4-TOTAL DIFFERENCE CORDIAL LABELING OF CORONA OF SNAKE GRAPHS WITH $K_1$

R. PONRAJ$^{1,*}$, S. YESU DOSS PHILIP$^{2,†}$, R. KALA$^2$

$^1$Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India

$^2$Department of Mathematics, Manonmaniam Sundarnar University, Abishekapatti, Tirunelveli, 627012, Tamilnadu, India

Copyright © 2020 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. Let $G$ be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called $k$-total difference cordial labeling of $G$ if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with $x$. A graph with admits a $k$-total difference cordial labeling is called $k$-total difference cordial graphs. In this paper we investigate the 4-total difference cordial labeling behaviour of corona of snake graphs with $K_1$.

Keywords: $T_n \odot K_1$; $Q_n \odot K_1$; $A(T_n \odot K_1)$.

2010 AMS Subject Classification: 05C78.

1. INTRODUCTION

All graphs in this paper are finite, simple and undirecte. The $k$-total difference cordial graph was introduced in [3]. In [3, 4], 3-total difference cordial labeling behaviour of path, complete graph, comb, armed crown, crown, wheel, star etc have been investigated. Also 4-total difference cordial labeling of path, star, bistar, comb, crown, $P_n \cup K_1, n$, $S(P_n \cup K_{1,n})$, $P_n \cup B_{n,n}$...
etc., have been investigated [5]. In this paper we investigate 4-total difference of cordial labeling of Corona of triangular snake and quadrilateral snake graphs with $K_1$.

2. Preliminaries

**Definition 2.1.** Let $G$ be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, k - 1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called $k$-total difference cordial labeling of $G$ if $|tdf(i) - tdf(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k - 1\}$ where $tdf(x)$ denotes the total number of vertices and the edges labelled with $x$. A graph with a $k$-total difference cordial labeling is called $k$-total difference cordial graph.

**Definition 2.2.** The Triangular snake $T_n$ is obtained from the path $P_n : u_1 u_2 \ldots u_n$ with $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n - 1\}$ and $E(T_n) = E(P_n) \cup \{u_i v_i, u_i v_{i+1} : 1 \leq i \leq n - 1\}$.

**Definition 2.3.** The Quadrilateral snake $Q_n$ is obtained from the path $P_n : u_1 u_2 \ldots u_n$ with $V(Q_n) = V(P_n) \cup \{v_i, w_i : 1 \leq i \leq n - 1\}$ and $E(Q_n) = E(P_n) \cup \{u_i v_i, u_i+1 w_i : 1 \leq i \leq n - 1\}$.

**Definition 2.4.** The The Alternate triangular snake of $A(T_n)$ is obtained from the path $P_n : u_1 u_2 \ldots u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to the vertex $v_i$. That is every alternate edge of a path is replaced by $C_3$.

**Definition 2.5.** Let $G_1, G_2$ respectively be $p_1, q_1, p_2, q_2$ graphs. The corona of $G_1$ with $G_2, G_1 \odot G_2$ is the graph is obtained by taking one copy of $G_1$ and $p_1$ copies of $G_2$ and joining the $i^{th}$ vertex of $G_1$ with an edge to every vertex in the $i^{th}$ copy of $G_2$.

3. Main Results

**Theorem 3.1.** The corona of triangular snake $T_n$ with $K_1$, $T_n \odot K_1$ is 4-total difference cordial.

**Proof.** Take the vertex set and edge set of $T_n$ as in definition 2.1. Let $x_i (1 \leq i \leq n - 1)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n - 1)$ and $y_i (1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n - 1)$. Clearly $|V(T_n)| + |E(T_n)| = 9n - 6$.

Case 1. $n > 3$. Fix the labels 1, 1, 3, 3, 3, 3, 1, 1 and 1 to the vertices $x_1, x_2, v_1, v_2,$ $u_1, u_2, u_3, y_1, y_2$ and $y_3$. Next assign the label 3 to the all path vertices $u_1 u_2 \ldots u_n$. Next assign the labels 1, 2, 1 and 3 to the vertices $v_3, v_4, v_5$ and $v_6$. Similarly assign the labels 1, 2, 1 and 3 to
the next four vertices \( v_7, v_8, v_9 \) and \( v_{10} \). Continue in this pattern until we reach the vertex \( v_{n-1} \). Clearly the vertex \( v_{n-1} \) receive the label 1 when \( n \equiv 1, 3 \pmod{4} \) and 2 or 3 according as \( n \equiv 0 \pmod{4} \) or \( n \equiv 2 \pmod{3} \).

Next assign the labels 2, 3, 2 and 1 to the vertices \( x_3, x_4, x_5 \) and \( x_6 \). Assign the labels 2, 3, 2 and 1 to the next four vertices \( x_7, x_8, x_9 \) and \( x_{10} \). Proceeding in this way until we reach the vertex \( x_{n-1} \). Clearly the vertex \( x_{n-1} \) receive the label 2 when \( n \equiv 1, 3 \pmod{4} \) and 3 or 1 according as \( n \equiv 0 \pmod{4} \) or \( n \equiv 2 \pmod{3} \).

Next assign the labels 1, 3, 3 and 3 to the vertices \( y_3, y_4, y_5 \) and \( y_6 \). Assign the labels 1, 3, 3 and 3 to the next four vertices \( y_7, y_8, y_9 \) and \( y_{10} \). Proceeding like this until we reach the vertex \( y_n \). Clearly the vertex \( y_n \) receive the label 3 or 1 when \( n \equiv 0, 1, 2 \pmod{4} \) or \( n \equiv 3 \pmod{4} \).

Case 2. \( n \leq 3 \).

Table 1 gives a 4-total difference cordial labeling for this case.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{TABLE 1} \]

The table 2 shows that this vertex labeling is a 4-total difference cordial labeling.

<table>
<thead>
<tr>
<th>Values of ( n )</th>
<th>( t_{df}(0) )</th>
<th>( t_{df}(1) )</th>
<th>( t_{df}(2) )</th>
<th>( t_{df}(3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \equiv 0 \pmod{4} )</td>
<td>( \frac{9n-4}{4} )</td>
<td>( \frac{9n-8}{4} )</td>
<td>( \frac{9n-4}{4} )</td>
<td>( \frac{9n-8}{4} )</td>
</tr>
<tr>
<td>( n \equiv 1 \pmod{4} )</td>
<td>( \frac{9n-5}{4} )</td>
<td>( \frac{9n-9}{4} )</td>
<td>( \frac{9n-9}{4} )</td>
<td>( \frac{9n-5}{4} )</td>
</tr>
<tr>
<td>( n \equiv 2 \pmod{4} )</td>
<td>( \frac{9n-6}{4} )</td>
<td>( \frac{9n-6}{4} )</td>
<td>( \frac{9n-6}{4} )</td>
<td>( \frac{9n-6}{4} )</td>
</tr>
<tr>
<td>( n \equiv 3 \pmod{4} )</td>
<td>( \frac{9n-7}{3} )</td>
<td>( \frac{9n-7}{4} )</td>
<td>( \frac{9n-7}{4} )</td>
<td>( \frac{9n-7}{4} )</td>
</tr>
</tbody>
</table>

\[ \text{TABLE 2} \]

\[ \Box \]

**Example 3.1.** A 4-total difference cordial labeling of \( T_6 \odot K_1 \) is shown in Figure 1
Theorem 3.2. The corona of quadrilateral snake $Q_n$ with $K_1$, $Q_n \odot K_1$ is 4-total difference cordial.

Proof. Take the vertex set and edge set of $Q_n$ as in definition 2.2. Let $x_i$ be the pendent vertices adjacent to $v_i$ and $z_i$ be the pendent vertices adjacent to $w_i (1 \leq i \leq n - 1)$. Let $y_i (1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i (1 \leq i \leq n)$. It is easy to verify that $|V(Q_n)| + |E(Q_n)| = 13n - 10.$

Assign the label 3 to the all the path vertices $u_1 u_2 \ldots u_n$. Next assign the labels 3, 3, 1 and 1 to the vertices $v_1, v_2, v_3$ and $v_4$. Assign the labels 3, 3, 1 and 1 to the vertices $v_5, v_6, v_7$ and $v_8$. Continue in this pattern until we reach the vertex $v_{n-1}$. Clearly the vertex $v_{n-1}$ receive the label 3 or 1 according as $n \equiv 1, 2 \pmod{4}$ or $n \equiv 0, 3 \pmod{4}$.

We now consider the vertices $w_i$. Assign the labels 3, 3, 1 and 2 to the vertices $w_1, w_2, w_3$ and $w_4$. Next assign the labels 3, 3, 1 and 2 to the vertices $w_5, w_6, w_7$ and $w_8$. Proceeding like this until we reach the vertex $w_{n-1}$. Clearly the vertex $w_{n-1}$ receive the label 3 when $n \equiv 1, 2 \pmod{4}$ and 1 or 2 when $n \equiv 0, 3 \pmod{4}$.

Now we consider the vertices $x_i$. Assign the labels 1, 1, 2 and 3 to the vertices $x_1, x_2, x_3$ and $x_4$. Next assign the labels 1, 1, 2 and 3 to the vertices $x_5, x_6, x_7$ and $x_8$. Proceeding like this until we reach the vertex $x_{n-1}$. Clearly the vertex $x_{n-1}$ receive the label 1 when $n \equiv 1, 2 \pmod{4}$ and 2 or 3 when $n \equiv 3, 0 \pmod{4}$.

We now move to the vertices $z_i$. Assign the labels 1, 1, 3 and 3 to the vertices $z_1, z_2, z_3$ and $z_4$. Next assign the labels 1, 1, 3 and 3 to the vertices $z_5, z_6, z_7$ and $z_8$. Proceeding like this until
we reach the vertex \( z_{n-1} \). Clearly the vertex \( z_{n-1} \) receive the labels 1 or 3 according as \( n \equiv 1, 2 \pmod{4} \) or \( n \equiv 3, 0 \pmod{4} \).

Next we move to the pendant vertices of path. Fix the label 1 to the vertex \( y_i \). Assign the labels 1, 1, 3 and 3 to the vertices \( y_2, y_3, y_4 \) and \( y_5 \). Next assign the labels 1, 1, 3 and 3 to the vertices \( y_6, y_7, y_8 \) and \( y_9 \). Proceeding like this until we reach the vertex \( y_n \). Clearly the vertex \( y_n \) receive the label 1 or 3 according as \( n \equiv 2, 3 \pmod{4} \) or \( n \equiv 0, 1 \pmod{4} \).

The table 3 shows that this vertex labeling is a 4-total difference cordial labeling.

<table>
<thead>
<tr>
<th>Values of ( n )</th>
<th>( t_{df}(0) )</th>
<th>( t_{df}(1) )</th>
<th>( t_{df}(2) )</th>
<th>( t_{df}(3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \equiv 0 \pmod{4} )</td>
<td>( \frac{13n-8}{4} )</td>
<td>( \frac{13n-12}{4} )</td>
<td>( \frac{13n-8}{4} )</td>
<td>( \frac{13n-12}{4} )</td>
</tr>
<tr>
<td>( n \equiv 1 \pmod{4} )</td>
<td>( \frac{13n-13}{4} )</td>
<td>( \frac{13n-9}{4} )</td>
<td>( \frac{13n-9}{4} )</td>
<td>( \frac{13n-9}{4} )</td>
</tr>
<tr>
<td>( n \equiv 2 \pmod{4} )</td>
<td>( \frac{13n-10}{4} )</td>
<td>( \frac{13n-10}{4} )</td>
<td>( \frac{13n-10}{4} )</td>
<td>( \frac{13n-10}{4} )</td>
</tr>
<tr>
<td>( n \equiv 3 \pmod{4} )</td>
<td>( \frac{13n-7}{4} )</td>
<td>( \frac{13n-11}{4} )</td>
<td>( \frac{13n-11}{4} )</td>
<td>( \frac{13n-11}{4} )</td>
</tr>
</tbody>
</table>

**Table 3**

**Example 3.2.** A 4-total difference cordial labeling of \( Q_5 \odot K_1 \) is shown in Figure 2

![Figure 2](image-url)

**Theorem 3.3.** The corona of alternate triangular snake \( A(T_n) \) with \( K_1 \), \( A(T_n) \odot K_1 \) is 4-total difference cordial.
**Proof.** Take the vertex set and edge set of $A(T_n)$ as in definition 2.3.

Case 1. The edge $u_1u_2$ lies in a triangle and the edge $u_{n−1}u_n$ lies in a triangle.

Let $x_i (1 ≤ i ≤ n − 1)$ be the pendent vertices adjacent to $v_i (1 ≤ i ≤ n − 1)$ and $y_i (1 ≤ i ≤ n)$ be the pendent vertices adjacent to $u_i (1 ≤ i ≤ n − 1)$. Clearly $n$ is even. In this case $|V(A(T_n))∩K_1|+|E(A(T_n))| = \frac{13n−2}{2}$.  

Assign the label 3 to the path vertices $u_1u_2 \ldots u_n$. Next fix the label 3 and 3 to the vertices $v_1$ and $v_2$. Fix the label 1 to the vertices $x_1, x_2, y_1$ and $y_2$. Next assign the labels 2, 3, 2 and 1 to the vertices $x_3, x_4, x_5$ and $x_6$. Assign the labels 2, 3, 2 and 1 to the next four vertices $x_7, x_8, x_9$ and $x_{10}$. Continue in this pattern until we reach the vertex $x_2$. Clearly the vertex $x_2$ receive the label 2 when $n ≡ 1, 3$ (mod 4) and 3 or 1 according as $n ≡ 0$ (mod 4) or $n ≡ 2$ (mod 3).

We now consider the vertices $v_i$. Assign the labels 1, 2, 1 and 3 to the vertices $v_3, v_4, v_5$ and $v_6$. Similarly assign the labels 1, 2, 1 and 3 to the next four vertices $v_7, v_8, v_9$ and $v_{10}$. Continue in this pattern until we reach the vertex $v_2$. Clearly the vertex $v_2$ receive the label 1 when $n ≡ 1, 3$ (mod 4) and 2 or 3 according as $n ≡ 2$ (mod 4) or $n ≡ 0$ (mod 3).

Consider the vertices $y_j$. Assign the labels 1, 1, 1, 3, 1, 3 and 1 to the vertices $y_3, y_4, y_5, y_6, y_7, y_8, y_9$ and $y_{10}$. Next assign the labels 1, 1, 1, 3, 1, 3 and 1 to the vertices $y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, y_{17}$ and $y_{18}$. Continue in this pattern until we reach the vertex $y_n$. Clearly the vertex $y_n$ receive the label 1 or 3 according as $n ≡ 1, 3, 5, 7$ (mod 8) or $n ≡ 0, 2, 6$ (mod 4).

The table 4 shows that this vertex labeling is a 4-total difference cordial labeling.

<table>
<thead>
<tr>
<th>Values of $n$</th>
<th>$t_{df}(0)$</th>
<th>$t_{df}(1)$</th>
<th>$t_{df}(2)$</th>
<th>$t_{df}(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n ≡ 0$ (mod 8)</td>
<td>$\frac{13n}{8}$</td>
<td>$\frac{13n}{8}$</td>
<td>$\frac{13n−8}{8}$</td>
<td>$\frac{13n−8}{8}$</td>
</tr>
<tr>
<td>$n ≡ 2$ (mod 8)</td>
<td>$\frac{13n−2}{8}$</td>
<td>$\frac{13n−2}{8}$</td>
<td>$\frac{13n−2}{8}$</td>
<td>$\frac{13n−2}{8}$</td>
</tr>
<tr>
<td>$n ≡ 4$ (mod 8)</td>
<td>$\frac{13n+4}{8}$</td>
<td>$\frac{13n−4}{8}$</td>
<td>$\frac{13n−4}{8}$</td>
<td>$\frac{13n−4}{8}$</td>
</tr>
<tr>
<td>$n ≡ 6$ (mod 8)</td>
<td>$\frac{13n+2}{8}$</td>
<td>$\frac{13n−6}{8}$</td>
<td>$\frac{13n+2}{8}$</td>
<td>$\frac{13n−6}{8}$</td>
</tr>
</tbody>
</table>

**TABLE 4**

Case 2. The edge $u_1u_2$ lies in a triangle and the edge $u_{n−2}u_{n−1}$ lies in a triangle. In this case $n$ is odd.
Clearly removal of the edge $u_{n-1}u_n$ is the graph as in case(i). Assign the label to the vertices $u_i (1 \leq i \leq n-1)$ and $v_i (1 \leq i \leq \frac{n-1}{2})$ as in case (i). Finally assign the labels 3 and 1 respect to the vertices $u_n$ and $v_n$. The table 5 shows that this vertex labeling is a 4-total difference cordial labeling.

<table>
<thead>
<tr>
<th>Values of $n$</th>
<th>$t_{df}(0)$</th>
<th>$t_{df}(1)$</th>
<th>$t_{df}(2)$</th>
<th>$t_{df}(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 1 \pmod{8}$</td>
<td>$\frac{13n-5}{8}$</td>
<td>$\frac{13n-5}{8}$</td>
<td>$\frac{13n-13}{8}$</td>
<td>$\frac{13n-5}{8}$</td>
</tr>
<tr>
<td>$n \equiv 3 \pmod{8}$</td>
<td>$\frac{13n-7}{8}$</td>
<td>$\frac{13n-7}{8}$</td>
<td>$\frac{13n-7}{8}$</td>
<td>$\frac{13n-7}{8}$</td>
</tr>
<tr>
<td>$n \equiv 5 \pmod{8}$</td>
<td>$\frac{13n-9}{8}$</td>
<td>$\frac{13n-17}{8}$</td>
<td>$\frac{13n-17}{8}$</td>
<td>$\frac{13n-17}{8}$</td>
</tr>
<tr>
<td>$n \equiv 7 \pmod{8}$</td>
<td>$\frac{13n-11}{8}$</td>
<td>$\frac{13n-19}{8}$</td>
<td>$\frac{13n-11}{8}$</td>
<td>$\frac{13n-19}{8}$</td>
</tr>
</tbody>
</table>

TABLE 5

Case 3. The edge $u_2u_3$ lies in a triangle and the edge $u_{n-2}u_{n-1}$ lies in a triangle. Obviously removal of the edge $u_1u_2$ as in case(ii). Assign the label to the vertices $u_i (2 \leq i \leq n)$ and $v_i (2 \leq i \leq \frac{n-2}{2})$ as in case (i). Next assign the labels 3 and 1 respect to the vertices $u_1$ and $v_1$. The table 6 shows that this vertex labeling is a 4-total difference cordial labeling.

<table>
<thead>
<tr>
<th>Values of $n$</th>
<th>$t_{df}(0)$</th>
<th>$t_{df}(1)$</th>
<th>$t_{df}(2)$</th>
<th>$t_{df}(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 \pmod{8}$</td>
<td>$\frac{13n-8}{8}$</td>
<td>$\frac{13n-16}{8}$</td>
<td>$\frac{13n-8}{8}$</td>
<td>$\frac{13n-16}{8}$</td>
</tr>
<tr>
<td>$n \equiv 2 \pmod{8}$</td>
<td>$\frac{13n-10}{8}$</td>
<td>$\frac{13n-10}{8}$</td>
<td>$\frac{13n-18}{8}$</td>
<td>$\frac{13n-10}{8}$</td>
</tr>
<tr>
<td>$n \equiv 4 \pmod{8}$</td>
<td>$\frac{13n-12}{8}$</td>
<td>$\frac{13n-12}{8}$</td>
<td>$\frac{13n-12}{8}$</td>
<td>$\frac{13n-12}{8}$</td>
</tr>
<tr>
<td>$n \equiv 6 \pmod{8}$</td>
<td>$\frac{13n-6}{8}$</td>
<td>$\frac{13n-14}{8}$</td>
<td>$\frac{13n-14}{8}$</td>
<td>$\frac{13n-14}{8}$</td>
</tr>
</tbody>
</table>

TABLE 6
**Theorem 3.4.** The corona of alternate quadrilateral snake $A(Q_n)$ with $K_1, A(Q_n) \odot K_1$ is 4-total difference cordial.

*Proof.* Take the vertex set and edge set of $A(Q_n)$ as in definition 2.3.

Case 1. The edge $u_1u_2$ lies in a Quadrilateral and the edge $u_{n-1}u_n$ lies in a Quadrilateral.

Let $x_i(1 \leq i \leq n)$ be the pendent vertices adjacent to $v_i(1 \leq i \leq n)$ and $z_i(1 \leq i \leq \frac{n}{2})$ be the pendent vertices adjacent to $w_i(1 \leq i \leq n)$ and $y_i(1 \leq i \leq n)$ be the pendent vertices adjacent to $u_i1 \leq i \leq n$. Clearly $n$ is even. In this case $|V(A(Q_n)) \odot K_1| + |E(A(Q_n))| = \frac{17n-11}{2}$.

Assign the label 3 to the path vertices $u_1u_2\ldots u_n$. Next fix the label 3 to the vertices $v_1$ and $w_1$. Fix the label 1 to the vertices $x_1,z_1$ and $y_i(1 \leq i \leq n)$. Next assign the labels 1, 3, 3 and 3 to the vertices $x_2,x_3,x_4$ and $x_5$. Assign the labels 1, 3, 3 and 3 to the next four vertices $x_6,x_7,x_8$ and $x_9$. Continue in this pattern until we reach the vertex $x_{\frac{n}{2}}$. Clearly the vertex $x_{\frac{n}{2}}$ receive the label 1 when $n \equiv 2 \pmod{4}$ and 3 when $n \equiv 0,1,3 \pmod{4}$.

We now consider the vertices $z_i(1 \leq i \leq \frac{n}{2})$. Assign the labels 1, 2, 3 and 2 to the vertices $z_2,z_3,z_4$ and $z_5$. Similarly assign the labels 1, 2, 3 and 2 to the next four vertices $z_6,z_7,z_8$ and $z_9$. Continue in this pattern until we reach the vertex $z_{\frac{n}{2}}$. Clearly the vertex $z_{\frac{n}{2}}$ receive the label 2 when $n \equiv 1,3 \pmod{4}$ and 1 or 3 according as $n \equiv 0,2 \pmod{4}$.

Consider the vertices $v_i(1 \leq i \leq \frac{n}{2})$. Assign the label 3 to the vertices $v_1,v_2,v_{\frac{n}{2}}$. Next assign the labels 3, 1, 2 and 1 to the vertices $w_2,w_3,w_4$ and $w_5$. Assign the labels 3, 1, 2 and 1 to the next four vertices $w_6,w_7,w_8$ and $w_9$. Continue in this way until we reach the vertex $w_{\frac{n}{2}}$. Clearly the vertex $w_{\frac{n}{2}}$ receive the label 1 when $n \equiv 1,3 \pmod{4}$ and 3 or 2 when $n \equiv 0,2 \pmod{4}$.

The table 7 shows that this vertex labeling is a 4-total difference cordial labeling.

Case 2. The edge $u_1u_2$ lies in a quadrilateral and the edge $u_{n-2}u_{n-1}$ lies in a quadrilateral. In this case $n$ is odd.

Clearly removal of the edge $u_{n-1}u_n$ is the graph as in case(i). Assign the label to the vertices
Values of $n$ & $t_{df}(0)$ & $t_{df}(1)$ & $t_{df}(2)$ & $t_{df}(3)$
\hline
$n \equiv 0 \pmod{8}$     & $\frac{17n}{8}$ & $\frac{17n}{8}$ & $\frac{17n-8}{8}$ & $\frac{17n}{8}$ \\
$n \equiv 2 \pmod{8}$     & $\frac{17n-2}{8}$ & $\frac{17n-2}{8}$ & $\frac{17n-2}{8}$ & $\frac{17n-2}{8}$ \\
$n \equiv 4 \pmod{8}$     & $\frac{17n+4}{8}$ & $\frac{17n-4}{8}$ & $\frac{17n-4}{8}$ & $\frac{17n-4}{8}$ \\
$n \equiv 6 \pmod{8}$     & $\frac{17n+2}{8}$ & $\frac{17n-6}{8}$ & $\frac{17n+2}{8}$ & $\frac{17n-6}{8}$ \\
\hline
TABLE 7

$u_i(1 \leq i \leq n-1)$ and $v_i(1 \leq i \leq \frac{n}{2})$ and $w_i(1 \leq i \leq \frac{n}{2})$ as in case (i). Finally assign the labels 3 and 1 respect to the vertices $u_n$ and $v_{\frac{n}{2}}$.

The table 8 shows that this vertex labeling is a 4-total difference cordial labeling.

Values of $n$ & $t_{df}(0)$ & $t_{df}(1)$ & $t_{df}(2)$ & $t_{df}(3)$
\hline
$n \equiv 1 \pmod{8}$     & $\frac{17n-9}{8}$ & $\frac{17n-9}{8}$ & $\frac{17n-17}{8}$ & $\frac{17n-9}{8}$ \\
$n \equiv 3 \pmod{8}$     & $\frac{17n-11}{8}$ & $\frac{17n-11}{8}$ & $\frac{17n-11}{8}$ & $\frac{17n-11}{8}$ \\
$n \equiv 5 \pmod{8}$     & $\frac{17n-5}{8}$ & $\frac{17n-13}{8}$ & $\frac{17n-13}{8}$ & $\frac{17n-13}{8}$ \\
$n \equiv 7 \pmod{8}$     & $\frac{17n-7}{8}$ & $\frac{17n-15}{8}$ & $\frac{17n-7}{8}$ & $\frac{17n-15}{8}$ \\
\hline
TABLE 8

Case 3. The edge $u_2u_3$ lies in a Quadrilateral and the edge $u_{n-2}u_{n-1}$ lies in a Quadrilateral. Obviously removal of the edge $u_1u_2$ as in case(ii). Assign the label to the vertices $u_i(2 \leq i \leq n)$ and $v_i(2 \leq i \leq n-1)$ and $w_i(1 \leq i \leq \frac{n}{2})$ as in case (i). Next assign the labels 3 and 1 respect to the vertices $u_1$ and $v_1$.

The table 9 shows that this vertex labeling is a 4-total difference cordial labeling. 

□
Values of $n$ & $tdf(0)$ & $tdf(1)$ & $tdf(2)$ & $tdf(3)$
\hline
$n \equiv 0 \pmod{8}$ & $\frac{17n-32}{8}$ & $\frac{17n-40}{8}$ & $\frac{17n-32}{8}$ & $\frac{17n-40}{8}$ \\
$n \equiv 2 \pmod{8}$ & $\frac{17n-34}{8}$ & $\frac{17n-34}{8}$ & $\frac{17n-42}{8}$ & $\frac{17n-34}{8}$ \\
$n \equiv 4 \pmod{8}$ & $\frac{17n-20}{8}$ & $\frac{17n-20}{8}$ & $\frac{17n-20}{8}$ & $\frac{17n-20}{8}$ \\
$n \equiv 6 \pmod{8}$ & $\frac{17n-30}{8}$ & $\frac{17n-38}{8}$ & $\frac{17n-38}{8}$ & $\frac{17n-38}{8}$ \\
\hline
\textbf{TABLE 9}

\textbf{CONFLICT OF INTERESTS}

The author(s) declare that there is no conflict of interests.

\textbf{REFERENCES}


