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A STUDY OF INVENTORY MODEL FOR DETERIORATING ITEMS WITH PRICE AND STOCK DEPENDENT DEMAND UNDER THE JOINED EFFECT OF PRESERVATION TECHNOLOGY AND PRICE DISCOUNT FACILITY

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Abstract: The purpose of this paper is to develop an economic order quantity model for deteriorating items with price and stock dependent demand, deterioration rate with preservation technology and price discount facility. The proposed model focused on two things. The first one is the consideration of the fact that the deterioration rate can be reduced by the use of preservation technology investment and the second one is using the assumption that the unit purchase cost has a hostile relationship with the order size to maximize the total profit. The concept of salvage value is considered and incorporated in this model. The solution procedure of proposed optimization model is illustrated by a couple of numerical examples. Concavity of the average profit function is shown by plotting graphs. Sensitivity analysis is performed to study the effect of changing the value of all parameters in the proposed maximization model.

Keywords: inventory; deteriorating items; controllable deterioration rate; preservation technology; price discount; stock dependent demand; price dependent demand.

2010 AMS Subject Classification: 54H25, 47H10.

1. INTRODUCTION

In daily problems, we observe that inventory or stock of some items is essential to provide better service and management in business enterprises and industries. That is why inventory control or management is one of important field of study in operations research. Various factors like display

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of the items/ level of inventory, selling price of the product, time, advertisement, inflation etc., are the major issues to determine customer's demand. Generally, customers like to buy from a shop which has large piles of goods in its shelf space. The reason behind of it is visibility, variety of items. On the other hand, low stock of goods might raise the question that they are not good. Although it does not necessary mean that high stock of goods induces very high number of customers to buy it. Therefore, the demand is often inventory-level dependent. In the last three decades, the inventory model involving inventory-level dependent demand have received much attention of several researchers. According to Levin et al. [18], "It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more. Yet many goods piled up will lead a negative impression on buyers". Stock dependent demand is generally suitable for different types of industrial sectors such as processed and raw food industry, garments (cloths and dresses) industry, automobile industry and electronics & electrical industry etc. Mondal and Phaujder [23] established an EPQ model with linear stock-dependent demand. Giri et al. [9] invented an EPQ model for deteriorating items with stock dependent consumption rate. Wu et al. [30] developed an EOQ model with stock-dependent demand for non-instantaneous deteriorating products and partial backlogging.

On the other hand, in so many sectors specially food sectors, the demand of raw food items such as vegetables, fruits, fishes, eggs, meat, dairy product, rice, wheat etc., in a shop and processed food in hotel or restaurant are price sensitive. Customers like to purchase from a shop which has low selling price. If the retailer increases the selling price of the product, the consumers would move other shopping places to satisfy their demand. There are several research works have been done on the effect of price variations. Kotler [13] incorporated marketing policies into inventory decisions and discussed the relationship between economic order quantity and pricing decision. Ladany and Sterleib [16] studied the effect of selling price variation on EOQ. Goyal & Gunasekaran [11], Bhunia and Shaikh [2] developed EOQ models considering the effect of price variations on EOQ. Ranganayki et al. [24] invented an inventory model with demand dependent on price under fuzzy environment. Again, thinking the importance of sock and price both, Mashud et al. [21] developed a non-instantaneous model with stock and price dependent demand under partially backlogged shortages.

Deterioration of product is a major issue in the inventory control policy. We cannot neglect this in the present study. There are some items like milk, ice-cream, vegetables, dairy product, grocery items which deteriorates over time. Whitin [29] was the pioneer, who first studied an EOQ model of deteriorating items. An inventory model of deteriorating items which deteriorates exponentially is developed by Ghare and Schrader [8]. An EOQ model for items with variable rate of deterioration has been developed by Covert and Philips [4] by introducing two parameters Weibull distribution. A discrete in time model for deteriorating items has been developed by Shah and Shah [26], where they considered random demand and trade credit financing. De and Goswami [6] extended Shah and Shah's [26] model for deteriorating items by considering continuous cycle time as well. Goyal and Giri [10] described a note on the recent trends of modelling in deteriorating items in inventory. Kumar and Keerthika [15] developed a model considering the fact that deterioration follows various probabilistic distribution. Kumar [14] developed another model for deteriorating items where the holding cost is quadratic increasing with time.

So, inventory system of deteriorating items has been studied for a long time, but little is known about the effect of investing in reducing the rate deterioration. Hsu et al. [12] developed an inventory model where the retailer invests on the preservation technology. Lee and Dye [17] first formulated a deteriorating inventory model with stock dependent demand by allowing preservation technology cost as a decision variable. Maihami & Kamalabadi [19] developed a joint pricing and inventory control system for non-instantaneous deteriorating items with price and time dependent demand. Mishra [22] developed an inventory model of instantaneous deteriorating items with controllable deterioration rate using preservation technology for a time dependent demand. Mushud et al. [20] developed an inventory model under the joint effect of trade credit and preservation technology. So, in this paper preservation technology is used to reduce deterioration rate by which retailer can gain his profit, satisfy customer's demand and increase business competitiveness.

As in most of the research work, demand depends upon selling price and stock, so the order size of the customer will be affected directly. So, retailer can offer discount and other promotional offer to attract large number of customers, and hence demand. Less selling price creates high demand and large no. of orders will be placed to customers and in this situation, all-unit discount offer will be provided to the customer according to the size of the order. Taleizadeh and Pentico [27] introduced price discount concept in their model. Alfair and Ghaithan [1] developed an inventory model for no shortage case under price discount facility. Shaikh et al. [25] modified Alfair and Ghaithan's [1] model considering shortages.

The present work is developed under the following considerations: i) use of preservation technology to reduce deterioration of items ii) demand is price and stock dependent iii) price discount facility. Our aim is to determine the maximum profit of this model. The next part of the paper is designed to organize as cited. The assumption and notations of the model are introduced in section 2. Following the section 3, we have developed a mathematical optimization problem of this model. In section 4, theorical result for concavity of the profit function is presented. In section 5, we give numerical solution procedure an algorithm for the proposed model. In section 6, some numerical examples and graphical representation are carried out. The sensitivity analysis is recorded in section 7. In the last, we conclude and give some future research scope in section 8.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are presented to formulate the proposed model.

2.1 Assumptions:

i) Only one item inventory model is considered.

ii) The replenishment rate is infinite and lead time is zero.

iii) Shortages are not allowed

iv) The demand D of this model is linearly price and stock-dependent i.e., $D = \alpha - \beta p + \gamma I(t)$, where $\alpha, \beta, \gamma > 0$ and $\alpha - \beta p > 0$.

v) θ is the constant deterioration rate without preservation technology investment ($0 \leq \theta \leq 1$).

vi) To reduce the deterioration effect, preservation technology is used. The resultant rate of deterioration with the investment of preservation technology is $\theta\omega(\xi)$, where $\omega(\xi)$ is a decreasing function with $\omega''(\xi) > 0$. Here we have considered $\omega(\xi) = e^{-\upsilon\xi}, \upsilon > 0$ and $\omega(\xi) = \frac{1}{1+\upsilon\xi}$, $\upsilon > 0$.

vii) Unit purchase cost is decreasing step function based on the ordering quantity(Q). $Q_i, i = 1,2,3 \ldots, n+1$ ($Q_1 < Q_2 < Q_3 < \cdots < Q_n < Q_{n+1} = \infty$) are quantities that determine the n price breaks with in the unit purchase cost C_i,$i = 1,2,3, \ldots, n$($C_1 > C_2 > \cdots > C_n$) .

2.2 Notations:

i) K: replenishment cost per cycle

ii) α: constant part of the demand rate($\alpha > 0$)

iii) β: price dependent demand rate parameter ($\beta > 0$)

iv) γ: stock-dependent demand rate parameter ($\gamma > 0$)

v) θ: deterioration rate without preservation technology investment ($0 \leq \theta \leq 1$)

vi) p: selling price per unit item

vii) C_h: holding cost for per unit item per unit time

viii) C_d: deterioration cost per unit item

ix) C_i: purchase cost per unit item

x) Q: number of ordering quantity per replenishment cycle

xi) η: salvage coefficient ($0 \leq \eta \leq 1$)

xii) $I(t)$: Inventory level at time t

xiii) AP: average profit per unit time

Decision Variables

i) T: Replenishment time per cycle

ii) ξ: preservation cost per unit time

3. MATHEMATICAL MODEL FORMULATION

The objective of the model is to determine the optimum profit for items having above mentioned demand and deterioration. The replenishment of inventory is carried out at the beginning and ultimately at the end of cycle stock level exhausted. The level of inventory depletes as a result of demand as well as the deterioration. The governing differential equation is as follows:

$$\frac{dI(t)}{dt} + \theta\omega(\xi)I(t) = -\big(\alpha - \beta p + \gamma I(t)\big), \qquad 0 \leq t \leq T \tag{1}$$

With boundary conditions $I(T) = 0$ and $I(0) = Q$.

The solution of equation (1) is

$$I(t) = \frac{\alpha-\beta p}{\gamma+\theta\omega(\xi)}\left(e^{(\gamma+\theta\omega(\xi))(T-t)} - 1\right) \tag{2}$$

Using the initial condition $I(0) = Q$, from equation (2) we get

$$Q = \frac{\alpha-\beta p}{\gamma+\theta\omega}\left(e^{(\gamma+\theta\omega)T} - 1\right) \tag{3}$$

So, the cycle length T can be expressed as follows:

$$T = \frac{1}{\gamma+\theta\omega}\log\left[1 + \frac{\gamma+\theta\omega}{\alpha-\beta p}Q\right] \tag{4}$$

Now the total sales revenue (SR) per cycle $=\int_0^T p\{\alpha - \beta p + \gamma I(t)\}\, dt$

$$= (\alpha - \beta p)pT + \gamma p \frac{\alpha-\beta p}{\gamma+\theta\omega}[\frac{\left(e^{(\gamma+\theta\omega)T}-1\right)}{\gamma+\theta\omega} - T] \tag{5}$$

Replenishment cost (RC) per cycle=K

Total inventory cost (IHC)per cycle=$C_h \int_0^T I(t)dt = C_h \frac{\alpha-\beta p}{\gamma+\theta\omega}[\frac{\left(e^{(\gamma+\theta\omega)T}-1\right)}{\gamma+\theta\omega} - T]$ (6)

Total purchase cost (PC) per cycle =$C_i Q = C_i \frac{\alpha-\beta p}{\gamma+\theta\omega}\left(e^{(\gamma+\theta\omega)T} - 1\right)$ (7)

Total deterioration cost (DC) per cycle=$C_d \left(Q - \int_0^T \{\alpha - \beta p + \gamma I(t)\}\, dt\right)$

$$=C_d \left\{\frac{\alpha-\beta p}{\gamma+\theta\omega}\left(e^{(\gamma+\theta\omega)T} - 1\right) - (\alpha - \beta p)T - \gamma \frac{\alpha-\beta p}{\gamma+\theta\omega}\left[\frac{\left(e^{(\gamma+\theta\omega)T}-1\right)}{\gamma+\theta\omega} - T\right]\right\} \tag{8}$$

Total salvage value for the deteriorating items (SV) per cycle= $\eta.DC$

$$=\eta C_d \left\{\frac{\alpha-\beta p}{\gamma+\theta\omega}\left(e^{(\gamma+\theta\omega)T} - 1\right) - (\alpha - \beta p)T - \gamma \frac{\alpha-\beta p}{\gamma+\theta\omega}\left[\frac{\left(e^{(\gamma+\theta\omega)T}-1\right)}{\gamma+\theta\omega} - T\right]\right\} \tag{9}$$

Total preservation cost (PRC) per cycle=ξT (10)

Average profit ($AP(\xi, T)$) per unit time =$\frac{1}{T}(SR + SV - RC - IHC - PC - DC - PRC)$

$$=\frac{1}{T}\left[\begin{array}{c} (\alpha - \beta p)pT + \{\gamma(p + C_d - \eta C_d) - C_h\}\left(\frac{\alpha-\beta p}{\gamma+\theta\omega}\right)\left\{\frac{\left(e^{(\gamma+\theta\omega)T}-1\right)}{\gamma+\theta\omega} - T\right\} - \\ (C_i + C_d - \eta C_d)\left(\frac{\alpha-\beta p}{\gamma+\theta\omega}\right)\left(e^{(\gamma+\theta\omega)T} - 1\right) + (C_d - \eta C_d)(\alpha - \beta p)T - \xi T - K \end{array}\right] \tag{11}$$

As stock-dependent parameter (γ) and deterioration(θ) is small, so we expand the exponential term by Taylor expansion formula and ignoring third and higher order term, which is earlier used by Chung [5], Teng [28], etc. we write:

$$AP(\xi, T) = \frac{1}{T}\{(\alpha - \beta p)\{\gamma(p - C_i) - C_h - \theta\omega(\xi)(C_i + C_d - \eta C_d)\}\frac{T^2}{2} + (\alpha - \beta p)(p - C_i - \xi)T - K\} \tag{12}$$

Now our objective is to obtain optimal cycle time T^* and preservation cost ξ^* in order to maximize the average profit ($AP(\xi, T)$) per unit time.

In order to find out necessary conditions to maximize profit, we have

$$\frac{\partial(AP)}{\partial T} = T^2(\alpha - \beta p)\{\gamma(p - C_i) - C_h - \theta\omega(\xi)(C_i + C_d - \eta C_d)\} - 2K = 0 \tag{13}$$

and $\frac{\partial(AP)}{\partial \xi} = \{\gamma(p - C_i) - C_h - \theta\omega'(\xi)(C_i + C_d - \eta C_d)\}T - 2 = 0$ (14)

Solving equations (13) and (14) simultaneously, we can get ξ^* and T^*. Next, we shall show that average profit function ($AP(\xi, T)$) attains the global maximum value at (ξ^*, T^*) under certain condition by the help of result of Cambini and Martein [3] and Dye [7].

4. THEORETICAL RESULT FOR OPTIMALITY

The average total profit function $AP(\xi,T)$ is strictly pseudo-concave in ξ and T, if $\gamma(p-C_i)-C_h-\theta(C_i+C_d-\eta C_d)>0$, hence $AP(\xi,T)$ attains the global maximum value at (ξ^*,T^*).

Proof. To prove this theorem, we first write average total profit function as $AP(\xi,T)=\frac{\varphi(\xi,T)}{\psi(\xi,T)}$,

where

$$\varphi(\xi,T)=(\alpha-\beta p)\{\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)\}\frac{T^2}{2}+(\alpha-\beta p)(p-C_i-\xi)T-K \tag{15}$$

and $\psi(\xi,T)=T$ (16)

$$\frac{\partial^2\varphi}{\partial\xi^2}=-(\alpha-\beta p)\theta\omega''(\xi)(C_i+C_d-\eta C_d)\frac{T^2}{2} \tag{17}$$

$$\frac{\partial^2\varphi}{\partial T^2}=(\alpha-\beta p)\{\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)\} \tag{18}$$

$$\frac{\partial^2\varphi}{\partial T\partial\xi}=\frac{\partial^2\varphi}{\partial\xi\partial T}=-(\alpha-\beta p)\{\theta\omega'(\xi)(C_i+C_d-\eta C_d)T+1\} \tag{19}$$

The Hessian matrix for $\varphi(\xi,T)$ is $H=\begin{bmatrix}\frac{\partial^2\varphi}{\partial\xi^2} & \frac{\partial^2\varphi}{\partial\xi\partial T}\\ \frac{\partial^2\varphi}{\partial T\partial\xi} & \frac{\partial^2\varphi}{\partial T^2}\end{bmatrix}$

The first principal minor is $|H_{11}|=\frac{\partial^2\varphi}{\partial\xi^2}=-(\alpha-\beta p)\theta\omega''(\xi)(C_i+C_d-\eta C_d)\frac{T^2}{2}$ (20)

Since $(\alpha-\beta p)>0,\omega''(\xi)>0$ and $0\le\eta\le1$, so $|H_{11}|<0$

The second principal minor is

$$|H_{22}|=\frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\varphi}{\partial T^2}-\frac{\partial^2\varphi}{\partial T\partial\xi}\frac{\partial^2\varphi}{\partial\xi\partial T}$$

$$=-\frac{(\alpha-\beta p)^2}{2}\Big[\big\{\theta\omega''(\xi)(C_i+C_d-\eta C_d)\big\{\{\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)\}\big\}+2\theta^2(\omega'(\xi))^2(C_i+C_d-\eta C_d)^2\big\}T^2+4\theta\omega'(\xi)(C_i+C_d-\eta C_d)T+2\Big] \tag{21}$$

$|H_{22}|$ will be less than zero if the quadratic expression

$$\big\{\theta\omega''(\xi)(C_i+C_d-\eta C_d)\big\{\{\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)\}\big\}+2\theta^2(\omega'(\xi))^2(C_i+C_d-\eta C_d)^2\big\}T^2+4\theta\omega'(\xi)(C_i+C_d-\eta C_d)T+2>0$$

The quadratic expression will be greater than zero if the discriminant is less than zero i.e., if

$\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)>0$(after simplifying)

Now as $0 < \omega(\xi) < 1$, so if $\gamma(p - C_i) - C_h - \theta(C_i + C_d - \eta C_d) > 0$ then $\gamma(p - C_i) - C_h - \theta\omega(\xi)(C_i + C_d - \eta C_d) > 0$ and hence $|H_{22}| < 0$.

Thus if $\gamma(p - C_i) - C_h - \theta(C_i + C_d - \eta C_d) > 0$ then all the principal minors of the Hessian matrix for $\varphi(\xi, T)$ are negative and hence, the Hessian matrix is negative definite. Therefore $\varphi(\xi, T)$ is negative, differentiable and strictly concave.

Moreover $\psi(\xi, T) = T$ is positive, differentiable, and convex function, so the average profit function per unit time $AP(\xi, T)$ is pseudo-concave function in ξ and T. Therefore, if $\gamma(p - C_i) - C_h - \theta(C_i + C_d - \eta C_d) > 0$, then $AP(\xi, T)$ attains the global maximum value at the point (ξ^*, T^*).

This completes the proof of the theorem.

5. SOLUTION PROCEDURE AND ALGORITHM

In this section, we have sketched solution procedure as well as algorithm to find out maximum profit and optimal solution in price discount environment.

5.1 Solution procedure.

Supplier offers unit purchasing cost C_i $(C_1 > C_2 > \cdots > C_n)$ if the number of replenishment quantity (Q) lies in between Q_i and Q_{i+1}, $i = 1, 2, 3, \ldots, n (Q_1 < Q_2 < \cdots < Q_n < Q_{n+1} = \infty)$. As supplier offers less purchasing cost if retailer buys more products that mentioned in assumption (vii) of section 2, so at first retailer will try to purchase items by the lowest purchasing cost in order to get maximum profit. So, he/she will find T and ξ from the equations (13) and (14) by inserting all parameter values and lowest purchasing cost C_n, and then ordering quantity Q from the equation (3). If Q lies in between Q_n, Q_{n+1}, the solution (T^*, ξ^*) is optimal and maximum profit will be $AP(T^*, \xi^*)$ (from equation (11)).

If Q does not belong in between Q_n, Q_{n+1}, then the retailer will try to take benefit of minimum purchasing cost by taking ordering quantity $Q = Q_n$. So, setting $Q = Q_n$, in equation (4) he/she will get T, then the average profit function AP will be a function of single variable ξ by substituting T in equation (11). Consequently, there will be single decision variable ξ under which the profit function $AP(\xi)$ has to be maximized.

To obtain optimal ξ , the necessary condition is $\frac{d(AP(\xi)}{d\xi}=0$. Calculating $\frac{d(AP(\xi)}{d\xi}$ by the help of equation (4) and $\frac{dT}{d\xi}=\frac{\theta\omega'(\xi)}{(\gamma+\theta\omega)^2}\left[\frac{\frac{\gamma+\theta\omega}{\alpha-\beta p}Q}{1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q}-\log\left[1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q\right]\right]$ (22)

and setting equal to zero, the necessary condition becomes

$$\frac{\theta\omega'(\xi)}{(\gamma+\theta\omega)^2}\left[\frac{\frac{\gamma+\theta\omega}{\alpha-\beta p}Q}{1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q}-\log\left[1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q\right]\right]$$

$$\times\left[\frac{(\alpha-\beta p)}{2}\{\gamma(p-C_i)-C_h-\theta\omega(\xi)(C_i+C_d-\eta C_d)\}+\frac{K}{\left(\frac{1}{\gamma+\theta\omega}\log\left[1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q\right]\right)^2}\right]$$

$$+\frac{(\alpha-\beta p)}{2}\{\gamma(p-C_i)-C_h-\theta\omega'(\xi)(C_i+C_d-\eta C_d)\}\times\left(\frac{1}{\gamma+\theta\omega}\log\left[1+\frac{\gamma+\theta\omega}{\alpha-\beta p}Q\right]\right)-(\alpha-\beta p)=0$$

(23)

Using equations (23) and (4) retailer can find out T,ξ and $AP(T,\xi)$. If the profit is greater than the profit of the retailer by choosing the next unit purchasing cost C_{n-1}, then this T,ξ will be optimal solution (T^*,ξ^*), otherwise repeat this process for the purchasing cost C_{n-1} and so on.

5.2 Algorithm

Here we draft the algorithm for the proposed model.

Step1. Set $MaxATP(T,\xi)=-\infty$ and $i=n$.

Step 2. Insert all values of parameters including C_i in equations (13) and (14) to find out T and ξ. Evaluate Q by substituting T and ξ in equation (3).

a) If $Q\in[Q_i,Q_{i+1})$ then calculate $AP(T,\xi)$(from equation (11)) and if $AP(T,\xi)$> $MaxAP(T,\xi)$, set $MaxAP(T,\xi)$= $AP(T,\xi)$. Go to step 5.

b) If $Q\notin[Q_i,Q_{i+1})$ then go to step 3.

Step 3. Set $Q=Q_i$ and put all values of parameters including C_i in equation (23) and solve for ξ. Calculate corresponding T from equation (4) and finally calculate $AP(T,\xi)$(from equation 11) with the help of these T and ξ. if $AP(T,\xi)$> $MaxAP(T,\xi)$, set $MaxAP(T,\xi)$= $AP(T,\xi)$. Go to step 4.

Step 4. If $i\geq 2$, go to step 2 with $i=i-1$, otherwise go to Step 5.

Step 5. Optimal profit is $MaxAP(T,\xi)$ with the corresponding T and ξ.

6. NUMERICAL ILLUSTRATIONS

To exemplify different cases of the established model, three numerical examples are taken with their appropriate values.

Example 1. Let us take the following parameters in appropriate units as follows:

Price breaks:

Ordering quantity	0=$Q_1 \leq Q < Q_2 = 300$	300=$Q_2 \leq Q < Q_3 = 550$	550=$Q_3 \leq Q <= \infty$
Unit purchase cost	$C_1 = \$2$	$C_2 = \$1.75$	$C_3 = \$1.5$

& all other parameters are $K = \$500$, $\alpha = 100$, $\beta = 1.5$, $\gamma = 0.3$, $C_h = \$0.80$ per unit item per month, $p = \$20$ per unit item, $C_d = \$1.0$ per unit item, $\theta = 0.025$, $\eta = 0.08$ and $\omega(\xi) = e^{-v\xi}$, where $v = 2$. The values of the parameters are considered here are realistic, though these values are not taken from any case study of an existing inventory problem. The computational work has been done by Mathematica 12.

Maximum profit per month for this example can be evaluated by the help of algorithm discussed in section 5.2 as follows:

Step 1: Set $MaxAP(T, \xi) = -\infty$ and $i = 3$

Step 2: Inserting all values of parameters including $C_3 = \$1.5$ in equations (13) and (14), we obtain $T = 7.07762$ and $\xi = 1.84816$ and the corresponding ordering quantity $Q = 553.216$. Inserting these values of T and ξ in equation (11), we get $AP(T, \xi) = 1156.78$. Since $Q = 553.216 \in [550, \infty)$ and $AP(T, \xi) = 1156.78 > MaxAP(T, \xi) = -\infty$, so we set $MaxAP(T, \xi) = 1156.78$ and go to step 5.

Step 5. Optimal solution is $T^* = 7.07762$ month, $\xi^* = \$1.84816$, $Q^* = 553.216$ unit and $MaxAP(T, \xi) = \$1156.78$.

The concavity of the profit function against T and ξ is observed in Fig 1. In addition, concavity of the profit function against T and concavity of the profit function against ξ are shown separately in Fig 2 and Fig 3 respectively.

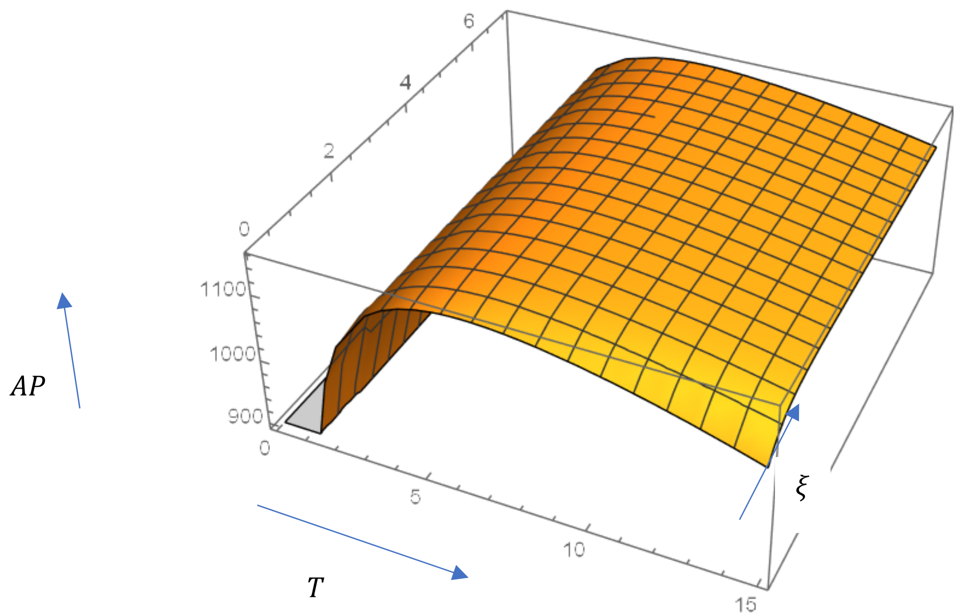

Fig 1. Profit per unit time vs T and ξ of example 1

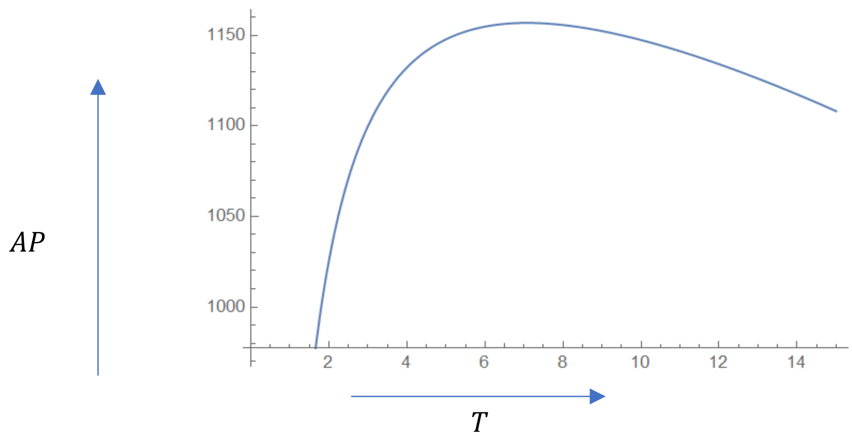

Fig 2. Profit per unit time vs T of example 1

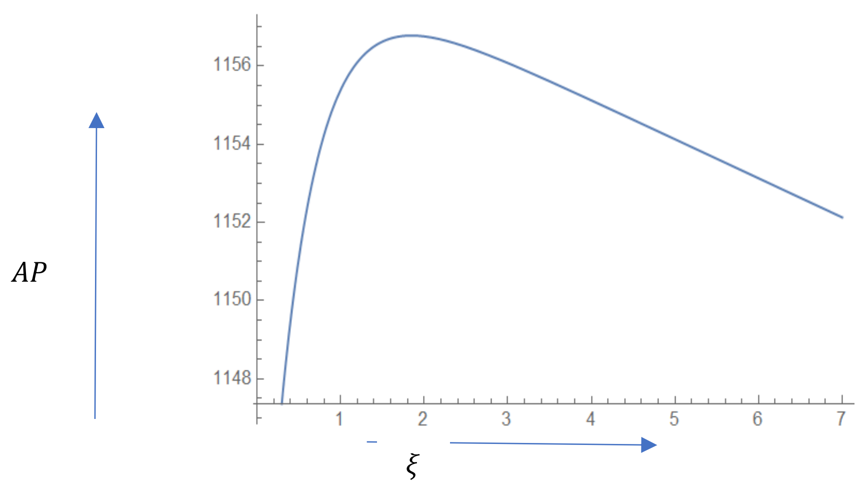

Fig 3. Profit per unit time vs ξ of example 1

Example 2. Let us take the following parameters in appropriate units as follows:

Price breaks:

Ordering quantity	0=$Q_1 \leq Q \leq Q_2 = 200$	200=$Q_2 \leq Q \leq Q_3 = 350$	350=$Q_3 \leq Q \leq Q_4 = 500$	500=$Q_4 \leq Q <= \infty$
Unit purchase cost	$C_1 = \$2.25$	$C_2 = \$2.0$	$C_3 = \$1.75$	$C_4 = \$1.5$

& all other parameters are $K = \$400$, $\alpha = 80$, $\beta = 1.8$, $\gamma = 0.3$, $C_h = \$0.80$ per unit item per month, $p = \$20$ per unit item, $C_d = \$1.0$ per unit item, $\theta = 0.025$, $\eta = 0.08$ and $\omega(\xi) = e^{-v\xi}$, where $v = 2$.

Maximum profit per month for this example can be evaluated by the help of algorithm discussed in section 5.2 as follows:

Step 1: Set $MaxATP(T, \xi) = -\infty$ and $i = 4$

Step 2: Inserting all values of parameters including $C_4 = \$1.5$ in equations (13) and (14), we obtain $T = 7.90239$ and $\xi = 1.68804$ and the corresponding ordering quantity $Q = 393.761$. Inserting these values of T and ξ in equation (11), we get $AP(T, \xi) = 715.026$. Since $Q = 393.761 \notin [500, \infty)$, it is not feasible. Hence go to step 3.

Step3. Set $Q = 500$ and put all values of parameters including $C_4 = \$1.5$ in equation (23) and we get $\xi = 1.78861$. Putting this value of ξ in equation (4), we get $T = 9.7481$. Finally, using these T and ξ in equation (11) we have found $AP(T, \xi) = 702.317$.

Since $AP(T, \xi) = 702.317 > MaxAP(T, \xi)$, set $MaxAP(T, \xi)$= 702.317. Go to step 4.

Step 4. Set $i = 3$. Using all parameters including $C_3 = \$1.75$ in equations (13) and (14), we obtain $T = 7.79404$ and $\xi = 1.72089$ and the corresponding ordering quantity $Q = 387.602$. Inserting these values of T and ξ in equation (11), we get $AP(T, \xi) = 702.582$. Since $Q = 387.602 \in [350, 500)$ and $AP(T, \xi) = 702.582 > MaxAP(T, \xi) = 702.317$, so we set $MaxAP(T, \xi) = 702.582$ and optimal solution is corresponding $T = 7.79404$, $\xi = 1.72089$ and $Q = 387.602$. Go to step 5.

Step 5. Optimal solution is $T^* = 7.79404$ month , $\xi^* = \$1.72089$, $Q^* = 387.602$unit and $MaxAP(T, \xi) = \$702.582$.

Example 3. All data are same as of example 2 except $\omega(\xi) = \frac{1}{1+v\xi}$, $v = 2$.

Here also maximum profit per month can be computed by the help of algorithm discussed in section 5.2 as follows:

Step 1: Set $MaxAP(T,\xi) = -\infty$ and $i = 4$

Step 2: Inserting all values of parameters including $C_4 = \$1.5$ in equations (13) and (14), we obtain $T = 7.69876$ and $\xi = 2.18425$ and the corresponding ordering quantity $Q = 388.239$. Inserting these values of T and ξ in equation (11), we get $AP(T,\xi) = 712.322$. Since $Q = 388.239 \notin [500, \infty)$, it is not feasible. Hence go to step 3.

Step3. Set $Q = 500$ and put all values of parameters including $C_4 = \$1.5$ in equation (23) and we get $\xi = 1.69281$. Putting this value of ξ in equation (5), we get $T = 9.53843$. Finally, using these T and ξ in equation (11) we have found $AP(T,\xi) = 705.224$.

Since $ATP(T,\xi) = 705.224 > MaxAP(T,\xi)$, set $MaxAP(T,\xi) = 705.224$. Go to step 4.

Step 4. Set $i = 3$. Using all parameters including $C_3 = \$1.75$ in equations (13) and (14), we obtain $T = 7.59014$ and $\xi = 2.22728$ and the corresponding ordering quantity $Q = 381.785$. Inserting these values of T and ξ in equation (11), we get $AP(T,\xi) = 699.731$. Since $Q = 381.785 \in [350, 500)$ and $AP(T,\xi) = 699.731 \ngtr MaxAP(T,\xi) = 705.224$, so we set $MaxAP(T,\xi) = 705.224$ and optimal solution is corresponding $T = 9.53843$, $\xi = 1.69281$ and $Q = 500$. Go to step 5.

Step 5. Optimal solution is $T^* = 9.53843$ month, $\xi^* = \$1.69281$, $Q^* = 500$ unit and $MaxAP(T,\xi) = \$705.224$.

7. Sensitivity Analysis

To test the flexibility of the model, we study the impact of changes in different parameters against optimal solutions (T,ξ), optimal order quantities and average profit for the example1. Changing the value on one parameter by -20% to +20% at a time and fixing other remaining parameters, the analysis has been done. Table 1 presents the observed results with various parameters.

Parameter	Original value	New value	T^*	ξ^*	Q^*	AP
K	500	600	7.70637	1.90300	608.252	1143.26
		550	7.40013	1.87675	581.311	1149.88
		450	6.73631	1.81673	523.771	1164.02
		400	6.47293	1.80182	500.000	1171.65
α	100	120	6.29544	1.89950	624.762	1509.19
		110	6.65143	1.87530	590.110	1332.71
		90	7.60098	1.81720	513.580	981.535
		80	8.86580	1.80120	500.000	807.144
β	1.5	1.80	7.37767	1.83012	529.804	1051.54
		1.65	7.22285	1.83932	541.641	1104.13
		1.35	6.94103	1.85665	564.546	1209.50
		1.20	6.81225	1.86486	575.646	1262.27
γ	0.3	0.36	9.18012	1.97066	763.319	1189.85
		0.33	7.91123	1.89941	634.263	1172.05
		0.27	6.51810	1.81050	500.000	1143.22
		0.24	6.57868	1.80225	500.000	1130.20
C_h	0.80	0.96	6.64333	1.78133	500.000	1119.44
		0.88	6.40332	1.82304	500.000	1136.95
		0.72	8.49362	1.92795	678.755	1180.21
		0.64	11.4587	2.06799	959.551	1210.44
p	20	24	10.0358	1.96652	751.397	1343.22
		22	8.22280	1.89547	626.342	1254.88
		18	6.26626	1.81358	504.342	1048.05
		16	5.78123	1.80687	500.000	928.231
C_d	1.0	1.2	7.07790	1.87769	553.403	1156.75
		1.1	7.07770	1.86314	553.436	1156.77
		0.9	7.07740	1.83271	553.502	1156.80
		0.8	7.07730	1.81677	553.539	1156.81
η	0.08	0.096	7.07760	1.84550	553.221	1156.79
		0.088	7.07761	1.84683	553.219	1156.78
		0.072	7.07764	1.84948	553.215	1156.78
		0.064	7.07765	1.85079	553.212	1156.78
θ	0.025	0.03	7.07762	1.93932	553.216	1156.69
		0.0275	7.07762	1.89581	553.216	1156.74
		0.0225	7.07762	1.79548	553.216	1156.84
		0.02	7.07762	1.73658	553.216	1156.89
v	2	2.4	7.08262	1.61627	553.440	1157.10
		2.2	7.08035	1.72357	553.339	1156.95
		1.8	7.07429	1.99482	553.067	1156.58
		1.6	7.07014	2.17033	552.881	1156.34
C_i	2, 1.75, 1.5	2.4, 2.1, 1.80	6.96034	1.88815	542.945	1133.36
		2.21, 1.925, 1.65	7.01824	1.86871	548.009	1145.06
		1.8, 1.575, 1.35	7.13857	1.82634	558.577	1168.51
		1.6, 1.4, 1.20	7.20115	1.80313	564.099	1180.26

Table 1. Sensitivity Analysis

From Table 1, the following observations can be made.

(i) There is positive effects on the average profit per unit time (AP) with respect to the value of the parameter α, γ, p, v that is AP increases when the values of α, γ, p, v increase, while for the parameters $K, \beta, C_h, C_d, \theta$ there is negative impact on AP.

(ii) The ordering quantity Q increases when the values of the parameters $K, \alpha, \gamma, p, \eta, v$ increase, on the hand Q decreases when β, C_h, C_i, C_d increase. The parameter θ does not impact on the ordering quantity. Furthermore, parameters C_d, η and v have little impact on the ordering quantity.

(iii) The optimal cycle time T^* depends on parameters $K, \beta, \gamma, C_h, p, C_d$ and v in a positive way but it depends on parameters C_h, α, η, C_i in negative way. Changing of the value of the parameter θ does not change the optimal cycle time. The parameters γ and p have the greatest impact on T^*.

(iv) The optimal preservation cost ξ^* increases when the values of the parameters $K, \alpha, \gamma, p, C_d, \theta, C_i$ increase, on the hand ξ^* decreases when β, C_h, η, v increase. Furthermore, parameters p, θ, C_h and v have greatest impact on the preservation cost.

Simple economic interpretations can be drawn from the sensitivity analysis that can be suggested to the retailer.

- If the replenishment cost per order is high, then the retailer will try to replenish big number of ordering quantity at a time to take the benefit of price discount and to avoid deterioration he/she will increase the investment of preservation technology.
- If the holding cost is high the manager or retailer will not be interested to take the benefit of price discount and will not invest so much to preserve products. He/she will prefer to order smaller quantity.
- If the demand is very much stock sensitive, the manager should go all out to reduce the unit purchase cost by making higher order size to the manufacturer/supplier. He/she should also invest higher cost for preservation technology to preserve items.
- If the product deteriorates at a high rate, then the retailer is suggested to reduce the deterioration rate by increasing investment cost for preservation technology.

8. CONCLUSIONS

In this work, we discussed an optimum inventory control problem according to stock and price depending on demand with the joint effect of preservation technology and price discount facility.

An expression of the average profit is derived as an EOQ problem. The concavity of the profit function is being shown. The solution procedure and algorithm are introduced to determine optimal cycle time and optimal preservation cost. With the help of MATHEMATICA software three different numerical example are demonstrated for illustration purpose. The concave nature of the profit function is justified by drawing graphs in three and two dimensions. To check the changes in the decision variables for changes in different parameters, a sensitivity analysis is also carried out.

This model can be extended by considering several realistic features such as time dependent holding cost, shortages and probabilistic demand. One can also extend it under fuzzy environment.

CONFLICT OF INTERESTS

The author declares that there is no conflict of interests.

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