MODIFIED FORMS OF SOFT NANO CONTRA CONTINUOUS FUNCTIONS

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Abstract: The objective of this paper is to present two such contexts where the results obtained for soft nano contra continuous functions widely differ from those known for soft nano continuous functions. Significant facts concerned with soft nano contra continuous, soft nano contra $g_{\Omega}$-continuous, soft nano contra $g_{\Omega}$-irresolute functions, $R_{\text{gsm}}$-maps, soft nano almost contra $g_{\Omega}$ - continuous functions are developed. Also, the stronger forms of soft nano contra continuous and soft nano contra $g_{\Omega}$-continuous functions called the soft nano-bi-contra continuous functions, soft nano-bi-contra $g_{\Omega}$ - continuous functions. Soft nano- strongly-bi-contra $g_{\Omega}$ - continuous functions are studied with their notable properties.

Keywords: soft nano $g_{\Omega}$-open maps; soft nano $g_{\Omega}$-closed maps; soft nano almost contra $g_{\Omega}$-continuous; soft nano-bi-contra-continuous functions; soft nano-bi-contra $g_{\Omega}$ - continuous maps; soft nano-strongly-bi-contra $g_{\Omega}$-continuous maps.

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1. INTRODUCTION

Generalized closed maps in topological spaces were proposed by Malghan[12]. After the introduction of these functions, extensive work was continued and it includes some of the
concepts of generalized open, β-open, almost open, α-open, gs-closed, ga-closed, sg-closed, wg-closed maps. Dontchev[7] introduced the concept of contra continuity. Later, strongly contra g#p-continuous functions, almost-contra g#p-continuous functions and related maps were studied by Salih[18]. Several researchers followed the theory of nano topology to introduce and study the properties of nano open maps and nano closed maps and related aspects in [2], [3], [4], [9], [10], [13],[14], [17], [20], [21].

Patil et.al. [16] introduced soft nano go-continuous functions in soft nano topological spaces and analyzed the properties of weaker forms of soft nano continuous functions with soft nano generalized continuous functions and in specific soft nano go-continuous functions and its compositions. They also introduced soft nano go-closed, soft nano go-open maps and soft nano go-irresolute functions. The characterizations of such functions in comparison with the generalized soft nano functions are derived. In this paper, the contrasting character of soft nano continuous functions gave the idea of soft nano contra continuous functions. The modified forms of such functions are soft nano strongly contra continuous, soft nano contra go-continuous, soft nano contra strongly go-continuous and soft nano contra go-irresolute functions in soft nano topological spaces. Various properties of soft nano contra g-closedness and soft nano g-closedness, soft nano contra wg-closedness, soft nano wg-closedness, soft nano contra go-closedness and soft nano go-CLOSEDNESS are independent notions. Important results involving soft nano almost contra go-continuous, soft nano perfectly continuous, soft nano-bi-contra continuous maps, R∞-maps and several others.

2. PRELIMINARIES

Definition 2.1: [1] Let set of objects be denoted by U, R is a soft equivalence relation and \( \tau_R(X_1) = \{ U, \emptyset, (L_R(X_1), O_1), (U_R(X_1), O_1), (B_R(X_1), O_1) \} \) satisfies the following axioms:

i) \( U \) and \( \emptyset \) \( \in \tau_R(X_1) \)

ii) the union of the elements of any finite subcollection \( \tau_R(X_1) \) is in \( \tau_R(X_1) \).

iii) the intersection of the elements of any finite subcollection \( \tau_R(X_1) \) is in \( \tau_R(X_1) \).

Then, \( \tau_R(X_1) \) is soft nano topology on U with respect to \( X_1 \), elements of the soft nano topology are known as the soft nano open sets and \( (\tau_R(X_1), U, O_1) \) is called a soft nano topological space.
Definition 2.2:[16] In a soft nano topological space \( (\tau_{s_n}(X_1), U_1, O_1) \), \( B_{s_n} = \{ U_1, L_{s_n}(X_1), B_{s_n}(X_1) \} \) is soft nano basis for \( \tau_{s_n}(X_1) \).

Definition 2.3: [16] A function \( F: (\tau_{s_n}(X_1), U_1, O_1) \to (\tau_{s_n''}(X_2), U_2, O_2) \) is sn-gω-continuous if the inverse image of every sn-open in \( U_2 \) is sn-gω-open in \( U_1 \).

3. TYPES OF SOFT NANO CONTRA-CONTINUOUS FUNCTIONS (Sn-C-CONTINUOUS FUNCTIONS)

Definition 3.1: A mapping \( F: (\tau_{s_n}(X_1), U_1, O_1) \to (\tau_{s_n''}(X_2), U_2, O_2) \) is

i) soft nano contra continuous (briefly, sn-c-continuous) if \( F^{-1}(E_1^*, O_2) \) is \( sn-C(X_1, O_1) \) for every \( s_n-O(X_2, O_2) \).

ii) soft nano contra-gω-continuous (briefly, sn-c-gω-continuous) if \( F^{-1}(E_1^*, O_1) \) is \( sn-C(X_1, O_1) \) for every \( sn-g\omega-O(X_2, O_2) \).

iii) soft nano contra strongly-gω-continuous (briefly, sn-c\( g\omega \)-continuous), if \( F^{-1}(E_1^*, O_1) \) is soft nano clopen in \( U_1 \) for every \( sn-g\omega-O(X_2, O_2) \).

Remark 3.2: Soft nano continuity is independent of soft nano contra continuity.

Example 3.3: Let \( U_1 = \{ \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \}, X_1 = \{ \epsilon_1, \epsilon_4 \} \leq U_1 \) with \( \tau_{s_n}(X_1) = \{ U_1, \emptyset, (\kappa_1, \{ \epsilon_3, \epsilon_4 \}), (\kappa_2, \{ \epsilon_3, \epsilon_4 \}), (\kappa_3, \{ \epsilon_3, \epsilon_4 \}) \} \) and \( U_2 = \{ \epsilon'_1, \epsilon'_2, \epsilon'_3, \epsilon'_4 \}, X_2 = \{ \epsilon'_1, \epsilon'_3 \} \) with \( \tau_{s_n''}(X_2) = \{ U_2, \emptyset, (\kappa'_1, \{ \epsilon'_1, \epsilon'_3 \}), (\kappa'_2, \{ \epsilon'_1, \epsilon'_3 \}), (\kappa'_3, \{ \epsilon'_1, \epsilon'_3 \}) \} \). Here, sn-closed sets are \( \{ \epsilon_2, \epsilon_3 \} \) and sn-closed sets are \( \{ \epsilon'_2, \epsilon'_3 \} \) respectively. We define the function as \( F(\epsilon_1) = \epsilon'_2, F(\epsilon_2) = \epsilon'_1, F(\epsilon_3) = \epsilon'_3 \) and \( F(\epsilon_4) = \epsilon'_4 \). Then \( F^{-1}(E_1^*) = \{ \epsilon_2 \}, F^{-1}(E_1^*) = \{ \epsilon_1 \}, F^{-1}(E_1^*) = \{ \epsilon_3 \} \) and \( F^{-1}(E_1^*) = \{ \epsilon_4 \} \). Here \( F^{-1}(E_1^*) = \{ \epsilon_1, \epsilon_4 \} \), that is inverse image of every \( sn-C(X_2, O_2) \) is \( sn-O(X_1, O_1) \).

Theorem 3.4: Every soft nano perfectly continuous is soft nano contra continuous function.

Proof: As \( F \) is soft nano perfectly continuous, \( F^{-1}(V^*, O_1) \) is soft nano clopen in \( U_1 \). Let \( (V^*, O_1) \) be \( sn-O(X_2, O_2) \), then \( F^{-1}(V^*, O_1) \) is \( sn-cl(X_1, O_1) \). Therefore \( F \) is soft nano contra continuous.

Remark 3.5: Converse of the theorem need not be true in general.
Example 3.6: Let $U_1 = \{e_1, e_2, e_3, e_4\}$, $U_1' = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$ and let $X_1 = \{e_1, e_4\} \subseteq U_1$, then $\tau'_{X_1}(X_1) = \{U_1, \emptyset, (\kappa_1, \{e_1, e_4\}), (\kappa_2, \{e_1, e_4\}), (\kappa_3, \{e_1, e_4\})\}$. Let $U_2 = \{e_1, e_2, e_3, e_4\}$, with $U_2' = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$. Let $X_2 = \{e_1, e_3\} \subseteq U_2$, then $\tau''_{X_2}(X_2) = \{U_2, \emptyset, (\kappa_1', \{e_1, e_3\}), (\kappa_2', \{e_1, e_3\}), (\kappa_3', \{e_1, e_3\})\}$. Then a function $F$ is defined as $F(e_1) = e_2'$, $F(e_2) = e_3'$, $F(e_3) = e_3'$, $F(e_4) = e_4'$. Here $F^{-1}(\{e_1', e_2'\}) = \{e_2, e_3\}$ in $\text{sn-c}(X_1, O_1)$ but not in $\text{sn-O}(X_2, O_2)$. Therefore, $F$ is $\text{sn}$ contra continuous and not soft nano perfectly continuous.

Theorem 3.7: Every soft nano contra continuous is soft nano contra $g_\omega$-continuous function.

Proof: Let $(V^*, O_1)$ be $\text{sn-O}(X_2, O_2)$, then It is $\text{sn-}g_\omega\text{-O}(X_2, O_2)$ [15]. As $F$ is soft nano contra continuous, then $F^{-1}(V^*, O_1)$ is $\text{sn-cl}(X_1, O_1)$. So it is $\text{sn-}g_\omega\text{-C}(X_1, O_1)$ [15]. Hence, $F$ is soft nano contra $g_\omega$-continuous.

Remark 3.8: The converse of the theorem 3.7 need not be true in general.

Theorem 3.9: For a function $F: (\tau'_{X_1}(X_1), U_1, O_1) \rightarrow (\tau''_{X_2}(X_2), U_2, O_2)$, we have

i) every soft nano strongly $g_\omega$-continuous is soft nano continuous.

ii) every soft nano contra strongly $g_\omega$-continuous is soft nano contra continuous.

iii) every soft nano contra strongly $g_\omega$-continuous is soft nano contra $g_\omega$-continuous.

iv) every soft nano contra strongly $g_\omega$-continuous is soft nano contra $g_\omega$-irresolute.

Proof: i) Let $(P^*, O_1)$ be $\text{sn-O}(X_2, O_2)$. Then it is $\text{sn-}g_\omega\text{-O}(X_2, O_2)$ [17]. As $F$ is $\text{sn-c}g_\omega$-continuous, $F^{-1}(P^*, O_1)$ is $\text{sn-O}(X_1, O_1)$. Hence $F$ is $\text{sn}$-continuous.

ii) Let $(P^*, O_1)$ as a $\text{sn-O}(X_2, O_2)$ and it is $\text{sn-}g_\omega\text{-O}(X_2, O_2)$[17]. Here $F^{-1}(P^*, O_1)$ is $\text{sn-O}(X_1, O_1)$ is $\text{sn-C}(X_1, O_1)$ as $F$ is $\text{sn-c}g_\omega$-continuous.

iii) Follows from the fact that every soft nano contra continuous function is soft nano contra $g_\omega$-continuous, the proof is obvious

iv) Obvious.

Theorem 3.10: A bijective function $F: (\tau'_{X_1}(X_1), U_1, O_1) \rightarrow (\tau''_{X_2}(X_2), U_2, O_2)$ is $\text{sn-contra–}g_\omega$-continuous function if and only if the inverse image of every $\text{sn-C}(X_2, O_2)$ is $\text{sn-}g_\omega\text{-O}(X_1, O_1)$. 


Proof: Let $F$ be a $sn$-$go\omega$-continuous and $(V_1^*, O_1)$ is $sn-C(X_2, O_2)$. Now $U_2 - (V_1^*, O_1)$ is $sn-O(X_2, O_2)$ and $F^{-1}(U_2 - (V_1^*, O_1))$ is $sn$-$go\omega$-$cl(X_1, O_1)$ as $F$ is sn-c–go-continuous. That implies $F^{-1}(U_2) - F^{-1}(V_1^*, O_1) = U_1 - F^{-1}(V_1^*, O_1)$ is $sn$-$go\omega$-$cl(X_1, O_1)$. Thus $F^{-1}(V_1^*, O_1)$ is $sn$-$go\omega$-$O(X_1, O_1)$, hence $F$ is sn-c–go-continuous on $U_2$.

Conversely, let $(S_1^*, O_1)$ be $sn-O(X_2, O_2)$. So $U_2 - (S_1^*, O_1)$ is $sn-C(X_2, O_2)$. By the hypothesis $F^{-1}(U_2 - (S_1^*, O_1))$ is $sn$-$go\omega$-$O(X_1, O_1)$, which implies $F^{-1}(U_2) - F^{-1}(S_1^*, O_1) = U_1 - F^{-1}(S_1^*, O_1)$ is $sn$-$go\omega$-$O(X_1, O_1)$. Thus $F^{-1}(S_1^*, O_1)$ is $sn$-$go\omega$-$cl(X_1, O_1)$. Therefore the inverse image of every $sn-O(X_2, O_2)$ is $sn$-$go\omega$-$cl(X_1, O_1)$. Hence $F$ is sn-c–go-continuous on $U_1$.

Remark 3.11: Composition of two soft nano contra $go\omega$ - continuous functions is a soft nano contra $go\omega$ - continuous function

Example 3.12: Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, $X_1 = \{\varepsilon_4, \varepsilon_5\} \subseteq U_1$, $O_1 = \{\kappa_1, \kappa_2, \kappa_3\}$, $U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, $O_2 = \{\kappa_1', \kappa_2', \kappa_3'\}$, $X_2 = \{\varepsilon_3, \varepsilon_4, \varepsilon_5'\} \subseteq U_2$, $U_3 = \{\varepsilon_1'', \varepsilon_2'', \varepsilon_3'', \varepsilon_4'', \varepsilon_5''\}$, $O_3 = \{\kappa_1'', \kappa_2'', \kappa_3''\}$, $X_3 = \{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\} \subseteq U_3$.

$U_2/\mathbb{R}$ = $\{\{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}\}$, $U_3/\mathbb{R}''$ = $\{\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3'', \varepsilon_4'', \varepsilon_5''\}\}$.

Define the function $F: (\tau_{\mathbb{R}}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}}(X_2), U_2, O_2)$ as $F(\varepsilon_1) = \varepsilon_3'$, $F(\varepsilon_2) = \varepsilon_4'$, $F(\varepsilon_3) = \varepsilon_5'$, $F(\varepsilon_4) = \varepsilon_1'$, and $F(\varepsilon_5) = \varepsilon_2'$. And a function $F_1: (\tau_{\mathbb{R}}(X_2), U_2, O_2) \rightarrow (\tau_{\mathbb{R}}(X_3), U_3, O_3)$ defined as $F_1(\varepsilon_1') = \varepsilon_5''$, $F_1(\varepsilon_2') = \varepsilon_4''$, $F_1(\varepsilon_3') = \varepsilon_5''$, $F_1(\varepsilon_4') = \varepsilon_2''$ and $F_1(\varepsilon_5') = \varepsilon_1''$. Here $F$ and $F_1$ are contra sn-$go\omega$-continuous. sn-$go\omega$ closed sets in $U_1$ are $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $\{\varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$ and $t_n$ $U_2$ $\{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, $\{\varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$, $\mathcal{F}_1^{-1}(\varepsilon_1', \varepsilon_2', \varepsilon_3') = \{\varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\}$ is sn-$go\omega$ closed in $U_2$. Here $F_1$ is contra sn-$go\omega$ continuous. Let $(B_1^+, O_1) = (\{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}, (\{\varepsilon_1'', \varepsilon_2'', \varepsilon_3''\}), (\{\varepsilon_1', \varepsilon_2'', \varepsilon_3''\}))$ then $(\mathcal{F}_1 \circ \mathcal{F}) = \mathcal{F}^{-1}(\mathcal{F}_1^{-1}(\{\varepsilon_1', \varepsilon_2', \varepsilon_3'\})) = \mathcal{F}^{-1}(\{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is again sn-$go\omega$ continuous.
Theorem 3.13: Let $\mathcal{F}: (\tau_{\mathbb{R}}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ be a soft nano contra $g\omega$-continuous function, then soft nano restriction map induced by soft nano contra $g\omega$-continuous function:

i) $(H^*, O_1) \subseteq sn-O(X_1, O_1)$ and $\mathcal{F}$ is $sn-c-g\omega$-open, then the restriction $\mathcal{F}_{(H^*, O_1)}: ((H^*, O_1), U_{(H^*, O_1)}, O_1) \to (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ is $sn-c-g\omega$-open.

ii) $(H^*, O_1) \subseteq sn-C(X_2, O_2)$ and $\mathcal{F}$ is $sn-c-g\omega$-closed, then the restriction $\mathcal{F}_{(H^*, O_1)}: ((H^*, O_1), U_{(H^*, O_1)}, O_1) \to (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ is $sn-c-g\omega$-closed.

iii) $(H^*, O_1) = F^{-1}(G^*, O_1)$ for $sn-O(X_1, O_2)$ and $\mathcal{F}$ is $sn-c-g\omega$-closed bijective function, then the restriction $\mathcal{F}_{(H^*, O_1)}: ((H^*, O_1), U_{(H^*, O_1)}, O_1) \to (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ is $sn-c-g\omega$-closed.

Proof: i) Let $(G^*, O_1) \subseteq sn-O(H^*, O_1)$. Then for some $sn$-open set $(P^*, O_1)$ of $(\tau_{\mathbb{R}}'(X_1), U_1, O_1)$, $(G^*, O_1) = (H^*, O_1) \cap (P^*, O_1)$ and $(G^*, O_1)$ is $sn-O(X_1, O_1)$. Given, $\mathcal{F}(G^*, O_1)$ is $sn-g\omega$ closed. But $\mathcal{F}(G^*, O_1) = \mathcal{F}_{(H^*, O_1)}(G^*, O_1)$ and hence $\mathcal{F}_{(H^*, O_1)}$ is $sn-c-g\omega$-open.

ii) Let $(G^*, O_1) \subseteq sn-C(H^*, O_4)$. Then for some $sn$-closed set $(Q^*, O_1)$ of $(\tau_{\mathbb{R}}'(X_1), U_1, O_1)$, $(G^*, O_1) = (H^*, O_1) \cap (Q^*, O_1)$ and so $(G^*, O_1)$ is $sn-C(X_1, O_4)$. But given, $\mathcal{F}(G^*, O_1) = \mathcal{F}_{(H^*, O_1)}(G^*, O_1)$ and hence $\mathcal{F}_{(H^*, O_1)}$ is $sn-c-g\omega$-closed.

(iii) Let $(R^*, O_1) \subseteq sn-C(H^*, O_1)$. Then for some $sn$-closed set $(S^*, O_1)$ of $(\tau_{\mathbb{R}}'(X_1), U_1, O_1)$, $(R^*, O_1) = (H^*, O_1) \cap (S^*, O_1)$. But $\mathcal{F}(R^*, O_1) = \mathcal{F}_{(H^*, O_1)}(R^*, O_1) = \mathcal{F}((R^*, H_1) \cap (R^*, O_1)) = \mathcal{F}(F^{-1}(R^*, H_1) \cap (R^*, O_1)) = (G^*, O_1) \cap \mathcal{F}(R^*, O_1)$. Hence $\mathcal{F}(R^*, O_1)$ is $sn-g\omega$-open as $\mathcal{F}$ is $sn-c-g\omega$-closed and so $(G^*, O_1) \cap \mathcal{F}(R^*, O_1)$ is $sn-g\omega-O(X_2, O_2)$. Therefore $\mathcal{F}$ is $sn-c-g\omega$-closed.

4. NATURE OF INDEPENDENCY OF FEW SOFT NANO GENERALIZED CONTINUOUS WITH THEIR CORRESPONDING CONTRA CONTINUOUS FUNCTIONS

Definition 4.1: The mapping $\mathcal{F}: (\tau_{\mathbb{R}}(X_1), U_1, O_1) \to (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ is a

i) soft nano contra $g$-continuous (briefly, $sn-c-g$-continuous), if $\mathcal{F}^{-1}(A^*, O_1)$ is soft nano g-closed in $U_1$ for every soft nano open set $(A^*, O_1)$ in $U_2$ or $sn-O(X_2, O_2)$.

ii) soft nano contra $wg$-continuous (briefly, $sn-c-wg$-continuous), if $\mathcal{F}^{-1}(A^*, O_1)$ is soft nano contra $wg$-closed in $U_1$ for every soft nano open set $(A^*, O_1)$ in $U_2$. 
iii) soft nano contra $gw$-continuous (briefly, $sn-c-gw$-continuous), if $\mathcal{F}^{-1}(A^{*}, O_{1})$ is soft nano $gw$-closed in $U_{1}$ for every soft nano open set $(A^{*}, O_{1})$ in $U_{2}$.

**Remark 4.2:** Consider the following statements:

i) The concepts of $\text{sn-c-}g\text{-continuous}$ and $\text{sn-c-}g\omega\text{-continuous}$ functions are independent.

ii) $\text{sn-c-wg-}c\text{-continuity}$ and $\text{sn-wg-}c\text{-continuity}$ are independent concepts.

iii) $\text{sn-c-}g\omega\text{-closedness}$ and $\text{sn-c-}g\omega\text{-closedness}$ are independent notions.

The following examples are proposed to show that the above statements are valid.

**Example 4.3:** Let $U_{1} = \{\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}\}$ with $U_{1}/_{\mathbb{R}}' = \{\{\varepsilon_{1}\}, \{\varepsilon_{2}, \varepsilon_{3}\}\}$, $O_{1} = \{\kappa_{1}, \kappa_{2}, \kappa_{3}\}$. $X_{1} = \{\varepsilon_{2}, \varepsilon_{3}\} \subseteq U_{1}$ then $\tau_{\mathbb{R}}'(X_{1}) = \{U_{1}, \emptyset, (\kappa_{1}, \varepsilon_{2}, \varepsilon_{3}), (\kappa_{2}, \{\varepsilon_{2}, \varepsilon_{3}\}), (\kappa_{3}, \{\varepsilon_{2}, \varepsilon_{3}\})\}$. Let $U_{2} = \{\varepsilon_{1}', \varepsilon_{2}', \varepsilon_{3}'\}$ with $U_{2}/_{\mathbb{R}}'' = \{\{\varepsilon_{1}'\}, \{\varepsilon_{2}', \varepsilon_{3}'\}\}$, $O_{2} = \{\kappa_{1}', \kappa_{2}', \kappa_{3}'\}$. $X_{2} = \{\varepsilon_{1}'\} \subseteq U_{2}$ then $\tau_{\mathbb{R}}''(X_{2}) = \{U_{2}, \emptyset, (\kappa_{1}', \{\varepsilon_{1}'\}), (\kappa_{2}', \{\varepsilon_{1}'\}), (\kappa_{3}', \{\varepsilon_{1}'\})\}$. Define a function $\mathcal{F}: (\tau_{\mathbb{R}}'(X_{1}), U_{1}, O_{1}) \rightarrow (\tau_{\mathbb{R}}''(X_{2}), U_{2}, O_{2})$ as $\mathcal{F}(\varepsilon_{1}) = \varepsilon_{1}', \mathcal{F}(\varepsilon_{2}) = \varepsilon_{3}', \mathcal{F}(\varepsilon_{3}) = \varepsilon_{3}'$. Here $\mathcal{F}$ is $\text{sn-c-}g\text{-continuous}$ and $\text{sn-c-}g\omega\text{-continuous}$ but not $\text{sn-}g\text{-continuous}$ and $\text{sn-g-}\omega\text{-continuous}$. Here we define $\mathcal{F}_{1}: (\tau_{\mathbb{R}}'(X_{1}), U_{1}, O_{1}) \rightarrow (\tau_{\mathbb{R}}''(X_{2}), U_{2}, O_{2})$ by $\mathcal{F}_{1}(\varepsilon_{1}) = \varepsilon_{1}', \mathcal{F}_{1}(\varepsilon_{2}) = \varepsilon_{1}', \mathcal{F}_{1}(\varepsilon_{3}) = \varepsilon_{1}'. Then $\mathcal{F}_{1}$ is $\text{sn-}g\text{-continuous}$ but not $\text{sn-}c\text{-continuous}$. Also $\mathcal{F}_{1}$ is $\text{sn-g-}\omega\text{-continuous}$ but not $\text{sn-c-}g\omega\text{-continuous}$.  

**Example 4.4:** Let $U_{1} = \{\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4}, \varepsilon_{5}\}$, $O_{1} = \{\kappa_{1}, \kappa_{2}, \kappa_{3}\}$. $X_{1} = \{\varepsilon_{1}, \varepsilon_{3}\}$, $U_{1}/_{\mathbb{R}}' = \{\{\varepsilon_{1}, \varepsilon_{3}\}, \{\varepsilon_{2}\}, \{\varepsilon_{4}\}, \{\varepsilon_{5}\}\}$ then $\tau_{\mathbb{R}}'(X_{1}) = \{U_{1}, \emptyset, (\kappa_{1}, \{\varepsilon_{1}, \varepsilon_{3}\}), (\kappa_{2}, \{\varepsilon_{1}, \varepsilon_{3}\}), (\kappa_{3}, \{\varepsilon_{1}, \varepsilon_{3}\})\}$. Let $U_{2} = \{\varepsilon_{1}', \varepsilon_{2}', \varepsilon_{3}', \varepsilon_{4}', \varepsilon_{5}'\}$, $O_{2} = \{\kappa_{1}', \kappa_{2}', \kappa_{3}'\}$. $X_{2} = \{\varepsilon_{1}', \varepsilon_{3}'\}$, $U_{2}/_{\mathbb{R}}'' = \{\{\varepsilon_{1}', \varepsilon_{2}', \varepsilon_{3}'\}, \{\varepsilon_{4}', \varepsilon_{5}'\}\}$ then $\tau_{\mathbb{R}}''(X_{2}) = \{U_{2}, \emptyset, (\kappa_{1}', \{\varepsilon_{1}', \varepsilon_{3}'\}), (\kappa_{2}', \{\varepsilon_{1}', \varepsilon_{3}'\}), (\kappa_{3}', \{\varepsilon_{1}', \varepsilon_{3}'\})\}$. Then $\mathcal{F}: (\tau_{\mathbb{R}}'(X_{1}), U_{1}, O_{1}) \rightarrow (\tau_{\mathbb{R}}''(X_{2}), U_{2}, O_{2})$ is defined as $\mathcal{F}(\varepsilon_{1}) = \varepsilon_{2}', \mathcal{F}(\varepsilon_{2}) = \varepsilon_{1}', \mathcal{F}(\varepsilon_{3}) = \varepsilon_{3}', \mathcal{F}(\varepsilon_{4}) = \varepsilon_{4}'$ and $\mathcal{F}(\varepsilon_{5}) = \varepsilon_{5}'. Here $\mathcal{F}$ is $\text{sn-g-}w\text{-continuous}$ but not $\text{sn-c-}w\text{-continuous}$.  


Example 4.5: Let $U_1 = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5\}$, $X_1 = \{\epsilon_1, \epsilon_2\}$, $O_1 = \{\kappa_1, \kappa_2, \kappa_3\}$, $U_1/\mathbb{R} = \{\{\epsilon_1\}, \{\epsilon_2\}, \{\epsilon_3\}, \{\epsilon_4\}, \{\epsilon_5\}\}$ then $\tau_{\mathbb{R}}'(X_1) = \{U_1, \phi, (\kappa_1, \{\epsilon_1, \epsilon_2\}), (\kappa_2, \{\epsilon_1, \epsilon_2\})\}$, $(\kappa_3, \{\epsilon_1, \epsilon_2\}))$. Let $U_2 = \{\epsilon'_1, \epsilon'_2, \epsilon'_3, \epsilon'_4, \epsilon'_5\}$, $O_2 = \{\kappa'_1, \kappa'_2, \kappa'_3\}, X_2 = \{\epsilon'_2, \epsilon'_3\}$, $U_2/\mathbb{R}' = \{\{\epsilon'_1, \epsilon'_3\}, \{\epsilon'_2\}, \{\epsilon'_4, \epsilon'_5\}\}$ then $\tau_{\mathbb{R}}''(X_2) = \{U_2, \emptyset, (\kappa'_1, \{\epsilon'_1, \epsilon'_2, \epsilon'_3\}), (\kappa'_2, \{\epsilon'_1, \epsilon'_2, \epsilon'_3\})\}$, $(\kappa'_3, \{\epsilon'_1, \epsilon'_2, \epsilon'_3\})$. Then $F: (\tau_{\mathbb{R}}'(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ is defined as $F(\epsilon_1) = \epsilon'_4$, $F(\epsilon_2) = \epsilon'_5$, $F(\epsilon_3) = \epsilon'_1$, $F(\epsilon_4) = \epsilon'_2$ and $F(\epsilon_5) = \epsilon'_3$. Here $F$ is sn-c-\textit{wg}-continuous but not sn \textit{wg}-continuous.

5. SOFT NANO CONTRA \textit{gω}-IRRRESOLUTE FUNCTIONS

Definition 5.1: A function $F: (\tau_{\mathbb{R}}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}}'(X_2), U_2, O_2)$ is said to be soft nano contra \textit{gω}-irresolute (briefly, sn-c- \textit{gω}-irresolute), if $F^{-1}(M^*_1, O_1)$ is sn-\textit{gω}-C$(X_1, O_1)$ for each sn-\textit{gω}-C$(X_2, O_2)$ of $U_2$.

Theorem 5.2: Let a soft nano-\textit{gω}-continuous function $F_1: (\tau_{\mathbb{R}}'(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ and soft nano $S_{\textit{gω}}$-continuous function $F_2: (\tau_{\mathbb{R}}''(X_2), U_2, O_2) \rightarrow (\tau_{\mathbb{R}}'''(X_3), U_3, O_3)$, then $F_2 \circ F_1$ is sn-\textit{gω}-irresolute.

Proof: Let $(P^*, O_1)$ be sn-\textit{gω}-C$(X_3, O_3)$. Here $F_2^{-1}(P^*, O_1)$ is sn-C$(X_2, O_2)$ as $F_2$ is $S_{\textit{sn-gω}}$-continuous and $F_1^{-1}(F_2^{-1}(P^*, O_1))$ is sn-\textit{gω}-C$(X_1, O_1)$. But $F_1^{-1}(F_2^{-1}(P^*, O_1)) = (F_2 \circ F_1)^{-1}(P^*, O_1)$ is sn-\textit{gω}-C$(X_1, O_1)$. Thus $F_2 \circ F_1$ is sn-\textit{gω}-irresolute.

Theorem 5.3: Let $F_1: (\tau_{\mathbb{R}}'(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}}''(X_2), U_2, O_2)$ be soft nano contra strongly-\textit{gω}-continuous and $F_2: (\tau_{\mathbb{R}}''(X_2), U_2, O_2) \rightarrow (\tau_{\mathbb{R}}'''(X_3), U_3, O_3)$ be sn-\textit{gω}-continuous, $F_2 \circ F_1$ is sn-c-continuous.

Proof: Let $(M^*, O_1)$ be sn-O$(X_3, O_3)$. As $F_2$ is sn-\textit{gω}-continuous, $F_2^{-1}(M^*, O_1)$ is sn-\textit{gω}-O$(X_2, O_2)$. Hence $F_1^{-1}(F_2^{-1}(M^*, O_1))$ is sn-\textit{gω}-O$(X_1, O_1)$. Also, $F_1$ is sn-contra-strongly-\textit{gω}-continuous, $F_1^{-1}(F_2^{-1}(M^*, O_1)) = (F_2 \circ F_1)^{-1}(M^*, O_1)$ is sn-C$(X_1, O_1)$. Therefore $F_2 \circ F_1$ is sn-contra continuous.
**Theorem 5.4:** Consider a soft nano contra $g\omega$-continuous function $F_1: \tau_\mathcal{R}'(X_1, U_1, O_1) \to \tau_\mathcal{R}''(X_2, U_2, O_2)$ and sn-$c\delta_g\omega$-continuous function $F_2: \tau_\mathcal{R}''(X_2, U_2, O_2) \to \tau_\mathcal{R}'''(X_3, U_3, O_3)$, then $F_2 \circ F_1$ is sn-$c\omega$-irresolute.

**Proof:** Let $(M^*, O_1)$ be $sn-g\omega-O(X_3, O_3)$. As $F_2$ is sn-$g\omega$-continuous, $F_2^{-1}(M^*, O_1)$ is sn-$O(X_2, O_2)$. Hence $F_2^{-1}(F_2^{-1}(M^*, O_1))$ is sn-$g\omega-C(X_1, O_1)$ as $F_1$ is sn-$c\omega$-continuous. Therefore $F_1^{-1}(F_2^{-1}(M^*, O_1)) = (F_2 \circ F_1)^{-1}(M^*, O_1)$ is sn-$g\omega-C(X_1, O_1)$. Therefore $F_2 \circ F_1$ is sn-$c\omega$-irresolute.

**Theorem 5.5:** Let $F_1: \tau_\mathcal{R}'(X_1, U_1, O_1) \to \tau_\mathcal{R}''(X_2, U_2, O_2)$ and $F_2: \tau_\mathcal{R}''(X_2, U_2, O_2) \to \tau_\mathcal{R}'''(X_3, U_3, O_3)$ be functions, if

i) $F_1$ is sn-$c\omega$-irresolute and $F_2$ is sn-$c\omega$-continuous, then $F_2 \circ F_1$ is sn-$c\omega$-continuous.

ii) $F_1$ is sn-$c\omega$-irresolute and $F_2$ is sn-$g\omega$-irresolute, then $F_2 \circ F_1$ is sn-$c\omega$-irresolute.

iii) $F_1$ is sn-$g\omega$-irresolute and $F_2$ is sn-continuous, then $F_2 \circ F_1$ is sn-$c\omega$-continuous.

iv) $F_1$ is sn-$g\omega$-irresolute and $F_2$ is sn-$g\omega$-irresolute, then $F_2 \circ F_1$ is sn-$c\omega$-irresolute.

v) $F_1$ is sn-$c\omega$-continuous and $F_2$ is sn-continuous, then $F_2 \circ F_1$ is sn-$c\omega$-continuous.

**Proof:** i) Consider a sn-$O(X_3, O_3)$, $(V^*, O_1)$ and a sn-$g\omega$-continuous function $F_2$. So $F_2^{-1}(V^*, O_1)$ is sn-$g\omega-O(X_2, O_2)$. As $F_1$ is sn-$c\omega$-irresolute, $F_1^{-1}(F_2^{-1}(V^*, O_1))$ is sn-$g\omega$-C$(X_1, O_1)$. Hence $F_1^{-1}(F_2^{-1}(V^*, O_1)) = (F_2 \circ F_1)^{-1}(V^*, O_1)$ is sn-$C(X_1, O_1)$. Therefore $F_2 \circ F_1$ is sn-$c\omega$-continuous.

ii) Let $(V^*, O_1)$ be a sn-$g\omega-O(X_3, O_3)$. As $F_2$ is sn-$g\omega$-irresolute, $F_2^{-1}(V^*, O_1)$ is sn-$g\omega-O(X_2, O_2)$. As $F_1$ is sn-$c\omega$-irresolute, $F_1^{-1}(F_2^{-1}(V^*, O_1)) = (F_2 \circ F_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C$(X_1, O_1)$. Therefore $F_2 \circ F_1$ is sn-$c\omega$-irresolute.

iii) Let $(V^*, O_1)$ be a sn-$O(X_3, O_3)$, it is sn-$g\omega-O(X_2, O_2)$. As $F_2$ is sn-$g\omega$-irresolute, $F_2^{-1}(V^*, O_1)$ is sn-$g\omega-O(X_2, O_2)$. Here $F_1$ is sn-$c\omega$-continuous. Hence $F_1^{-1}(F_2^{-1}(V^*, O_1)) = (F_2 \circ F_1)^{-1}(V^*, O_1)$ is sn-$g\omega$-C$(X_1, O_1)$. Therefore $F_2 \circ F_1$ is sn-$c\omega$-continuous.
iv) Let \((V^*, O_1)\) be a \(s\text{-}g\omega\text{-}O(X_3, O_3)\). As \(F_2\) is \(s\text{-}c\text{-}g\omega\) irresolute, \(F_2^{-1}(V^*, O_1)\) is \(s\text{-}g\omega\text{-}C(X_2, O_2)\). As \(F_1\) is \(s\text{-}g\omega\) irresolute, \(F_1^{-1}(F_2^{-1}(V^*, O_1)) = (F_2 \circ F_1)^{-1}(V^*, O_1)\) is \(s\text{-}g\omega\text{-}C(X_1, O_1)\). Therefore \(F_2 \circ F_1\) is \(s\text{-}c\text{-}g\omega\) irresolute.

v) Let \((V^*, O_1)\) be a \(s\text{-}O(X_3, O_3)\). As \(F_2\) is \(s\text{-}O\) continuous, \(F_2^{-1}(V^*, O_1)\) is \(s\text{-}O(X_2, O_2)\). As \(F_1\) is \(s\text{-}c\text{-}g\omega\) continuous, \(F_1^{-1}(F_2^{-1}(V^*, O_1)) = (F_2 \circ F_1)^{-1}(V^*, O_1)\) is \(s\text{-}g\omega\text{-}C(X_1, O_1)\). Therefore \(F_2 \circ F_1\) is \(s\text{-}c\text{-}g\omega\) continuous.

6. SOFT NANO ALMOST CONTRA \(g\omega\)-CONTINUOUS FUNCTIONS

In this section, soft nano almost contra \(g\omega\)-continuous functions are introduced and their properties are studied.

**Definition 6.1:** A function \(F: (\tau(X_1), U_1, O_1) \rightarrow (\tau(X_2), U_2, O_2)\) is soft nano almost contra \(g\omega\)-continuous, if \(F^{-1}(V^*, O_1)\) is \(s\text{-}g\omega\)-closed in \(U_1\) for each \(s\text{-}r\text{-}C\) \((V, O_1)\) in \(U_2\).

**Theorem 6.2:** If the function \(F: (\tau(X_1), U_1, O_1) \rightarrow (\tau(X_2), U_2, O_2)\) is soft nano almost contra \(g\omega\)-continuous, then \(F^{-1}(M^*, O_1)\) is \(s\text{-}g\omega\)-open in \(U_1\) for each \(s\text{-}n\text{-}r\text{-}C\) \((M, O_1)\) in \(U_2\).

**Proof:** Let \((M^*, O_1)\) be \(s\text{-}r\text{-}C(X_2, O_2)\). Then \(U_2 - (M^*, O_1)\) is \(s\text{-}r\text{-}C\). As \(F\) is \(s\text{-}n\text{-}a\text{-}g\omega\)-continuous, then \(F^{-1}(U_2 - (M^*, O_1)) = U_1 - F^{-1}(M^*, O_1)\) is \(s\text{-}g\omega\text{-}C(X_1, O_1)\). Hence \(F^{-1}(M^*, O_1)\) is \(s\text{-}g\omega\text{-}O(X_1, O_1)\).

**Theorem 6.3:** Let \((P^*, O_1)\) be soft nano-subset of \(U_1\) and if the function \(F: (\tau(X_1), U_1, O_1) \rightarrow (\tau(X_2), U_2, O_2)\) is soft nano-almost contra \(g\omega\)-continuous function, then the restriction \(F(P^*, O_1): (P^*, O_1) \rightarrow (\tau(X_2), U_2, O_2)\) is also soft nano-almost contra \(g\omega\)-continuous.

**Proof:** Let \((M^*, O_1)\) be \(s\text{-}r\text{-}C(X_2, O_2)\). As \(F\) is \(s\text{-}n\text{-}a\text{-}g\omega\)-continuous, \(F^{-1}(M^*, O_1)\) is \(s\text{-}g\omega\text{-}O(X_1, O_1)\). Now \((F(P^*, O_1))^{-1}(M^*, O_1) = (P^*, O_1) \cap F^{-1}(M^*, O_1)\) where \(F^{-1}(M^*, O_1)\) is \(s\text{-}g\omega\)-open set in \((P^*, O_1)\). Hence \(F(P^*, O_1)\) is \(s\text{-}a\text{-}g\omega\)-continuous.
Theorem 6.4: Let \( \mathcal{F}_1: (\tau_{\text{n}}(X_1), U_1, O_1) \rightarrow (\tau_{\text{n}}'(X_2), U_2, O_2) \) is soft nano-almost-contra-\( g \)-continuous and \( \mathcal{F}_2: (\tau_{\text{n}}''(X_2), U_2, O_2) \rightarrow (\tau_{\text{n}}'''(X_3), U_3, O_3) \) is soft nano-almost-contra-\( g \)-continuous, then \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is soft nano-almost-contra-\( g \)-continuous.

**Proof:** Let \((P^*, O_1)\) be sn-r-O\((X_3, O_3)\). As \( \mathcal{F}_2 \) is sn-almost-continuous, \( \mathcal{F}_{2}^{-1}(P^*, O_1) \) is sn-O\((X_2, O_2)\). Since \( \mathcal{F}_1 \) is sn-almost-contra-\( g \)-continuous, \( \mathcal{F}_{1}^{-1}(\mathcal{F}_{2}^{-1}(P^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1) \) is sn-\( g \)-C\((X_1, O_1)\). Hence \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is soft nano-almost-contra-\( g \)-continuous.

Theorem 6.5: Let \( \mathcal{F}_1: (\tau_{\text{n}}(X_1), U_1, O_1) \rightarrow (\tau_{\text{n}}'(X_2), U_2, O_2) \) is soft nano-almost contra-\( g \)-continuous and \( \mathcal{F}_2: (\tau_{\text{n}}''(X_2), U_2, O_2) \rightarrow (\tau_{\text{n}}'''(X_3), U_3, O_3) \) is soft nano perfectly continuous, then \( \mathcal{F}_2 \circ \mathcal{F}_1 \) soft nano-contra-\( g \)-continuous.

**Proof:** Let \((P^*, O_1)\) be sn-O\((X_3, O_3)\). As \( \mathcal{F}_2 \) is sn-perfectly continuous function, \( \mathcal{F}_{2}^{-1}(P^*, O_1) \) is sn-clopen in \( U_2 \). As \( \mathcal{F}_1 \) is sn-almost contra \( g \)-continuous, \( \mathcal{F}_{1}^{-1}(\mathcal{F}_{2}^{-1}(P^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1) \) is sn-\( g \)-C\((X_1, O_1)\). Hence \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is soft nano-contra-\( g \)-continuous.

Theorem 6.6: Let \( \mathcal{F}_1: (\tau_{\text{n}}(X_1), U_1, O_1) \rightarrow (\tau_{\text{n}}'(X_2), U_2, O_2) \) is soft nano-almost contra-\( g \)-continuous and \( \mathcal{F}_2: (\tau_{\text{n}}''(X_2), U_2, O_2) \rightarrow (\tau_{\text{n}}'''(X_3), U_3, O_3) \) is \( R_{\text{sn}} \)-map, then \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is soft nano-almost contra-\( g \)-continuous.

**Proof:** Let \((P^*, O_1)\) be sn-r-O\((X_3, O_3)\). As \( \mathcal{F}_2 \) is \( R_{\text{sn}} \)-map, \( \mathcal{F}_{2}^{-1}(P^*, O_1) \) is sn-r-O\((X_2, O_2)\). As \( \mathcal{F}_1 \) is sn-almost contra \( g \)-continuous. Thus \( \mathcal{F}_{1}^{-1}(\mathcal{F}_{2}^{-1}(P^*, O_1)) = (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(P^*, O_1) \) is sn-\( g \)-C\((X_1, O_1)\). Hence \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is soft nano almost contra \( g \)-continuous.

7. SOFT NANO-BI-CONTRA-CONTINUOUS FUNCTIONS

**Definition 7.1:** A surjective function \( \mathcal{F}: (\tau_{\text{n}}(X_1), U_1, O_1) \rightarrow (\tau_{\text{n}}'(X_2), U_2, O_2) \) is called a soft nano bi-contra continuous if \( \mathcal{F} \) is soft nano contra \( g \)-continuous and \( \mathcal{F}^{-1}(P^*, O_1) \) is soft nano open in \( U_1 \) implied \((P^*, O_1)\) is soft nano \( g \)-closed in \( U_2 \).

**Example 7.2:** Let \( U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\} \) with \( U_1/\tau_{\text{n}'} = \{\{\varepsilon_1, \varepsilon_2\}, \{\varepsilon_3, \varepsilon_4\}, \{\varepsilon_5\}\} \).

\( X_1 = \{\varepsilon_3, \varepsilon_4\} \subseteq U_1 \) then \( \tau_{\text{n}'}(X_1) = \{U_1, \emptyset, \{\varepsilon_3, \varepsilon_4\}\} \).

\( U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4', \varepsilon_5'\} \)
with \( U_2 / \mathbb{R}'' = \{ (\varepsilon_1', \varepsilon_2'), (\varepsilon_3', \varepsilon_4', \varepsilon_5') \} \) for \( X_2 = \{ \varepsilon_3', \varepsilon_4', \varepsilon_5' \} \subseteq U_2 \) then \( \tau_{\mathbb{R}''}(X_2) = \{ U_2, \emptyset, (\varepsilon_3', \varepsilon_4', \varepsilon_5') \} \).

Then \( \mathcal{F} : (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \) is defined as \( \mathcal{F}(\varepsilon_1) = \varepsilon_4', \mathcal{F}(\varepsilon_2) = \varepsilon_5' \), \( \mathcal{F}(\varepsilon_3) = \varepsilon_1' \), \( \mathcal{F}(\varepsilon_4) = \varepsilon_2' \) and \( \mathcal{F}(\varepsilon_5) = \varepsilon_3' \). Here \( \mathcal{F} \) is soft nano contra continuous as \( \mathcal{F}^{-1}(\{\varepsilon_3', \varepsilon_4', \varepsilon_5'\}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_5\} \) that is inverse image of is \( \text{sn-O}(X_2, O_2) \) is \( \text{sn-c}(X_1, O_1) \). Also, \( \mathcal{F}^{-1}(\{\varepsilon_1', \varepsilon_2'\}) = \{\varepsilon_3, \varepsilon_4\} \) is \( \text{sn-O}(X_1, O_1) \). Therefore \( \mathcal{F} \) is soft nano bi-contra continuous function.

**Theorem 7.3:** Composition of two soft nano bi-contra continuous map is again soft nano bi-contra continuous map.

**Proof:** Let \( \mathcal{F}_1 : (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \) and
\[
\mathcal{F}_2 : (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \rightarrow (\tau_{\mathbb{R}'''}(X_3), U_3, O_3)
\]
be soft nano bi-contra continuous. Let \( (V^*, O_1) \) be a \( \text{sn-O}(X_3, O_3) \). As \( \mathcal{F}_2 \) is soft nano continuous, \( \mathcal{F}_2^{-1}(V^*, O_1) \) is \( \text{sn-O}(X_2, O_2) \) and as \( \mathcal{F}_1 \) is soft nano-bi-contra continuous \( \mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) = \text{sn-C}(X_1, O_1) \). \( (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1) = \mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) \) and thus \( (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1) \) is \( \text{sn-C}(X_1, O_1) \). Here \( \mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1)) \) is \( \text{sn-O}(X_1, O_1) \) as \( \mathcal{F}_1 \) is \( \text{sn-open} \), \( \mathcal{F}_1(\mathcal{F}_1^{-1}(\mathcal{F}_2^{-1}(V^*, O_1))) = \mathcal{F}_2^{-1}(V^*, O_1) \) is \( \text{sn-O}(X_2, O_2) \). Again \( \mathcal{F}_2 \) is \( \text{sn-c-continuous} \). Here \( (V^*, O_1) \) is \( \text{sn-C}(X_2, O_3) \). Here \( (\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(V^*, O_1) \) is soft nano-bi-contra continuous.

**Theorem 7.4:** The function \( \mathcal{F} : (\tau_{\mathbb{R}'}(X_1), U_1, O_1) \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \) is a soft nano bi-contra continuous map and onto-mapping. Let \( (M^*, O_1) \) be \( \text{sn-clopen} \) subset of \( U_1 \), then the restriction \( \mathcal{F}_{(M^*, O_1)} : [(M^*, O_1), (\tau_{\mathbb{R}'}(M^*, O_1), U_1, O_1)] \rightarrow (\tau_{\mathbb{R}''}(X_2), U_2, O_2) \) is soft nano bi-contra continuous map.

**Proof:** By hypothesis \( \mathcal{F} \) is \( \text{sn-bi-contra continuous} \). Let \( (P^*, O_1) \) be \( \text{sn-O}(X_2, O_2) \), then \( \mathcal{F}^{-1}(P^*, O_1) \) is \( \text{sn-O}(X_1, O_1) \). Since \( \mathcal{F} \) is \( \text{sn-contra continuous} \). As \( (M^*, O_1) \) is clopen, \( \mathcal{F}^{-1}(M^*, O_1) \cap (P^*, O_1) \) is \( \text{sn-C}(M^*, O_1) \). Here \( \mathcal{F}^{-1}(M^*, O_1) = \mathcal{F}^{-1}(P^*, O_1) \cap (M^*, O_1) \). Since \( (M^*, O_1) \) is \( \text{sn-clopen} \), \( \mathcal{F}^{-1}(P^*, O_1) \) is \( \text{sn-O}(X_1, O_1) \) and as \( \mathcal{F} \) is \( \text{sn-bi-contra continuous} \) \( (P^*, O_1) \) is \( \text{sn-C}(X_2, O_2) \).

**Remark 7.5:** The condition of assuming \( (M^*, O_1) \) as both \( \text{sn-closed} \) and \( \text{sn-open} \) cannot be given up in the above theorem as taken in the example 7.2.
8. SOFT NANO-BI-CONTRA gω - CONTINUOUS FUNCTIONS

Definition 8.1: An onto map \( F: (\tau_{\mathfrak{N}}(X_1), U_1, O_1) \rightarrow (\tau_{\mathfrak{N}'}(X_2), U_2, O_2) \) is called a soft nano-bi-contra-\( g\omega \)-continuous, if \( F \) is soft nano contra-\( g\omega \)-continuous and \( F^{-1}(M^*, O_1) \) is soft nano open in \( U_1 \) implies \( (M^*, O_1) \) is soft nano-\( g\omega \)-closed.

Theorem 8.2: Every soft nano-bi contra continuous is soft nano bi-contra \( g\omega \)-continuous.

Proof: Let \( F^{-1}(M^*, O_2) \) be \( sn\)-O\((X_1, O_1)\). We know that soft nano-contra-continuous implies soft-nano- contra-\( g\omega \)-continuous. As \( F \) is soft nano-bi-continuous, \((M^*, O_1)\) is soft nano-\( g\omega \)-closed in \( U_2 \). Therefore, \((M^*, O_1)\) is soft nano- \( g\omega \)-C\((X_2, O_2)\).

Remark 8.3: The converse of above theorem is not true always.

Example 8.4: Let \( U_1 = \{e_1, e_2, e_3, e_4, e_5\} \), \( O_1 = \{k_1, k_2, k_3\}, X_1 = \{e_3, e_4\} \), \( \tau_{\mathfrak{N}}(X_1) = \{(k_1, \{e_3, e_4\}), (k_2, \{e_3, e_4\}), (k_3, \{e_3, e_4\})\} \), \( U_2 = \{e'_1, e'_2, e'_3, e'_4\} \), \( O_2 = \{k'_1, k'_2, k'_3\}, X_2 = \{e'_3, e'_4\} \), \( \tau_{\mathfrak{N}'}(X_2) = \{(k'_1, \{e'_3, e'_4\}), (k'_2, \{e'_3, e'_4\}), (k'_3, \{e'_3, e'_4\})\} \). Define a function

\[ F: (\tau_{\mathfrak{N}}(X_1), U_1, O_1) \rightarrow (\tau_{\mathfrak{N}'}(X_2), U_2, O_2) \] as \( F(e_1) = e'_1 \), \( F(e_3) = e'_2 \), \( F(e'_4) = e'_3 \) and \( F(e_5) = e'_4 \). Here \( sn \)-open sets in \( U_1 \) are \((A_1^*, O_1) = \{(k_1, \{e_1, e_2, e_5\}), (k_2, \{e_1, e_2, e_5\}), (k_3, \{e_1, e_2, e_5\})\}\), \( sn \)-closed sets in \( U_1 \) are \((A_1^*, O_1)^c = \{(k'_1, \{e'_1, e'_5\}), (k'_2, \{e'_1, e'_5\}), (k'_3, \{e'_1, e'_5\})\}, \) \( sn \)-\( g\omega \)-closed sets in \( U_1 \) are \((B_1^*, O_1) = \{(k_1, \{e_1, e_2, e_3, e_5\}), (k_2, \{e_1, e_2, e_3, e_5\}), (k_3, \{e_1, e_2, e_3, e_5\})\}\) and \( sn \)-open sets in \( U_2 \) is \((C_1^*, O_2) = \{(k'_1, \{e'_1, e'_3\}), (k'_2, \{e'_1, e'_3\}), (k'_3, \{e'_1, e'_3\})\}\), \( sn\)-\( g\omega \)-closed set in \( U_2 \) is \((C_1^*, O_2)^c = \{(k'_1, \{e'_2, e'_4\}), (k'_2, \{e'_2, e'_4\}), (k'_3, \{e'_2, e'_4\})\}\), \( sn\)-\( g\omega \)-closed sets in \( U_2 \) are \((D_1^*, O_2) = \{(k'_1, \{e'_1, e'_3\}), (k'_2, \{e'_1, e'_3\}), (k'_3, \{e'_1, e'_3\})\}, \) \( D_2^*, O_2) = \{(k'_1, \{e'_2, e'_4\}), (k'_2, \{e'_2, e'_4\}), (k'_3, \{e'_2, e'_4\})\}, \) \( D'_2^*, O_2) = \{(k'_1, \{e'_1, e'_3\}, e'_2, e'_4\}), (k'_2, \{e'_1, e'_3\}, e'_2, e'_4\}), (k'_3, \{e'_1, e'_3\}, e'_2, e'_4\})\), \( (D'_2^*, O_2) = \{(k'_1, \{e'_2, e'_3\}, e'_1, e'_4\}), (k'_2, \{e'_2, e'_3\}, e'_1, e'_4\}), (k'_3, \{e'_2, e'_3\}, e'_1, e'_4\})\}. Here \( F^{-1}(e_1, e_2) = (e_1, e_2, e_3) \) that is inverse image of \( sn \)-open set in \( U_2 \) is \( sn\)-\( g\omega \)-closed in \( U_1 \). Hence \( F \) is \( sn\)-contra \( g\omega \)-continuous and \( F^{-1}(e'_2, e'_4) \) is \( sn\)-open in \( U_1 \) implies \( \{e'_2, e'_4\} \) is \( sn\)-\( g\omega \)-closed in \( U_2 \). Therefore, \( F \) is \( sn\)-bi-contra-\( g\omega \)-continuous map. If we define \( F(e_1) = e'_1, F(e_2) = e'_2, F(e_3) = e'_3, F(e_4) = e'_4 \) and \( F(e_5) = e'_5 \), then
$\mathcal{F}^{-1}(\epsilon_1, \epsilon_2) = \{\epsilon_1, \epsilon_2, \epsilon_3\}$ that is inverse image of sn-open set in $U_2$ is not sn-open set in $U_1$.

Hence $\mathcal{F}$ is not sn-bi-contra continuous.

**Theorem 8.5:** If $\mathcal{F}: (\tau_\mathbb{R}'(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}''(X_2), U_2, O_2)$ is onto, soft-nano-contra $g\omega$-continuous and soft nano-contra $g\omega$-open, then $\mathcal{F}$ is soft nano-bi-contra $g\omega$-continuous.

**Proof:** Let $(M^*, O_1)$ be sn-cl$(X_2, O_2)$, $\mathcal{F}^{-1}(M^*, O_1)$ is sn-O$(X_1, O_1)$. As $\mathcal{F}$ is soft nano contra-$g\omega$-open, $\mathcal{F}(\mathcal{F}^{-1}(M^*, O_1)) = (M^*, O_1)$ is sn-$g\omega$-closed. Therefore $\mathcal{F}$ is soft nano-contra $g\omega$-continuous.

**Theorem 8.6:** Let $\mathcal{F}_1: (\tau_\mathbb{R}'(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}''(X_2), U_2, O_2)$ be an open surjective, sn-$g\omega$-irresolute and $\mathcal{F}_2: (\tau_\mathbb{R}''(X_2), U_2, O_2) \rightarrow (\tau_\mathbb{R}'''(X_3), U_3, O_3)$ be a soft nano-bi-contra $g\omega$-continuous map. Then $\mathcal{F}_2 \circ \mathcal{F}_1$ is soft nano-bi-contra $g\omega$-continuous map.

**Proof:** Let $(M^*, O_1)$ be a sn-O$(X_3, O_3)$. As $\mathcal{F}_2$ is soft nano-bi-contra $g\omega$-continuous, $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-$g\omega$-cl$(X_2, O_2)$. Since $\mathcal{F}_2$ is soft nano $g\omega$-irresolute, $\mathcal{F}_2^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ is sn-$g\omega$-cl$(X_1, O_1)$. Thus $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(M^*, O_1) = \mathcal{F}_2^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ is sn-$g\omega$-cl$(X_1, O_1)$. Therefore $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}$ is sn-contra $g\omega$-continuous.

Let $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}(M^*, O_1) = \mathcal{F}_2^{-1}(\mathcal{F}_2^{-1}(M^*, O_1))$ be sn-O$(X_1, O_1)$. As $\mathcal{F}_1$ is sn-open and surjective $\mathcal{F}_2^{-1}(M^*, O_1)$ is sn-O$(X_2, O_2)$. As $\mathcal{F}_2$ is soft nano-bi-contra-$g\omega$-continuous, $(M^*, O_1)$ is sn-$g\omega$-cl$(X_2, O_2)$.

Therefore $(\mathcal{F}_2 \circ \mathcal{F}_1)^{-1}$ is sn-bi-contra-$g\omega$-continuous.

**Theorem 8.7:** If $\mathcal{F}_1: (\tau_\mathbb{R}'(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}''(X_2), U_2, O_2)$ is soft nano-bi-contra $g\omega$-continuous map and $\mathcal{F}_2: (\tau_\mathbb{R}''(X_2), U_2, O_2) \rightarrow (\tau_\mathbb{R}'''(X_3), U_3, O_3)$ is a continuous map, where $(\tau_\mathbb{R}'''(X_3), U_3, O_3)$ is a space that is constant on each set $\mathcal{F}_1^{-1}((P^*, O_1))$, for $(P^*, O_1) \in (\tau_\mathbb{R}''(X_2), U_2, O_2)$, then $\mathcal{F}_2$ induces a soft nano-bi-contra $g\omega$-continuous map $\mathcal{F}_3: (\tau_\mathbb{R}''(X_2), U_2, O_2) \rightarrow (\tau_\mathbb{R}'''(X_3), U_3, O_3)$ such that $\mathcal{F}_3 \circ \mathcal{F}_1 = \mathcal{F}_2$.

### 9. Soft Nano Strongly-Bi-Contra $g\omega$-Continuous Maps

**Definition 9.1:** Let $\mathcal{F}: (\tau_\mathbb{R}'(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}''(X_2), U_2, O_2)$ be an onto map. Then $\mathcal{F}$ is called a soft nano strongly-bi-contra $g\omega$-continuous provided $\mathcal{F}$ is soft nano contra $g\omega$-continuous and $\mathcal{F}^{-1}(M^*, O_1)$ is soft nano open in $U_1$, if and only if $(M^*, O_1)$ is soft nano $g\omega$-closed in $U_2$. 
Definition 9.2: The function \( \mathcal{F}: (\tau_\mathbb{R}(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}(X_2), U_2, O_2) \) is called soft nano bi-contra \((\mathbb{g}\omega)^*\)-continuous, if \( \mathcal{F} \) is soft nano contra \( \mathbb{g}\omega \)-irresolute and \( \mathcal{F}^{-1}(M^*, O_1) \) is sn-\( \mathbb{g}\omega \) open in \( U_1 \), if and only if \((M^*, O_1)\) is soft nano closed in \( U_2 \).

Theorem 9.3: Each soft nano-bi contra-(\( \mathbb{g}\omega \)^*)-continuous map is soft nano-bi-contra \( \mathbb{g}\omega \)-continuous map.

Proof: Proof is obvious.

Theorem 9.4: Let \( \mathcal{F}_1: (\tau_\mathbb{R}(X_1), U_1, O_1) \rightarrow (\tau_\mathbb{R}(X_2), U_2, O_2) \) be a onto, soft nano strongly \( \mathbb{g}\omega \)-open and soft nano \( \mathbb{g}\omega \)-irresolute map and \( \mathcal{F}_2: (\tau_\mathbb{R}(X_2), U_2, O_2) \rightarrow (\tau_\mathbb{R}(X_3), U_3, O_3) \) be a soft nano-bi-contra \((\mathbb{g}\omega)^*\)-continuous map. Then \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is an soft nano-bi-contra \((\mathbb{g}\omega)^*\)-continuous map.

Proof: Let \((M^*, O_2)\) soft nano \( \mathbb{g}\omega \)-open in \( U_3 \). As \( \mathcal{F}_2 \) is soft nano-bi-contra \((\mathbb{g}\omega)^*\)-continuous, \( \mathcal{F}_2^{-1}(M^*, O_1) \) is soft nano \( \mathbb{g}\omega \)-closed in \( U_2 \). Since \( \mathcal{F}_1 \) is soft nano strongly-\( \mathbb{g}\omega \)-open and onto, \( \mathcal{F}_2^{-1}(M^*, O_1) \) is sn-\( \mathbb{g}\omega \)-O(\( X_2, O_2 \)). Also, \( \mathcal{F}_2 \) is soft nano-bi-contra \((\mathbb{g}\omega)^*\)-continuous and so \( \mathcal{F}_2^{-1}(M^*, O_1) \) is soft nano \( \mathbb{g}\omega \)-open if and only if \((M^*, O_1)\) is soft nano closed in \( U_3 \). Therefore \( \mathcal{F}_2 \circ \mathcal{F}_1 \) is an soft nano-bi-contra-(\( \mathbb{g}\omega \)^*)-continuous.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES


