RELIABILITY ANALYSIS OF COMMUNICATION NETWORK SYSTEM WITH REDUNDANT RELAY STATION UNDER PARTIAL AND COMPLETE FAILURE

IBRAHIM YUSUF1,*, ABDULKAREEM LAIDO ISMAIL2 AND U.A. ALI3

1Department of Mathematical Sciences, Bayero University, Kano, Nigeria
2Department of Mathematics, Kano State College of Arts, Science and Remedial Studies, Kano, Nigeria
3Department of Mathematics, Federal University, Dutse, Nigeria

Copyright © 2020 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract: The purpose of this paper is to study the performance of a communication network system consisting of a transmitter, two relay stations and a receiver arranged in series parallel. Through the transition diagram, the partial differential equations are derived, and Laplace transforms are then taken on these equations to derive system reliability, availability, the mean time to system failure (MTTF) and cost function. It is assumed that failure rates are constant and follows exponential distribution, repair rates of partial failure state are assumed to follow general distribution and complete failure states are repaired through Gumbel-Hougaard family copula. The system is analyzed through supplementary variable technique and Laplace transform. Different measures of testing system effectiveness which include reliability, availability, mean time to failure (MTTF) and profit function have been calculated for particular values of time, failure and repair rates. From the study, it is clear that time and failure rates of both transmitter, relay stations and receiver influence the reliability, availability, MTTF and profit function. Mathematical models developed in this paper can aid plant management for proper maintenance and system safety, avoiding incorrect reliability, availability and profit assessment and leading to inadequate maintenance decision making, which may result in unnecessary expenditures and reduction of safety standards.

Keywords: communication network; reliability; availability; mean time to failure.

2010 AMS Subject Classification: 90B25.

*Corresponding author
E-mail address: ibrahimyusuffagge@gmail.com
Received December 2, 2019
1. **Introduction**

From the literature study above, little or no attention is paid on the reliability analysis of communication network system with standby relay stations. The present paper is aim at reliability modelling and analysis of a communication network system with transmitter, two relay stations in parallel and a receiver. Explicit expressions for reliability, availability, mean time to failure and profit function have been obtained. The objectives of this paper are twofold: First is to derive the explicit expressions for the reliability, availability, mean time to failure and profit function. Second is to capture the effect of both passage time and failure rates on reliability, availability, MTTF and profit.

The organization of the paper is as follows. Section 2 presents the notations of the study. Section 3 contains a description of the system under study. Section 4 presents formulations of the models. The results of our analytical comparison between the systems are presented in section 5. Numerical examples are presented in section 5. Finally, we make some concluding remarks in Section 6.

2. Notations

- \( t \): Time variable
- \( s \): Laplace transform variable
- \( \lambda_A \): Failure rate of transmitter (subsystem A)
- \( \lambda_1 \): Failure rate of the first relay station (Subsystem B)
- \( \lambda_2 \): Failure rate of the second relay station (subsystem C)
- \( \lambda_D \): Failure rate of the receiver (subsystem D)
- \( \phi_1(x) \): Repair rate of the first relay station (Subsystem B)
- \( \phi_2(x) \): Repair rate of the second relay station (Subsystem C)
- \( \mu_0(x) \): Repair rate of complete failure state
- \( p_i(t) \): Probability that the system is in state \( S_i \) for \( i = 0, 1, 2, \ldots, 8 \)
- \( p(s) \): Laplace transformation of transition probability \( p(t) \)
- \( p_i(x,t) \): Probability that the system is in state \( S_i \) for \( i = 0, 1, 2, \ldots, 8 \), the system is under repair with elapse repair time \( (x,t) \) with variables \( x \) and \( t \) for repair and time respectively.
\( E_p(t) \): Revenue generated in the interval \([0,t]\)

\( k_1, k_2 \): Revenue and service cost

\[ S_f(x) = f(x) e^{-\int_0^x f(x) \, dx} \] Notation function with repair distribution \( f(x) \)

\[ \overline{S_f}(s) = \int_0^\infty e^{-sx} f(x) e^{-\int_0^x f(x) \, dx} \, dx \] Laplace transform of \( S_f(x) \)

3. **DESCRIPTION AND STATES OF THE SYSTEM**

![Reliability Block Diagram of the System](image)

The system consisting of four subsystems A, B, C and D arranged in series-parallel. Subsystems A and D are single unit while subsystem B and C are parallel to each other consisting of 2-out-of-3 units each. Signal is received from the transmitter by relay stations in subsystems B and C, is distributed to the receiver (subsystem D) for consumption as shown in Figure 1. Subsystem A is the transmitter, subsystems B and C are relay stations, while subsystem D is the receiver. However the two paths are parallel to each other. The system failed when any of the subsystem A or D failed or B and C.
Figure 2: State Transition Diagram
Table 1: States of the System

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>Initial state, transmitter, two consecutive relays from subsystems 2 and 3, receiver are working, one relay each from subsystems B and C are on standby. The system is working.</td>
</tr>
<tr>
<td>$S_1$</td>
<td>Transmitter, two consecutive relays from subsystems B and C, receiver are working, one relay has failed in subsystems B and one relay is on standby in subsystem C. The system is working.</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Transmitter, two consecutive relays from subsystems B and C, receiver are working, one relay has failed in subsystems C and one relay is on standby in subsystem B. The system is working.</td>
</tr>
<tr>
<td>$S_3$</td>
<td>Transmitter and one relay each from subsystems B and C have failed, receiver is idle. The system is down.</td>
</tr>
<tr>
<td>$S_4$</td>
<td>Receiver has failed, transmitter and relay stations are idle. The system is down.</td>
</tr>
<tr>
<td>$S_5$</td>
<td>Transmitter, two consecutive relays from subsystems C, receiver are working, two relays have failed in subsystems B and one relay is on standby in subsystem C. The system is working.</td>
</tr>
<tr>
<td>$S_6$</td>
<td>Relay stations in both subsystems B and C have failed. The system is down.</td>
</tr>
<tr>
<td>$S_7$</td>
<td>Two relays one each from subsystems B and C have failed, transmitter, relay stations from subsystems B and C, and receiver are working. The system is working.</td>
</tr>
<tr>
<td>$S_8$</td>
<td>Two relays one each from subsystems B and C and receiver have failed. The system is down.</td>
</tr>
</tbody>
</table>

### 4. Mathematical Models Formulation

By probability of considerations and continuity arguments, the following set of difference equations are associated with the present mathematical model.

\[
\begin{align*}
\left(\frac{\delta}{\delta t} + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D\right)p_0(t) &= \int_0^\infty \phi_1(x) p_1(x,t) \, dx + \int_0^\infty \phi_2(x) p_2(x,t) \, dx + \int_0^\infty \mu_0(x) p_3(x,t) \, dx + \\
&+ \int_0^\infty \mu_0(x) p_4(x,t) \, dx + \int_0^\infty \mu_0(x) p_5(x,t) \, dx + \int_0^\infty \mu_0(x) p_6(x,t) \, dx \\
\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + 2\lambda_2 + \lambda_A + \lambda_D + \phi_1(x)\right)p_1(x,t) &= 0 \\
\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + 2\lambda_A + \lambda_A + \lambda_D + \phi_2(x)\right)p_2(x,t) &= 0 \\
\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \mu_0(x\right)p_3(x,t) &= 0
\end{align*}
\]
\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \right) p_4(x,t) = 0
\]  \hspace{1cm} (5)

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \right) p_5(x,t) = 0
\]  \hspace{1cm} (6)

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \right) p_6(x,t) = 0
\]  \hspace{1cm} (7)

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_A + \lambda_D + \phi_2(x) \right) p_7(x,t) = 0
\]  \hspace{1cm} (8)

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \lambda_A + \lambda_D + \phi_1(x) \right) p_8(x,t) = 0
\]  \hspace{1cm} (9)

BOUNDARY CONDITIONS:

\[ p_1(0,t) = 2\lambda_0 p_0(t) \]  \hspace{1cm} (10)

\[ p_2(0,t) = 2\lambda_2 p_0(t) \]  \hspace{1cm} (11)

\[ p_3(0,t) = \lambda_A \left( p_0(t) + p_1(0,t) + p_2(0,t) + p_7(0,t) + p_8(0,t) \right) \]  \hspace{1cm} (12)

\[ p_4(0,t) = \lambda_D \left( p_0(t) + p_1(0,t) + p_2(0,t) + p_7(0,t) + p_8(0,t) \right) \]  \hspace{1cm} (13)

\[ p_5(0,t) = \lambda_1 \left( p_1(0,t) + p_8(0,t) \right) \]  \hspace{1cm} (14)

\[ p_6(0,t) = \lambda_2 \left( p_2(0,t) + p_7(0,t) \right) \]  \hspace{1cm} (15)

\[ p_7(0,t) = 2\lambda_2 p_1(0,t) \]  \hspace{1cm} (16)

\[ p_8(0,t) = 2\lambda_4 p_2(0,t) \]  \hspace{1cm} (17)

INITIALS CONDITIONS:

\[ p_k(t) = \begin{cases} 
1, & k = 0 \\
0, & k = 1, 2, ..., 8 
\end{cases} \text{ at } t = 0 \]  \hspace{1cm} (18)

SOLUTION OF THE MODEL

Taking Laplace transform of (1) to (17) using the initial condition above, to obtained

\[
\left( s + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D \right) p_0(s) = 1 + \int_0^\infty \phi_1(x) \bar{p}_1(x,s) dx + \int_0^\infty \phi_2(x) \bar{p}_2(x,s) dx + \int_0^\infty \mu_0(x) \bar{p}_3(x,s) dx +
\]
\[
\int_0^\infty \mu_0(x) p_1(x, s) \, dx + \int_0^\infty \mu_0(x) p_2(x, s) \, dx + \int_0^\infty \mu_0(x) p_3(x, s) \, dx
\]

\[
\left( s + \frac{\delta}{\delta x} + 2\lambda_2 + \lambda_1 + \lambda_A + \lambda_D + \phi_1(x) \right) \overline{p_1}(x, S) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + 2\lambda_1 + \lambda_2 + \lambda_A + \lambda_D + \phi_2(x) \right) \overline{p_2}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \mu_0(x) \right) \overline{p_3}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \mu_0(x) \right) \overline{p_4}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \mu_0(x) \right) \overline{p_5}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \mu_0(x) \right) \overline{p_6}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \lambda_2 + \lambda_A + \lambda_D + \phi_2(x) \right) \overline{p_7}(x, s) = 0
\]

\[
\left( s + \frac{\delta}{\delta x} + \lambda_1 + \lambda_A + \lambda_D + \phi_1(x) \right) \overline{p_8}(x, s) = 0
\]

The Laplace transform of the boundary conditions are:

\[
\overline{p_1}(0, s) = 2\lambda_1 \overline{p_0}(s)
\]

\[
\overline{p_2}(0, s) = 2\lambda_2 \overline{p_0}(s)
\]

\[
\overline{p_3}(0, s) = \lambda_A \left( \overline{p_0}(s) + \overline{p_1}(0, s) + \overline{p_2}(0, s) + \overline{p_7}(0, s) + \overline{p_8}(0, s) \right)
\]

\[
\overline{p_4}(0, s) = \lambda_D \left( \overline{p_0}(s) + \overline{p_1}(0, s) + \overline{p_2}(0, s) + \overline{p_7}(0, s) + \overline{p_8}(0, s) \right)
\]

\[
\overline{p_5}(0, s) = \lambda_1 \left( \overline{p_1}(0, s) + \overline{p_8}(0, s) \right)
\]

\[
\overline{p_6}(0, s) = \lambda_2 \left( \overline{p_2}(0, s) + \overline{p_7}(0, s) \right)
\]

\[
\overline{p_7}(0, t) = 2\lambda_2 \overline{p_1}(0, s)
\]
RELIABILITY ANALYSIS OF COMMUNICATION NETWORK SYSTEM

\[ \overline{p_8}(0,s) = 2\lambda_1 \overline{p_2}(0,s) \]  \hspace{1cm} (35)

Solving (19) – (27) with the help of (28) to (35), to obtained

\[ \overline{p_0}(s) = \frac{1}{D(s)} \]

\[ D(s) = \left( (s + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D) - \left\{ 2\lambda_1 \overline{S_{\phi_1}}(s + 2\lambda_2 + \lambda_1 + \lambda_A + \lambda_D) + 2\lambda_2 \overline{S_{\phi_2}}(s + 2\lambda_1 + \lambda_2 + \lambda_A + \lambda_D) + \lambda_A (1 + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D) \overline{S_{\rho_0}}(s) + \lambda_D (1 + 2\lambda_1 + 2\lambda_2 + 8\lambda_1\lambda_2) \overline{S_{\rho_2}}(s) \right\} + \left( 2\lambda_2^2 + 4\lambda_1\lambda_2^2 \right) \overline{S_{\rho_0}}(s) \right) \]

\[ \overline{p_1}(s) = 2\lambda_1 \left( 1 - \frac{\overline{S_{\phi_1}}(s + 2\lambda_2 + \lambda_1 + \lambda_A + \lambda_D)}{s + 2\lambda_2 + \lambda_1 + \lambda_A + \lambda_D} \right) \]  \hspace{1cm} (36)

\[ \overline{p_2}(s) = 2\lambda_2 \left( 1 - \frac{\overline{S_{\phi_2}}(s + 2\lambda_1 + \lambda_2 + \lambda_A + \lambda_D)}{s + 2\lambda_1 + \lambda_2 + \lambda_A + \lambda_D} \right) \]  \hspace{1cm} (37)

\[ \overline{p_3}(s) = \lambda_A \left( 1 + 2\lambda_1 + 2\lambda_2 + 8\lambda_1\lambda_2 \right) \left( 1 - \frac{\overline{S_{\rho_0}}(s)}{s} \right) \overline{p_0}(s) \]  \hspace{1cm} (38)

\[ \overline{p_4}(s) = \lambda_D \left( 1 + 2\lambda_1 + 2\lambda_2 + 8\lambda_1\lambda_2 \right) \left( 1 - \frac{\overline{S_{\rho_0}}(s)}{s} \right) \overline{p_0}(s) \]  \hspace{1cm} (39)

\[ \overline{p_5}(s) = \left( 2\lambda_1^2 + 4\lambda_1\lambda_2^2 \right) \left( 1 - \frac{\overline{S_{\rho_0}}(s)}{s} \right) \overline{p_0}(s) \]  \hspace{1cm} (40)

\[ \overline{p_6}(s) = \left( 2\lambda_2^2 + 4\lambda_1\lambda_2^2 \right) \left( 1 - \frac{\overline{S_{\rho_0}}(s)}{s} \right) \overline{p_0}(s) \]  \hspace{1cm} (41)

\[ \overline{p_7}(s) = 4\lambda_1\lambda_2 \left( 1 - \frac{\overline{S_{\phi_1}}(s + \lambda_2 + \lambda_A + \lambda_D)}{s + \lambda_2 + \lambda_A + \lambda_D} \right) \]  \hspace{1cm} (42)

\[ \overline{p_8}(s) = 4\lambda_1\lambda_2 \left( 1 - \frac{\overline{S_{\phi_1}}(s + \lambda_1 + \lambda_A + \lambda_D)}{s + \lambda_1 + \lambda_A + \lambda_D} \right) \]  \hspace{1cm} (43)

where
\[ D(s) = \left( (s + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D) - \left( 2\lambda_1 \overline{S_{\phi_1}}(s) + 2\lambda_2 \overline{S_{\phi_2}}(s) + 2\lambda_D \overline{S_{\phi_D}}(s) \right) \right) + \lambda_A \left( 1 + 2\lambda_1 + 2\lambda_2 + \lambda_A + \lambda_D \right) \overline{S_{\phi_A}}(s) + \lambda_D \left( 1 + 2\lambda_1 + 2\lambda_2 + 8\lambda_A\lambda_2 \right) \overline{S_{\phi_D}}(s) + \left( 2\lambda_2^2 + 4\lambda_A\lambda_2 \right) \overline{S_{\phi_D}}(s) \] 

\[ \overline{P_{up}}(s) = \left( \overline{P_0}(s) + \overline{P_1}(s) + \overline{P_2}(s) + \overline{P_A}(s) + \overline{P_D}(s) \right) = 1 + 2\lambda_1 \left( \frac{1 - \overline{S_{\phi_1}}(s) + 2\lambda_1 + \lambda_A + \lambda_D}{s + 2\lambda_1 + \lambda_A + \lambda_D} \right) + 2\lambda_2 \left( \frac{1 - \overline{S_{\phi_2}}(s) + 2\lambda_1 + \lambda_A + \lambda_D}{s + 2\lambda_1 + \lambda_A + \lambda_D} \right) + 4\lambda_A\lambda_2 \left( \frac{1 - \overline{S_{\phi_A}}(s) + 2\lambda_1 + \lambda_A + \lambda_D}{s + \lambda_1 + \lambda_A + \lambda_D} \right) \left( \frac{1 - \overline{S_{\phi_D}}(s) + 2\lambda_1 + \lambda_A + \lambda_D}{s + \lambda_1 + \lambda_A + \lambda_D} \right) \] 

\[ \text{(44)} \]

5. Numerical Computations

5.1 Availability Analysis

Setting \( \overline{S_{\phi_0}}(s) = \overline{S_{\exp\left[ x^\theta + \{\log\phi(x)\}^\theta \right]}^\frac{1}{\beta}}(s) = \frac{\exp\left[ x^\theta + \{\log\phi(x)\}^\theta \right]^{\frac{1}{\beta}}}{S + \exp\left[ x^\theta + \{\log\phi(x)\}^\theta \right]^{\frac{1}{\beta}}} \), \( S_{\phi_k}(s) = \frac{\phi_k}{s + \phi_k} \) and taking the values of different parameters as \( \lambda_1 = 0.11 \), \( \lambda_2 = 0.12 \), \( \lambda_A = 0.13 \), \( \lambda_D = 0.14 \) in (44), and then taking the inverse Laplace transform, one can obtain, the expression for availability \( P_{up} \) as:

For different values of time variable \( t=0,1,2,3,4,5,6,7,8,9,10 \) units of time to obtain \( P_{up}(t) \) using (44) depicted in Table 2 and Figure 3.

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.9576</td>
</tr>
<tr>
<td>2</td>
<td>0.9070</td>
</tr>
<tr>
<td>3</td>
<td>0.8419</td>
</tr>
<tr>
<td>4</td>
<td>0.7776</td>
</tr>
<tr>
<td>5</td>
<td>0.7174</td>
</tr>
<tr>
<td>6</td>
<td>0.6617</td>
</tr>
<tr>
<td>7</td>
<td>0.6103</td>
</tr>
<tr>
<td>8</td>
<td>0.5629</td>
</tr>
<tr>
<td>9</td>
<td>0.5191</td>
</tr>
<tr>
<td>10</td>
<td>0.4788</td>
</tr>
</tbody>
</table>
For, different values of time variable \( t = 0, 1, 2, 3, 4, \ldots, 10 \) units of time, one may get different values of availability from equation (44) as shown in Figure 2.

![Figure 3: Availability against time \( t \)]

### 5.2 RELIABILITY ANALYSIS

Taking all repair rates \((\alpha_i, i = 1, 2, 3)\) and \((\psi, \mu)\) in Eq. (44) to zero and then taking inverse Laplace transform, one may have the expression for reliability for system when all three subsystems are independent. Expression for reliability of independent subsystems is given as

\[
R(t) = 2e^{-0.6lt} + 0.1552941176e^{-0.39lt} + 0.1508571429e^{-0.38lt} + 2e^{-0.62lt} - 3.306151261e^{-0.73lt} \quad (45)
\]

For, different values of time \( t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots \) units of time, one may get different values of reliability that shown in Table 3 and graphical representation in Figure 4.

<table>
<thead>
<tr>
<th>Time</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.7776</td>
</tr>
<tr>
<td>2</td>
<td>0.5431</td>
</tr>
<tr>
<td>3</td>
<td>0.3586</td>
</tr>
<tr>
<td>4</td>
<td>0.2291</td>
</tr>
<tr>
<td>5</td>
<td>0.1435</td>
</tr>
<tr>
<td>6</td>
<td>0.0889</td>
</tr>
<tr>
<td>7</td>
<td>0.0547</td>
</tr>
<tr>
<td>8</td>
<td>0.0336</td>
</tr>
<tr>
<td>9</td>
<td>0.0207</td>
</tr>
<tr>
<td>10</td>
<td>0.0128</td>
</tr>
</tbody>
</table>
5.3 MTTF ANALYSIS

Taking all repairs zero in equation (45), and the limit, as \( s \) tends to zero one can obtain the expression for MTTF as:

\[
MTTF = \lim_{s \to 0} \frac{2\lambda_1}{2\lambda_2 + \lambda_1 + \lambda_A + \lambda_D} + \frac{2\lambda_2}{2\lambda_1 + \lambda_2 + \lambda_A + \lambda_D} + \frac{4\lambda_1\lambda_2}{\lambda_2 + \lambda_A + \lambda_D} + \frac{4\lambda_1\lambda_2}{\lambda_1 + \lambda_A + \lambda_D} \tag{46}
\]

Setting \( \lambda_1 = 0.11, \lambda_2 = 0.12, \lambda_A = 0.13, \lambda_D = 0.14 \) and varying \( \lambda_1, \lambda_2, \lambda_A, \lambda_D \) one by one respectively as (61), one may obtain the variation of MTTF with respect to failure rates as shown in Figure.

<table>
<thead>
<tr>
<th>Failure rate</th>
<th>MTTF ( \lambda_1 )</th>
<th>MTTF ( \lambda_2 )</th>
<th>MTTF ( \lambda_A )</th>
<th>MTTF ( \lambda_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>3.1193</td>
<td>3.1711</td>
<td>3.8170</td>
<td>3.9387</td>
</tr>
<tr>
<td>0.02</td>
<td>3.0751</td>
<td>3.1240</td>
<td>3.7024</td>
<td>3.8170</td>
</tr>
<tr>
<td>0.03</td>
<td>3.0339</td>
<td>3.0802</td>
<td>3.5941</td>
<td>3.7024</td>
</tr>
<tr>
<td>0.04</td>
<td>2.9953</td>
<td>3.0391</td>
<td>3.4917</td>
<td>3.5941</td>
</tr>
<tr>
<td>0.05</td>
<td>2.9588</td>
<td>3.0002</td>
<td>3.3947</td>
<td>3.4917</td>
</tr>
<tr>
<td>0.06</td>
<td>2.9241</td>
<td>2.9633</td>
<td>3.3026</td>
<td>3.3947</td>
</tr>
<tr>
<td>0.07</td>
<td>2.8910</td>
<td>2.9281</td>
<td>3.2152</td>
<td>3.3026</td>
</tr>
<tr>
<td>0.08</td>
<td>2.8592</td>
<td>2.8943</td>
<td>3.1321</td>
<td>3.2152</td>
</tr>
<tr>
<td>0.09</td>
<td>2.8287</td>
<td>2.8618</td>
<td>3.0529</td>
<td>3.1321</td>
</tr>
</tbody>
</table>
5.4 SENSITIVITY ANALYSIS

The sensitivity in MTTF of the system computed through the partial differentiation of MTTF with respect to the failure rates of the system. By applying the set of parameters as, $\lambda_A = 0.11$, $\lambda_1 = 0.12$, $\lambda_2 = 0.13$ and $\lambda_D = 0.14$, in the partial differentiation of MTTF, one can calculate the MTTF sensitivity as shown in Table 5 below.

Table 5: MTTF sensitivity for different failure rates

<table>
<thead>
<tr>
<th>Failure rate</th>
<th>$\frac{\partial (MTTF)}{\lambda_A}$</th>
<th>$\frac{\partial (MTTF)}{\lambda_1}$</th>
<th>$\frac{\partial (MTTF)}{\lambda_2}$</th>
<th>$\frac{\partial (MTTF)}{\lambda_D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-4.600</td>
<td>-4.900</td>
<td>-11.806</td>
<td>-12.537</td>
</tr>
<tr>
<td>0.02</td>
<td>-4.260</td>
<td>-4.536</td>
<td>-11.138</td>
<td>-11.806</td>
</tr>
<tr>
<td>0.03</td>
<td>-3.981</td>
<td>-4.238</td>
<td>-10.526</td>
<td>-11.138</td>
</tr>
<tr>
<td>0.04</td>
<td>-3.749</td>
<td>-3.990</td>
<td>-9.964</td>
<td>-10.526</td>
</tr>
<tr>
<td>0.05</td>
<td>-3.554</td>
<td>-3.782</td>
<td>-9.445</td>
<td>-9.964</td>
</tr>
<tr>
<td>0.06</td>
<td>-3.386</td>
<td>-3.603</td>
<td>-8.966</td>
<td>-9.445</td>
</tr>
<tr>
<td>0.07</td>
<td>-3.241</td>
<td>-3.449</td>
<td>-8.523</td>
<td>-8.966</td>
</tr>
<tr>
<td>0.08</td>
<td>-3.114</td>
<td>-3.313</td>
<td>-8.111</td>
<td>-8.523</td>
</tr>
<tr>
<td>0.09</td>
<td>-3.001</td>
<td>-3.192</td>
<td>-7.729</td>
<td>-8.111</td>
</tr>
</tbody>
</table>
5.5 COST ANALYSIS

If the service facility is always available, then expected profit during the interval \([0, t]\) is

\[
E_p(t) = K_1 \int_0^t P_{\text{up}}(t) \, dt - K_2 t
\]

(48)

Setting \(K_1 = 1\) and \(K_2 = 0.6, 0.5, 0.4, 0.3, 0.2\) and 0.1 respectively and varying \(t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\). Units of time, the results for expected profit can be obtained as shown in Table 6 below.
Table 6: Expected profit for different values of $k_2$

<table>
<thead>
<tr>
<th>Time</th>
<th>$k_2 = 0.6$</th>
<th>$k_2 = 0.5$</th>
<th>$k_2 = 0.4$</th>
<th>$k_2 = 0.3$</th>
<th>$k_2 = 0.2$</th>
<th>$k_2 = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3710</td>
<td>0.4710</td>
<td>0.5710</td>
<td>0.6710</td>
<td>0.7710</td>
<td>0.8710</td>
</tr>
<tr>
<td>1</td>
<td>0.7059</td>
<td>0.9059</td>
<td>1.1059</td>
<td>1.3059</td>
<td>1.5059</td>
<td>1.7059</td>
</tr>
<tr>
<td>2</td>
<td>0.9807</td>
<td>1.2807</td>
<td>1.5807</td>
<td>1.8807</td>
<td>2.1807</td>
<td>2.4807</td>
</tr>
<tr>
<td>3</td>
<td>1.1902</td>
<td>1.5902</td>
<td>1.9902</td>
<td>2.3902</td>
<td>2.7902</td>
<td>3.1902</td>
</tr>
<tr>
<td>4</td>
<td>1.3374</td>
<td>1.8374</td>
<td>2.3374</td>
<td>2.8374</td>
<td>3.3374</td>
<td>3.8374</td>
</tr>
<tr>
<td>5</td>
<td>1.4266</td>
<td>2.0266</td>
<td>2.6266</td>
<td>3.2266</td>
<td>3.8266</td>
<td>4.4266</td>
</tr>
<tr>
<td>6</td>
<td>1.4623</td>
<td>2.1623</td>
<td>2.8623</td>
<td>3.5623</td>
<td>4.2623</td>
<td>4.9623</td>
</tr>
<tr>
<td>7</td>
<td>1.4486</td>
<td>2.2486</td>
<td>3.0486</td>
<td>3.8486</td>
<td>4.6486</td>
<td>5.4486</td>
</tr>
<tr>
<td>8</td>
<td>1.3894</td>
<td>2.2894</td>
<td>3.1894</td>
<td>4.0894</td>
<td>4.9894</td>
<td>5.8894</td>
</tr>
<tr>
<td>9</td>
<td>1.2881</td>
<td>2.2881</td>
<td>3.2881</td>
<td>4.2881</td>
<td>5.2881</td>
<td>6.2881</td>
</tr>
</tbody>
</table>

Figure 7: Expected profit against time

Where

Series 1 for $k_i = 0.6$, Series 2 for $k_i = 0.5$, Series 3 for $k_i = 0.4$, Series 4 for $k_i = 0.3$,
Series 5 for $k_i = 0.2$, Series 6 for $k_i = 0.1$

6. DISCUSSION AND CONCLUSION

Tables 2 and 3 and corresponding Figures 3 and 4 displayed the impact of passage time on availability and reliability. It is evident from these Tables and figures that availability and reliability decreases as time $t$ increase for fixed values of failure and repair rates. On the other
hand, availability, profit and mean time to failure are higher value of repair rates and lower value of failure rates. Table 4 and the corresponding Figure 5 depicts the behavior of mean time to failure (MTTF) with respect to $\lambda_A$, $\lambda_1$, $\lambda_2$ and $\lambda_D$ fixing other parameters constant. It is evident from the Table and the figure that MTTF decreases with increase in the values of $\lambda_A$, $\lambda_1$, $\lambda_2$ and $\lambda_D$ respectively. However the MTTF is higher with respect to $\lambda_D$. Thus,

$$MTTF (w.r.t \lambda_D) > MTTF (w.r.t \lambda_2) > MTTF (w.r.t \lambda_1) > MTTF (w.r.t \lambda_A)$$

This sensitivity analysis implies that preventive and major maintenance should be invoked to the receiver, relay stations and the transmitter to minimize the system break down, prolong MTTF and maximizes the system reliability, availability as well as expected profit. Tables 5 and Figure 6 displayed the variation of sensitivity analysis with respect to change in $\lambda_A$, $\lambda_1$, $\lambda_2$ and $\lambda_D$ respectively. It is evident from the Table and figure that sensitivity of the MTTF increases with the increase in the value of $\lambda_A$, $\lambda_1$, $\lambda_2$ and $\lambda_D$. It is evident from the plots in Figure 6 that MTTF is sensitive to $\lambda_A$, $\lambda_1$, $\lambda_2$ and $\lambda_D$. Table 6 and Figure 7 displayed the results on revenue cost per unit time. From the table and the figure it is evident that expected profit increases with respect to time when $k_2$. The expected profit is lower when $k_2 = 0.6$ and higher for $k_2 = 0.1$. To achieve high quality, minimum production losses, higher production output as well as expected revenue, there should be failure free network and highest system reliability, availability and mean time to failure (MTTF).

**CONFLICT OF INTERESTS**

The authors declare that there is no conflict of interests.

**REFERENCES**

RELIABILITY ANALYSIS OF COMMUNICATION NETWORK SYSTEM


