+\lambda_D+\phi_1(x)\right)\overline{p_1}(x,S)=0 \quad (20)$$

$$\left(s+\frac{\delta}{\delta x}+2\lambda_1+\lambda_2+\lambda_A+\lambda_D+\phi_2(x)\right)\overline{p_2}(x,s)=0 \quad (21)$$

$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\overline{p_3}(x,s)=0 \quad (22)$$

$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\overline{p_4}(x,s)=0 \quad (23)$$

$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\overline{p_5}(x,s)=0 \quad (24)$$

$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\overline{p_6}(x,s)=0 \quad (25)$$

$$\left(s+\frac{\delta}{\delta x}+\lambda_2+\lambda_A+\lambda_D+\phi_2(x)\right)\overline{p_7}(x,s)=0 \quad (26)$$

$$\left(s+\frac{\delta}{\delta x}+\lambda_1+\lambda_A+\lambda_D+\phi_1(x)\right)\overline{p_8}(x,s)=0 \quad (27)$$

The Laplace transform of the boundary conditions are:

$$\overline{p_1}(0,s)=2\lambda_1\overline{p_0}(s) \quad (28)$$

$$\overline{p_2}(0,s)=2\lambda_2\overline{p_0}(s) \quad (29)$$

$$\overline{p_3}(0,s)=\lambda_A\left(\overline{p_0}(s)+\overline{p_1}(0,s)+\overline{p_2}(0,s)+\overline{p_7}(0,s)+\overline{p_8}(0,s)\right) \quad (30)$$

$$\overline{p_4}(0,s)=\lambda_D\left(\overline{p_0}(s)+\overline{p_1}(0,s)+\overline{p_2}(0,s)+\overline{p_7}(0,s)+\overline{p_8}(0,s)\right) \quad (31)$$

$$\overline{p_5}(0,s)=\lambda_1\left(\overline{p_1}(0,s)+\overline{p_8}(0,s)\right) \quad (32)$$

$$\overline{p_6}(0,s)=\lambda_2\left(\overline{p_2}(0,s)+\overline{p_7}(0,s)\right) \quad (33)$$

$$\overline{p_7}(0,t)=2\lambda_2\overline{p_1}(0,s) \quad (34)$$

$$\overline{p_8}(0,s) = 2\lambda_1 \overline{p_2}(0,s) \tag{35}$$

Solving (19) – (27) with the help of (28) to (35), to obtained

$$\overline{p_0}(s) = \frac{1}{D(s)}$$

$$D(s) = \Big((s+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D) - \Big\{2\lambda_1\overline{S_{\phi_1}}(s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D) + 2\lambda_2\overline{S_{\phi_2}}(s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D) +$$
$$\lambda_A(1+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D)\overline{S_{\mu_0}}(s) + \lambda_D(1+2\lambda_1+2\lambda_2+8\lambda_1\lambda_2)\overline{S_{\mu_0}}(s) + (2\lambda_1^2+4\lambda_1^2\lambda_2)\overline{S_{\mu_0}}(s)$$
$$+(2\lambda_2^2+4\lambda_1\lambda_2^2)\overline{S_{\mu_0}}(s)\Big)$$

$$\overline{p_1}(s) = 2\lambda_1\left(\frac{1-\overline{S_{\phi_1}}(s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D)}{s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D}\right) \tag{36}$$

$$\overline{p_2}(s) = 2\lambda_2\left(\frac{1-\overline{S_{\phi_1}}(s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D)}{s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D}\right) \tag{37}$$

$$\overline{p_3}(s) = \lambda_A(1+2\lambda_1+2\lambda_2+8\lambda_1\lambda_2)\left(\frac{1-\overline{S_{\mu_0}}(s)}{s}\right)\overline{p_0}(s) \tag{38}$$

$$\overline{p_4}(s) = \lambda_D(1+2\lambda_1+2\lambda_2+8\lambda_1\lambda_2)\left(\frac{1-\overline{S_{\mu_0}}(s)}{s}\right)\overline{p_0}(s) \tag{39}$$

$$\overline{p_5}(s) = (2\lambda_1^2+4\lambda_1^2\lambda_2)\left(\frac{1-\overline{S_{\mu_0}}(s)}{s}\right)\overline{p_0}(s) \tag{40}$$

$$\overline{p_6}(s) = (2\lambda_2^2+4\lambda_1\lambda_1^2)\left(\frac{1-\overline{S_{\mu_0}}(s)}{s}\right)\overline{p_0}(s) \tag{41}$$

$$\overline{p_7}(s) = 4\lambda_1\lambda_2\left(\frac{1-\overline{S_{\phi_2}}(s+\lambda_2+\lambda_A+\lambda_D)}{s+\lambda_2+\lambda_A+\lambda_D}\right) \tag{42}$$

$$\overline{p_8}(s) = 4\lambda_1\lambda_2\left(\frac{1-\overline{S_{\phi_1}}(s+\lambda_1+\lambda_A+\lambda_D)}{s+\lambda_1+\lambda_A+\lambda_D}\right) \tag{43}$$

where

$$D(s)=\Big((s+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D)-\Big\{2\lambda_1\overline{S_{\phi_1}}(s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D)+2\lambda_2\overline{S_{\phi_2}}(s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D)+$$
$$\lambda_A(1+2\lambda_1+2\lambda_2+\lambda_A+\lambda_D)\overline{S_{\mu_0}}(s)+\lambda_D(1+2\lambda_1+2\lambda_2+8\lambda_1\lambda_2)\overline{S_{\mu_0}}(s)+\left(2\lambda_1^2+4\lambda_1^2\lambda_2\right)\overline{S_{\mu_0}}(s)$$
$$+\left(2\lambda_2^2+4\lambda_1\lambda_2^2\right)\overline{S_{\mu_0}}(s)\Big)$$

$$\overline{p}_{up}(s)=\left(\overline{p_0}(s)+\overline{p_1}(s)+\overline{p_2}(s)+\overline{p_7}(s)+\overline{p_8}(s)\right)=1+2\lambda_1\left(\frac{1-\overline{S}_{\phi_1}(s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D)}{s+2\lambda_2+\lambda_1+\lambda_A+\lambda_D}\right)+$$
$$2\lambda_2\left(\frac{1-\overline{S}_{\phi_2}(s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D)}{s+2\lambda_1+\lambda_2+\lambda_A+\lambda_D}\right)+4\lambda_1\lambda_2\left(\frac{1-\overline{S}_{\phi_2}(s+\lambda_2+\lambda_A+\lambda_D)}{s+\lambda_2+\lambda_A+\lambda_D}\right) \tag{44}$$
$$+4\lambda_1\lambda_2\left(\frac{1-\overline{S}_{\phi_1}(s+\lambda_1+\lambda_A+\lambda_D)}{s+\lambda_1+\lambda_A+\lambda_D}\right)$$

5. NUMERICAL COMPUTATIONS

5.1 AVAILABILITY ANALYSIS

Setting $\overline{S}_{\mu_0}(s)=\overline{S}_{\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}(s)=\dfrac{\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}{S+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}$, $S_{\phi_S}(s)=\dfrac{\phi_s}{s+\phi_s}$ and taking the values of different parameters as $\lambda_1=0.11$, $\lambda_2=0.12$, $\lambda_A=0.13$, $\lambda_D=0.14$ in (44), and then taking the inverse Laplace transform, one can obtain, the expression for availability $\left(P_{up}\right)$ as:

For different values of time variable t=0,1,2,3,4,5,6,7,8,9,10 units of time to obtain $P_{up}(t)$ using (44) depicted in Table 2 and Figure 3.

Table 2: Variation of availability with respect of time

Time (t)	Availability
0	1.0000
1	0.9576
2	0.9070
3	0.8419
4	0.7776
5	0.7174
6	0.6617
7	0.6103
8	0.5629
9	0.5191
10	0.4788

For, different values of time variable t = 0, 1, 2, 3, 4,.., 10 units of time, one may get different values of availability from equation (44) as shown in Figure 2.

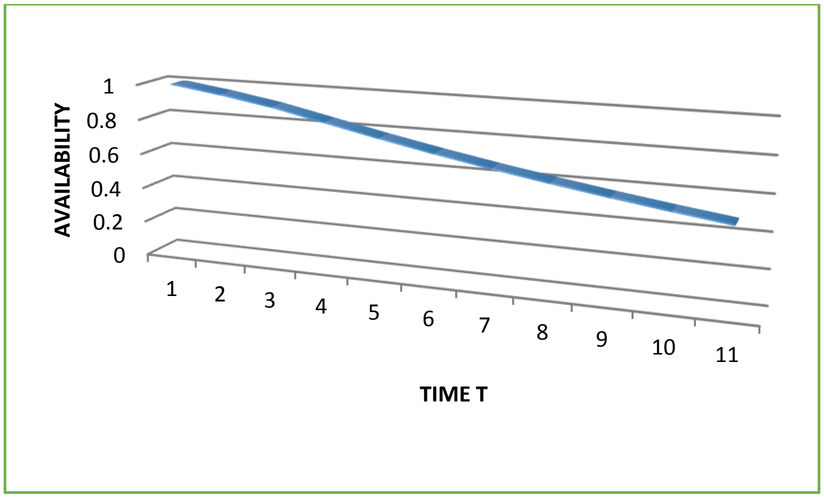


Figure 3: Availability against time t

5.2 RELIABILITY ANALYSIS

Taking all repair rates (αi, i = 1, 2, 3 and ψ, $\mu 0$) in Eq. (44) to zero and then taking inverse Laplace transform, one may have the expression for reliability for system when all three subsystems are independent. Expression for reliability of independent subsystems is given as

$$R(t) = 2e^{-0.61t} + 0.1552941176e^{-0.39t} + 0.1508571429e^{-0.38t} + 2e^{-0.62t} - 3.306151261e^{-0.73t} \quad (45)$$

For, different values of time t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.., units of time, one may get different values of reliability that shown in Table 3 and graphical representation in Figure 4.

Table 3: Variation of Reliability with respect of time

Time	Reliability
0	1.0000
1	0.7776
2	0.5431
3	0.3586
4	0.2291
5	0.1435
6	0.0889
7	0.0547
8	0.0336
9	0.0207
10	0.0128

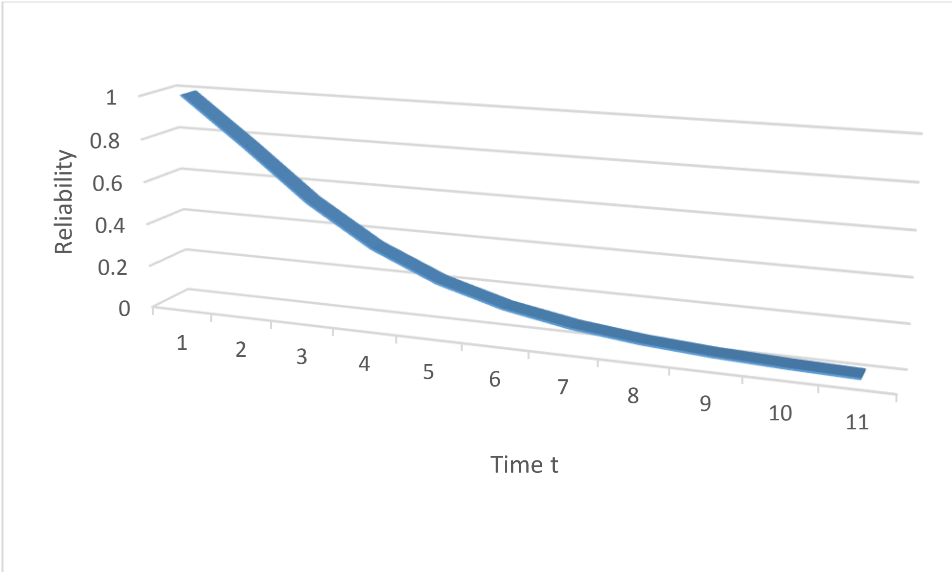


Figure 4: Reliability against time

5.3 MTTF ANALYSIS

Taking all repairs zero in equation (45), and the limit, as *s* tends to zero one can obtain the expression for MTTF as:

$$MTTF = \lim_{s\to 0} \overline{P}_{up}(s) = 1 + \frac{2\lambda_1}{2\lambda_2+\lambda_1+\lambda_A+\lambda_D} + \frac{2\lambda_2}{2\lambda_1+\lambda_2+\lambda_A+\lambda_D} + \frac{4\lambda_1\lambda_2}{\lambda_2+\lambda_A+\lambda_D} + \frac{4\lambda_1\lambda_2}{\lambda_1+\lambda_A+\lambda_D} \quad (46)$$

Setting $\lambda_1 = 0.11,\ \lambda_2 = 0.12,\ \lambda_A = 0.13,\ \lambda_D = 0.14$ and varying λ_1, λ_2, λ_A, λ_D one by one respectively as (61), one may obtain the variation of MTTF with respect to failure rates as shown in Figure .

Table 4: MTTF for various values of failure rates

Failure rate	MTTF λ_1	MTTF λ_2	MTTF λ_A	MTTF λ_D
0.01	3.1193	3.1711	3.8170	3.9387
0.02	3.0751	3.1240	3.7024	3.8170
0.03	3.0339	3.0802	3.5941	3.7024
0.04	2.9953	3.0391	3.4917	3.5941
0.05	2.9588	3.0002	3.3947	3.4917
0.06	2.9241	2.9633	3.3026	3.3947
0.07	2.8910	2.9281	3.2152	3.3026
0.08	2.8592	2.8943	3.1321	3.2152
0.09	2.8287	2.8618	3.0529	3.1321

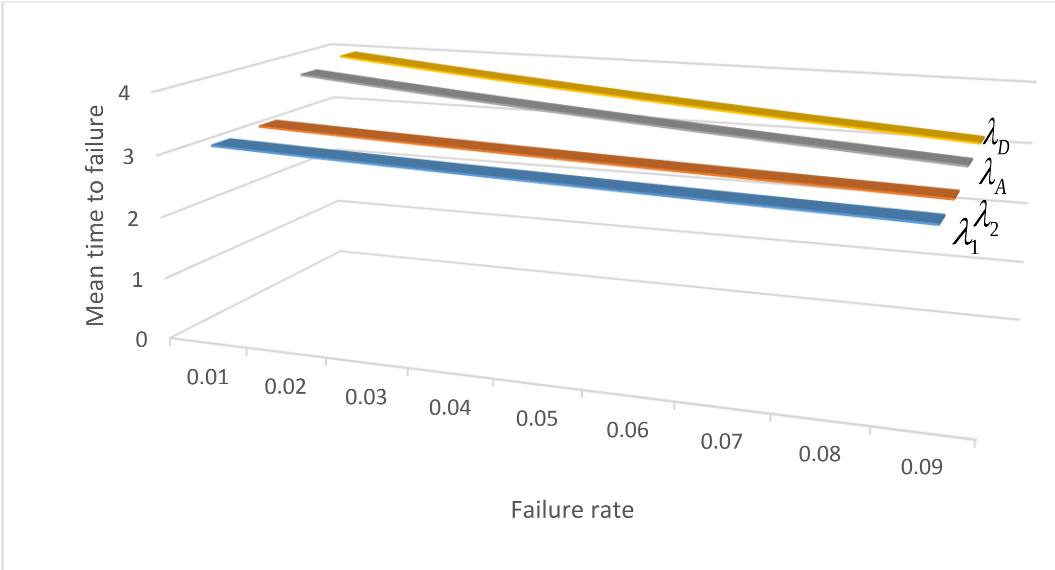


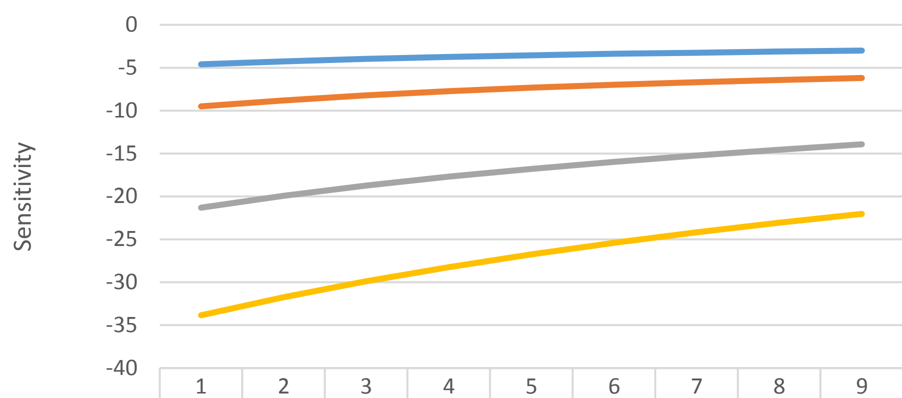
Figure 5: MTTF against failure rates

5.4 SENSITIVITY ANALYSIS

The sensitivity in MTTF of the system computed through the partial differentiation of MTTF with respect to the failure rates of the system. By applying the set of parameters as, $\lambda_A = 0.11$, $\lambda_1 = 0.12$, $\lambda_2 = 0.13$ and $\lambda_D = 0.14$, in the partial differentiation of MTTF, one can calculate the MTTF sensitivity as shown in Table 5 below.

Table 5: MTTF sensitivity for different failure rates

Failure rate	$\frac{\partial(MTTF)}{\lambda_A}$	$\frac{\partial(MTTF)}{\lambda_1}$	$\frac{\partial(MTTF)}{\lambda_2}$	$\frac{\partial(MTTF)}{\lambda_D}$
0.01	-4.600	-4.900	-11.806	-12.537
0.02	-4.260	-4.536	-11.138	-11.806
0.03	-3.981	-4.238	-10.526	-11.138
0.04	-3.749	-3.990	-9.964	-10.526
0.05	-3.554	-3.782	-9.445	-9.964
0.06	-3.386	-3.603	-8.966	-9.445
0.07	-3.241	-3.449	-8.523	-8.966
0.08	-3.114	-3.313	-8.111	-8.523
0.09	-3.001	-3.192	-7.729	-8.111



	1	2	3	4	5	6	7	8	9
Series4	-12.537	-11.806	-11.138	-10.526	-9.964	-9.445	-8.966	-8.523	-8.111
Series3	-11.806	-11.138	-10.526	-9.964	-9.445	-8.966	-8.523	-8.111	-7.729
Series2	-4.9	-4.536	-4.238	-3.99	-3.782	-3.603	-3.449	-3.313	-3.192
Series1	-4.6	-4.26	-3.981	-3.749	-3.554	-3.386	-3.241	-3.114	-3.001

Figure 6: MTTF sensitivity against failure rates

5.5 COST ANALYSIS

If the service facility is always available, then expected profit during the interval [0, t] is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{48}$$

Setting $\overline{S}_{\mu_0}(s) = \overline{S}_{\exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}(s) = \dfrac{\exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}{S + \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}$, $S_{\phi_S}(s) = \dfrac{\phi_s}{s+\phi_s}$ and taking the values of different parameters as $\lambda_1 = 0.11$, $\lambda_2 = 0.12$, $\lambda_A = 0.13$, $\lambda_D = 0.14$ in (44), and then taking the inverse Laplace transform, then taking inverse Laplace transform, one can obtain

$$\begin{aligned} E_p(t) = k_1\Big[& -5.783215296*10^{-8}\ e^{-161.0014976t} - 0.03743744964\, e^{-3.270708894t} + 0.08569776416\, e^{-1.715095953t} \\ & -13.29141820\, e^{-0.08089760293t} + 0.006189896096\, e^{-1.61t} + 0.01449771641\, e^{-1.39t} \\ & +0.01492951829\, e^{-1.38t} + 13.207\Big] - k_2(t) \end{aligned} \tag{49}$$

Setting K_1= 1and K_2= 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively and varying t =0, 1, 2, 3, 4, 5, 5, 7, 8, 9, 10. Units of time, the results for expected profit can be obtain as shown in Table 6 below.

Table 6: Expected profit for different values of k_2

Time	$k_2 = 0.6$	$k_2 = 0.5$	$k_2 = 0.4$	$k_2 = 0.3$	$k_2 = 0.2$	$k_2 = 0.1$
0	0	0	0	0	0	0
1	0.3710	0.4710	0.5710	0.6710	0.7710	0.8710
2	0.7059	0.9059	1.1059	1.3059	1.5059	1.7059
3	0.9807	1.2807	1.5807	1.8807	2.1807	2.4807
4	1.1902	1.5902	1.9902	2.3902	2.7902	3.1902
5	1.3374	1.8374	2.3374	2.8374	3.3374	3.8374
6	1.4266	2.0266	2.6266	3.2266	3.8266	4.4266
7	1.4623	2.1623	2.8623	3.5623	4.2623	4.9623
8	1.4486	2.2486	3.0486	3.8486	4.6486	5.4486
9	1.3894	2.2894	3.1894	4.0894	4.9894	5.8894
10	1.2881	2.2881	3.2881	4.2881	5.2881	6.2881

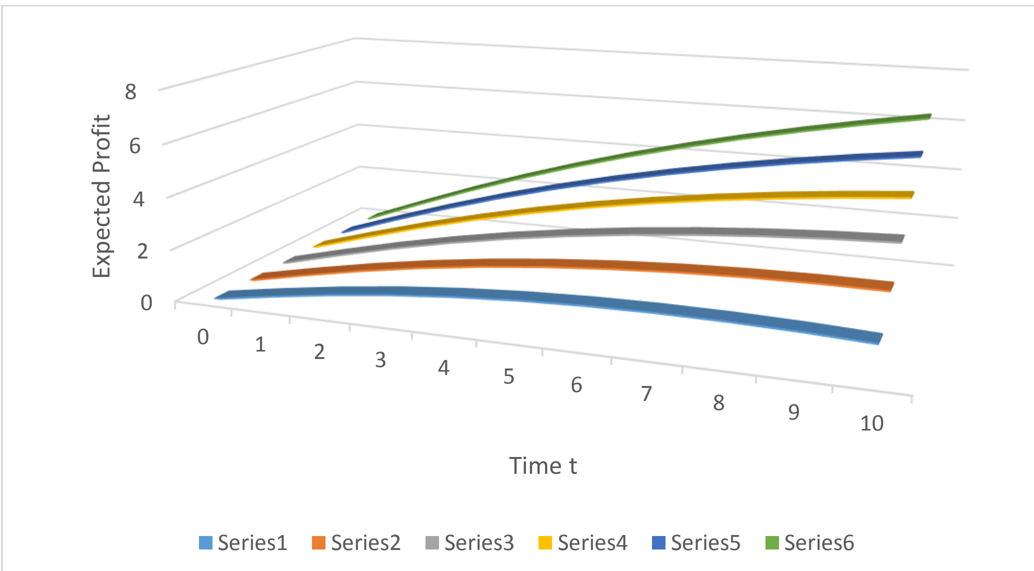


Figure 7: Expected profit againnst time

Where

Series 1 for $k_1 = 0.6$, Series 2 for $k_1 = 0.5$, Series 3 for $k_1 = 0.4$, Series 4 for $k_1 = 0.3$,

Series 5 for $k_1 = 0.2$, Series 6 for $k_1 = 0.1$

6. Discussion and Conclusion

Tables 2 and 3 and corresponding Figures 3 and 4 displayed the impact of passage time on availability and reliability. It is evident from these Tables and figures that availability and reliability decreases as time t increase for fixed values of failure and repair rates. On the other hand, availability, profit and mean time to failure are higher higher value of repair rates and lower value of failure rates. Table 4 and the corresponding Figure 5 depicts the behavior of mean time to failure (MTTF) with respect to λ_A, λ_1, λ_2 and λ_D fixing other parameters constant. It is evident from the Table and the figure that MTTF decreases with increase in the values of λ_A, λ_1, λ_2 and λ_D respectively. However the MTTF is higher with respect to λ_D. Thus,

$$MTTF\left(w.r.t\,\lambda_D\right) > MTTF\left(w.r.t\,\lambda_2\right) > MTTF\left(w.r.t\,\lambda_1\right) > MTTF\left(w.r.t\,\lambda_A\right)$$

This sensitivity analysis implies that preventive and major maintenance should be invoked to the receiver, relay stations and the transmitter to minimize the system break down, prolong MTTF and maximizes the system reliability, availability as well as expected profit. Tables 5 and Figure 6 displayed the variation of sensitivity analysis with respect to change in λ_A, λ_1, λ_2 and λ_D respectively. It is evident from the Table and figure that sensitivity of the MTTF increases with the increase in the value of λ_A, λ_1, λ_2 and λ_D. It is evident from the plots in Figure 6 that MTTF is sensitive to λ_A, λ_1, λ_2 and λ_D. Table 6 and Figure 7 displayed the results on revenue cost per unit time. From the table and the figure it is evident that expected profit increases with respect to time when k_2. The expected profit is lower when $k_2 = 0.6$ and higher for $k_2 = 0.1$. To achieve high quality, minimum production losses, higher production output as well as expected revenue, there should be failure free network and highest system *reliability*, availability and mean time to failure (MTTF).

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] E.K. Çetinkaya, J.P.G. Sterbenz, A taxonomy of network challenges. In Proceedings of the 9th IEEE/IFIP International Conference on Design of Reliable Communication Networks (DRCN), Budapest (pp. 322–330), 2013.

[2] M. Gahlot, V.V. Singh, H.I. Ayagi, and C.K. Goel, Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics, Int. J. Reliab. Safety, 12(4)(2018), 348-363.

[3] J. Gulati, V.V. Singh, D.K. Rawal, Performance assessment of two unit's redundant system under different failure and repair policies using copula, Int. J. Reliab. Appl. 19(2)(2018), 109-124.

[4] M. Kakkar, A. Chitkara, J. Bhatti, Reliability analysis of two unit parallel repairable industrial system, Decis. Sci. Lett. 4(4)(2015), 525-536.

[5] M. Kakkar, A.Chitkara, J. Bhatti, Reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance, Manage. Sci. Lett. 6(4)(2016), 285-296.

[6] A. Kumar, S.C. Malik, Reliability modeling of a computer system with priority to H/W repair over replacement oh H/W and up-gradation of S/W subject to MOT and MRT, Jordan J. Mech. Ind. Eng. 8(4)(2014), 233–241.

[7] A. Kumar, S. Malik, Reliability measures of a computer system with priority to PM over the H/W repair activities subject to MOT and MRT, Manage. Sci. Lett. 5(1)(2015), 29-38.

[8] A. Kumar, S. Pant, S.B. Singh, Reliability optimization of complex system by using cuckoos search algorithm, in Ram, M. and Davim, J.P. (Eds), Mathematical Concepts and Applications in Mechanical Engineering and Mechatronics, IGI Global Publisher, Hershey, PA, pp. 95-112, 2016.

[9] A. Kumar, S. Pant, S.B. Singh, Availability and cost analysis of an engineering system involving subsystems in series configuration, Int. J. Qual. Reliab. Manage. 34(6)(2017), 879-894.

[10] F. Ioan, T. Nicolae, T. Gavril, Network communication reliability analysis in terms of fault tree, J. Electric. Electron. Eng. 8(1) (2015), 13-16.

[11] A. Lado, V.V. Singh, I. Yusuf, Performance and cost assessment of repairable complex system with two subsystems connected in series configuration, Int. J. Reliab. Appl. 19(1)(2018), 27-42.

[12] A. Munjal, S.B. Singh, Reliability analysis of a complex repairable system composed of two 2-out-of-3: G subsystems connected in parallel, J. Reliab. Stat. Stud. 7(2014), 89–111.

[13] S. Negi, S.B. Singh, Reliability analysis of non-repairable complex system with weighted subsystems connected in series, Appl. Math. Comput. 262 (2015), 79–89.

[14] R. Niwas, M. S. Kadyan, J. Kumar, Probabilistic analysis of two reliability models of a single-unit system with preventive maintenance beyond warranty and degradation, Eksploatacja i Niezawodnosc - Maintenance and Reliability, 17(4)(2015), 535–543.

[15] R. Niwas, M.S. Kadyan, Reliability modeling of a maintained system with warranty and degradation, J. Reliab. Stat. Stud. 8(1) (2015), 63–75.

[16] S. Neumayer, E. Modiano, Network reliability with geographically correlated failures. In Proceedings of the IEEE International Conference on Computer Communications INFOCOM 2010 (pp. 1–9), 2010.

[17] P. Rafiee, G.L. Shabgahi, Evaluating the Reliability of Communication Networks (WAN) Using their Fuzzy Fault Tree Analysis - A Case Study. J. Math. Computer Sci. 2(2)(2011), 262-270.

[18] M. Ram, V. Singh, Modeling and availability analysis of internet data center with various maintenance policies, Int. J. Eng. 27(4)(2014), 599–608.

[19] M. Sadeghi, E. Roghanian, Reliability analysis of a warm standby repairable system with two cases of imperfect switching mechanism, Sci. Iran. E, 24(2)(2017), 808-822.

[20] V.V. Singh, N.P. Singh, Performance analysis of three unit redundant system with switch and human failure, Math. Eng., Sci. Aerosp. 6(2)(2015), 295–308.

[21] Singh, V.V., Gulati, J., Rawal, D.K and Goel, C.K. (2016). Performance analysis of complex system in series configuration under different failure and repair discipline using copula, Int. J. Reliab. Qual. Safety Eng. 23(2) (2016), 1650010.

[22] J. P. G. Sterbenz, D. Hutchison, E. K. Çetinkaya, A. Jabbar, J. P. Rohrer, M. Schöller, P. Smith, Redundancy, diversity, and connectivity to achieve multilevel network resilience, survivability, and disruption tolerance. Telecommun. Syst. 56(1) (2014)., 17–31.