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FUZZY CONTINUITY VIA FUZZY $\mathscr{C}$-OPEN SETS

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Abstract. In fuzzy topological space, the concepts of fuzzy continuity are developed by applying fuzzy open sets. In this paper the notions of fuzzy continuity are investigated using fuzzy $\mathscr{C}$-open sets, where $\mathscr{C}$ is an arbitrary complement function $\mathscr{C}:[0,1]\to[0,1]$ and some of their properties are studied.

Keywords: fuzzy sets; fuzzy complement functions; fuzzy $\mathscr{C}$-open sets; fuzzy $\mathscr{C}$-continuity.

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1. Introduction

Most of the concepts of general topology are developed using the notions of continuity. The idea of continuity in topological space are introduced by open sets. [5] generalized the continuity of a function in topological space to fuzzy topological space using fuzzy open sets. Some of the week forms of fuzzy continuity investigated by [1, 7, 9, 8, 4, 6, 10, 3]. A new type of fuzzy closed sets are introduced using arbitrary complement function named as fuzzy $\mathscr{C}$-closed sets by [2]. Continuing this fuzzy $\mathscr{C}$-open sets are defined with arbitrary complement function by [11] and verified that fuzzy $\mathscr{C}$-open sets and fuzzy $\mathscr{C}$-closed sets are independent with respect to arbitrary complement function $\mathscr{C}:[0,1]\to[0,1]$. A fuzzy subset λ is fuzzy $\mathscr{C}$-open if $\mathscr{C}\lambda$

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is fuzzy closed. In 2017, fuzzy $\mathscr{C}$-interior are characterized using fuzzy $\mathscr{C}$-open sets. Further fuzzy nearly open sets are investigated using fuzzy $\mathscr{C}$-interior and fuzzy closure operators. In this paper the concepts of fuzzy continuity are characterized using fuzzy nearly $\mathscr{C}$-open sets. Throughout this paper (X,τ) be a fuzzy topological space and $\mathscr{C}$ be an arbitrary complement function.

Definition 1. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,σ).*

(i) f is fuzzy continuous if the inverse image of every fuzzy open set in Y is fuzzy open in X. [5]

(ii) f is fuzzy semi continuous if $f^{-1}(\mu)$ is fuzzy semi open of X for each μ of Y. [1]

(iii) f is fuzzy pre continuous if $f^{-1}(\mu)$ is fuzzy pre open in X for each μ in Y. [4]

(iv) f is fuzzy α-continuous if $f^{-1}(\mu)$ is a fuzzy α-open set in X for each μ in Y. [4]

(v) f is fuzzy almost continuous function if $f^{-1}(\mu)$ is a fuzzy open set for each fuzzy regular open set μ in Y. [1]

(vi) f is fuzzy semi pre continuous function if $f^{-1}(\mu)$ is fuzzy semi pre open of X for each μ of Y. [6]

(vii) f is fuzzy b-continuous function if $f^{-1}(\mu)$ is fuzzy b-open of X for each μ of Y. [3]

Lemma 2. [11] *Let $\mathscr{C}S=S\mathscr{C}$ where $\mathscr{C}$ be an arbitrary complement function and S be standard complement function. Then*

(i) λ is fuzzy $\mathscr{C}$-closed iff $S\lambda$ is fuzzy $\mathscr{C}$-open.

(ii) λ is fuzzy $\mathscr{C}$-open iff $S\lambda$ is fuzzy $\mathscr{C}$-closed.

Definition 3. [13]

(i) If $int_{\mathscr{C}}(cl(\lambda))=\lambda$, then λ is fuzzy regular $\mathscr{C}$-open

(ii) If $\lambda\leq int_{\mathscr{C}}(cl(int_{\mathscr{C}}(\lambda)))$, then λ is fuzzy α-$\mathscr{C}$-open

(iii) If $\lambda\leq cl(int_{\mathscr{C}}(\lambda))$, then λ is fuzzy semi $\mathscr{C}$-open

(iv) If $\lambda\leq int_{\mathscr{C}}(cl(\lambda))$, then λ is fuzzy pre $\mathscr{C}$-open

(v) If $\lambda\leq cl(int_{\mathscr{C}}(cl(\lambda)))$, then λ is fuzzy semi pre $\mathscr{C}$-open

(vi) If $\lambda\leq cl(int_{\mathscr{C}}(\lambda))\vee int_{\mathscr{C}}(cl(\lambda))$, then λ is fuzzy b-$\mathscr{C}$-open

(vii) If $\lambda = cl(int_{\mathscr{C}}(\lambda)) \vee int_{\mathscr{C}}(cl(\lambda))$, *then* λ *is fuzzy* $b^{\#}$-$\mathscr{C}$*-open*

The fuzzy interior and fuzzy closure operators for the above sets are defined in the usual manner and denoted as $S_{\mathscr{C}}int$, $P_{\mathscr{C}}int$, $\beta_{\mathscr{C}}int$, $b_{\mathscr{C}}int$, $b^{\#}_{\mathscr{C}}int$, $S_{\mathscr{C}}cl$, $P_{\mathscr{C}}cl$, $\beta_{\mathscr{C}}cl$, $b_{\mathscr{C}}cl$ and $b^{\#}_{\mathscr{C}}cl$.

Lemma 4. [13] *Let* λ *be a fuzzy subset of a fuzzy topological space* (X,τ) *and* $\mathscr{C}$ *be a complement function that satisfies the monotonic and involutive conditions. Then* λ *is fuzzy* α-$\mathscr{C}$*-closed if and only if* $\mathscr{C}\lambda$ *is fuzzy* α-$\mathscr{C}$*-open.*

2. FUZZY $\mathscr{C}$-CONTINUITY

Definition 5. *A function* $f:(X,\tau)\to(Y,\sigma)$ *be a function and* $\mathscr{C}$ *be a complement function. Then* f *is said to be*

(i) fuzzy $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy* $\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(ii) fuzzy regular $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy regular* $\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(iii) fuzzy α-$\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy* α-$\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(iv) fuzzy semi $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy semi* $\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(v) fuzzy pre $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy pre* $\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(vi) fuzzy semi pre $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy semi pre* $\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(vii) fuzzy b-$\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy b-*$\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

(viii) fuzzy $b^{\#}$-$\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy* $b^{\#}$-$\mathscr{C}$*-open for each fuzzy open subset* μ *of* Y.

Definition 6. *A function* $f:(X,\tau)\to(Y,\sigma)$ *is said to be almost* $\mathscr{C}$*-continuous if* $f^{-1}(\mu)$ *is fuzzy* $\mathscr{C}$*-open for each fuzzy regular open subset* μ *of* Y.

Theorem 7. *A function* $f:(X,\tau)\to(Y,\sigma)$ *be a function and* $\mathscr{C}$ *be a complement function satisfies the monotonic and involutive conditions. Consider the following statements.*

(i) f *is a fuzzy almost* $\mathscr{C}$*-continuous function.*

(ii) f^{-1} *is a fuzzy* $\mathscr{C}$*-closed set for each regular closed set* μ *of* Y.

(iii) $int_{\mathscr{C}}f^{-1}(\mu) \leq f^{-1}(int(cl(\mu)))$ *for each* $\mu \in Y$.

(iv) $f^{-1}(cl(int(\mu))) \leq cl(f^{-1}(cl(\mu)))$

Then the implications $(i) \Rightarrow (ii)$, $(ii) \Rightarrow (i)$, $(i) \Rightarrow (iii)$ *and* $(i) \Rightarrow (iv)$ *holds.*

Proof. $(i) \Rightarrow (ii)$ Suppose (i) holds and λ is fuzzy regular closed. Then $S\lambda$ is fuzzy regular open. By using Definition 5, $f^{-1}(S\lambda)$ is fuzzy $\mathscr{C}$-open. $\Rightarrow Sf^{-1}(\lambda)$ is fuzzy $\mathscr{C}$-open. By using Lemma 2, $f^{-1}(\lambda)$ is fuzzy $\mathscr{C}$-closed.

$(ii) \Rightarrow (i)$ Suppose (ii) holds. Let λ be a fuzzy regular open, then $S\lambda$ is fuzzy regular closed. By (ii), $f^{-1}(S\lambda)$ is fuzzy $\mathscr{C}$-closed. $\Rightarrow S(f^{-1}(\lambda))$ is fuzzy $\mathscr{C}$-closed. Using Lemma 2, $f^{-1}(\lambda)$ is fuzzy $\mathscr{C}$-open. Thus f is fuzzy almost $\mathscr{C}$-continuous.

$(i) \Rightarrow (iii)$ Now $\lambda \leq cl(\lambda) \Rightarrow int(\lambda) \leq int(cl(\lambda)) \Rightarrow int_{\mathscr{C}} f^{-1}(int(\lambda)) \leq f^{-1}(int(cl(\lambda)))$.

$(i) \Rightarrow (iv)$ Now $int(\lambda) \leq \lambda \Rightarrow cl(int(\lambda)) \leq cl(\lambda) \Rightarrow f^{-1}(cl(int(\lambda))) \leq f^{-1}(cl(\lambda))$

$\Rightarrow f^{-1}(cl(int(\lambda))) \leq cl(f^{-1}(cl(\lambda)))$. □

Theorem 8. $f : (X, \tau) \to (Y, \sigma)$ *is fuzzy* α-$\mathscr{C}$*-continuous if and only if it is fuzzy semi* $\mathscr{C}$*-continuous and fuzzy pre* $\mathscr{C}$*-continuous where* $\mathscr{C}$ *be a complement function that satisfies the monotonic and involutive conditions.*

Proof. Let f be fuzzy semi $\mathscr{C}$-continuous and fuzzy pre $\mathscr{C}$-continuous. Let μ be a fuzzy open in Y. Then by Definition 5, $f^{-1}(\mu)$ is fuzzy semi $\mathscr{C}$-open and $f^{-1}(\mu)$ is fuzzy pre $\mathscr{C}$-open. Since $\mathscr{C}$ satisfies the monotonic and involutive conditions, using Lemma 4, $f^{-1}(\mu)$ is fuzzy α-$\mathscr{C}$-open set. Thus f is fuzzy α-$\mathscr{C}$-continuous.

Conversely, f is fuzzy α-$\mathscr{C}$-continuous. This implies that $f^{-1}(\mu)$ is fuzzy α-$\mathscr{C}$-open for each fuzzy open set μ in Y. We have every fuzzy α-$\mathscr{C}$-open set is fuzzy semi $\mathscr{C}$-open and fuzzy pre $\mathscr{C}$-open. Therefore f is fuzzy semi $\mathscr{C}$-continuous and fuzzy pre $\mathscr{C}$-continuous. □

Theorem 9. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function and* $\mathscr{C}$ *be a complement function that satisfies the monotonic and involutive conditions. Then the following statements are equivalent.*

(i) f *is a fuzzy b-*$\mathscr{C}$*-continuous function.*

(ii) $f^{-1}(\lambda)$ *is a fuzzy b-closed set of* X*, for each fuzzy closed set* λ *of* Y.

(iii) $f(b_{\mathscr{C}} cl \mu) \leq cl f(\mu)$ *for each fuzzy subset* μ *of* X.

(iv) $b_{\mathscr{C}} cl(f^{-1}(\lambda)) \leq f^{-1}(cl(\lambda))$ *for each fuzzy subset set* λ *of* Y.

(v) $f^{-1}(int(\lambda)) \leq b_{\mathscr{C}} int f^{-1}(\lambda)$ *for each fuzzy subset* λ *of* Y.

Then the implications (i)⇒(ii), (ii)⇒(iii), (iii)⇒(iv), (iv)⇒(v) and (v)⇒(vi) holds.

Proof. (i)⇒(ii)

Let λ be a fuzzy closed set of Y. Then $S\lambda$ is a fuzzy open set of Y. By Definition 5, $f^{-1}(S\lambda)$ is fuzzy b-$\mathscr{C}$-open in $X \Rightarrow Sf^{-1}(\lambda)$ is fuzzy b-$\mathscr{C}$-open in X. Since fuzzy b-$\mathscr{C}$-open and fuzzy b-open coincides when complement function is standard complement, $Sf^{-1}(\lambda)$ is fuzzy b-open in X. Thus $f^{-1}(\lambda)$ is a fuzzy b-closed set of X.

(ii)⇒(iii)

Let μ be a fuzzy subset of X. Then $clf(\mu)$ is a fuzzy closed set of Y. From(ii), $f^{-1}(clf(\mu))$ is a fuzzy b-$\mathscr{C}$-closed set of X

$\Rightarrow b_{\mathscr{C}}cl(\mu) \leq b_{\mathscr{C}}clf^{-1}(f(\mu)) \leq b_{\mathscr{C}}clf^{-1}(f(cl\mu)) = f^{-1}(f(cl\mu))$.

$\Rightarrow f(b_{\mathscr{C}}cl(\mu)) \leq ff^{-1}f(cl\mu) \leq f(cl(\mu))$.

(iii)⇒(iv)

Let λ be a fuzzy subset of Y. By assumptions $f(b_{\mathscr{C}}clf^{-1}(\lambda)) \leq cl(ff^{-1}(\lambda)) \leq cl(\lambda)$.

Thus $b_{\mathscr{C}}clf^{-1}(\lambda) \leq f^{-1}f(b_{\mathscr{C}}clf^{-1}(\lambda)) \leq f^{-1}(cl(\lambda))$

Therefore $b_{\mathscr{C}}clf^{-1}(\lambda) \leq f^{-1}(cl(\lambda))$.

(iv)⇒(v)

Let λ be a fuzzy subset of Y. From(iv), $f^{-1}(cl(\mathscr{C}\lambda)) \geq b_{\mathscr{C}}clf^{-1}(\mathscr{C}\lambda) = b_{\mathscr{C}}cl\mathscr{C}f^{-1}(\lambda)$

$\Rightarrow f^{-1}(int_{\mathscr{C}}(\lambda)) = \mathscr{C}f^{-1}(cl(\mathscr{C}\lambda)) \leq b_{\mathscr{C}}cl\mathscr{C}(f^{-1}(\mathscr{C}\lambda)) = b_{\mathscr{C}}intf^{-1}(\lambda)$.

Thus $f^{-1}(int_{\mathscr{C}}(\lambda)) = b_{\mathscr{C}}intf^{-1}(\lambda)$.

(v)⇒(vi)

Let λ be a fuzzy open set of Y. Then $\lambda = int(\lambda)$.

From (v)$f^{-1}(\lambda) = f^{-1}(int(\lambda)) \leq b_{\mathscr{C}}intf^{-1}(\lambda) \leq f^{-1}(\lambda)$

$\Rightarrow f^{-1}(\lambda) = b_{\mathscr{C}}int(f^{-1}(\lambda))$

Therefore f is a fuzzy b-$\mathscr{C}$-continuous function. □

Theorem 10. *Let $f : X \to Y$ be a function and $\mathscr{C}$ be a complement function. Consider the following statements.*

(i) f is a fuzzy b-$\mathscr{C}$-continuous.

(ii) $P_{\mathscr{C}}int(P_{\mathscr{C}}clf^{-1}(\lambda)) \leq f^{-1}(cl\lambda)$ for each λ of Y.

(iii) $f(P_{\mathscr{C}}int(P_{\mathscr{C}}cl\mu)) \leq clf(\mu)$ for each μ of X.

Then the implications (i)⇒(ii), (ii)⇒(iii) and (iii)⇒(i) holds.

Proof. (i)⇒(ii)

Let λ be a fuzzy subset of Y. Then $f^{-1}(cl\lambda)$ is a fuzzy b-$\mathscr{C}$-closed set.

Hence $f^{-1}(cl\lambda) \geq P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(cl\lambda)) \geq P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(\lambda))$.

(ii)⇒(iii)

Let μ be a fuzzy subset of X and $\lambda = f(\mu)$, then $\mu \leq f^{-1}(\lambda)$.

By assumption $P_{\mathscr{C}}int(P_{\mathscr{C}}cl(\lambda)) \leq P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(\lambda)) \leq f^{-1}(cl(\lambda))$.

Hence $f(P_{\mathscr{C}}int(P_{\mathscr{C}}cl\lambda)) \leq cl(\lambda) = cl(f(\mu))$.

(iii)⇒(i)

Let λ be a fuzzy closed set of Y.

By assumption, $f(P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(\lambda))) \leq cl f f^{-1}(\lambda) \leq cl(\lambda) = \lambda$.

$P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(\lambda)) \leq f^{-1} f(P_{\mathscr{C}}int(P_{\mathscr{C}}cl f^{-1}(\lambda))) \leq f^{-1}(\lambda)$

Therefore $f^{-1}(\lambda)$ is a fuzzy b-$\mathscr{C}$-closed set. □

Theorem 11. *Let $f : X \to Y$ be a bijective function and $\mathscr{C}$ be a complement function. Then the function f is fuzzy b-$\mathscr{C}$-continuous if and only if $int(f(\lambda)) \leq f(b_{\mathscr{C}}int(\lambda))$ for each fuzzy subset λ of X.*

Proof. Let f be fuzzy b-$\mathscr{C}$-continuous and λ be a fuzzy subset of X. Then $f^{-1}(int(f(\lambda)))$ is a fuzzy b-$\mathscr{C}$-open set of X. Since f is injective,

$f^{-1}(int(f(\lambda))) \leq b_{\mathscr{C}}int f^{-1}(int(f(\lambda)) \leq b_{\mathscr{C}}int f^{-1}(\lambda) = b_{\mathscr{C}}int(\lambda)$

Since f is surjective,

$int f(\lambda) = f f^{-1}(int f(\lambda)) \leq f(b_{\mathscr{C}}int\lambda)$.

Conversely, let μ be a fuzzy open set of Y. Then $int(\mu) = \mu$.

By assumption $f(b_{\mathscr{C}}int f^{-1}(\mu)) \geq int f f^{-1}(\mu) = int(\mu) = \mu$

$\Rightarrow f^{-1} f(b_{\mathscr{C}}int f^{-1}(\mu)) \geq f^{-1}(\mu)$

Since f is injective,

$b_{\mathscr{C}}int f^{-1}(\mu) = f^{-1} f(b_{\mathscr{C}}int f^{-1}(\mu)) \geq f^{-1}(\mu)$

$\Rightarrow b_{\mathscr{C}}int f^{-1}(\mu)) = f^{-1}(\mu)$.

Thus f is fuzzy b-$\mathscr{C}$-continuous. □

Theorem 12. *Let $f : X \to Y$ be a function and $\mathscr{C}$ be a complement function. Consider the following statements.*

(i) f is fuzzy semi pre $\mathscr{C}$-continuous.

(ii) the inverses of a fuzzy closed set of Y is a fuzzy semi pre closed set.

(iii) $f(\beta_{\mathscr{C}} cl(\lambda)) \leq clf(\lambda)$ for each λ of X.

(iv) $\beta_{\mathscr{C}} clf^{-1}(\mu) \leq f^{-1}(cl(\mu))$ for each μ of Y.

Then the implications (i)⇒(ii), (ii)⇒(iii), (iii)⇒(iv) holds.

Proof. (i)⇒(ii)

Let λ be a fuzzy closed set of Y. Then $S\lambda$ is a fuzzy open set of Y. By Definition 5, $f^{-1}(S\lambda)$ is fuzzy semi pre $\mathscr{C}$-open in $X \Rightarrow Sf^{-1}(\lambda)$ is fuzzy semi pre $\mathscr{C}$-open in X. Since fuzzy semi pre $\mathscr{C}$-open and fuzzy semi pre open coincides when complement function is standard complement, $Sf^{-1}(\lambda)$ is fuzzy semi pre open in X. Thus $f^{-1}(\lambda)$ is a fuzzy semi pre closed set of X.

(ii)⇒(iii)

Let λ be a fuzzy subset of X. Then $clf(\lambda)$ is fuzzy closed. By assumption $f^{-1}(clf(\lambda))$ is fuzzy semi pre $\mathscr{C}$-closed.

$\Rightarrow f^{-1}(clf(\lambda)) = \beta_{\mathscr{C}} clf^{-1}(clf(\lambda))$

Since $\lambda \leq f^{-1}f(\lambda), \beta_{\mathscr{C}} cl(\lambda) \leq \beta_{\mathscr{C}} clf^{-1}f(\lambda) \leq \beta_{\mathscr{C}} clf^{-1}(clf(\lambda)) = f^{-1}(clf(\lambda))$.

Thus $f(\beta_{\mathscr{C}} cl(\lambda)) \leq clf(\lambda)$.

(iii)⇒(iv)

Let μ be a fuzzy subset of Y. Then by (c) we have $f(\beta_{\mathscr{C}} clf^{-1}(\mu)) \leq clf(f^{-1}(\mu))$.

$\Rightarrow \beta_{\mathscr{C}} clf^{-1}(\mu) \leq f^{-1}(clf(f^{-1}(\mu))) \leq f^{-1}(cl\mu)$

Thus $\beta_{\mathscr{C}} clf^{-1}(\mu) \leq f^{-1}(cl\mu)$.

□

Theorem 13. *Let $f : (X,\tau) \to (Y,\sigma)$ is fuzzy pre $\mathscr{C}$-continuous, then f is b-$\mathscr{C}$-continuous.*

Proof. Let λ be a fuzzy open set in Y. Since f is fuzzy pre $\mathscr{C}$-continuous, $f^{-1}(\lambda)$ is fuzzy pre $\mathscr{C}$-open set in X which implies $f^{-1}(\lambda)$ is b-$\mathscr{C}$-open. Hence f is fuzzy b-$\mathscr{C}$-continuous. □

Theorem 14. *If $f : (X,\tau) \to (Y,\sigma)$ is fuzzy b-$\mathscr{C}$-continuous, then f is fuzzy semi pre $\mathscr{C}$-continuous.*

Proof. Let λ be a fuzzy open set in Y. Since f is fuzzy b-$\mathscr{C}$-continuous, $f^{-1}(\lambda)$ is fuzzy b-$\mathscr{C}$-open set and hence $f^{-1}(\lambda)$ is fuzzy semi pre $\mathscr{C}$-open in X. Therefore $f^{-1}(\lambda)$ is fuzzy semi pre $\mathscr{C}$-continuous. □

Theorem 15. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function and $\mathscr{C}$ be a complement function. Consider the following statements.*

(i) f is a fuzzy $b^{\#}$-$\mathscr{C}$-continuous function.

(ii) The inverse image of a closed set in Y is fuzzy $b^{\#}$-closed set in X.

(iii) $b^{\#}cl(f^{-1}(\mu))\leq(f^{-1}(cl(\mu)))$ for every fuzzy subset μ of Y.

(iv) $f(b^{\#}_{\mathscr{C}}cl(\lambda))\leq cl(f(\lambda))$ for every fuzzy subset λ of X.

(v) $f^{-1}(int(\mu))\leq b^{\#}_{\mathscr{C}}int(f^{-1}(\mu))$ for every fuzzy subset μ of Y.

Then the implications $(i)\Rightarrow(ii)$, $(ii)\Rightarrow(iii)$, $(iii)\Rightarrow(iv)$, $(iv)\Rightarrow(v)$ and $(v)\Rightarrow(i)$ holds

Proof. $(i)\Rightarrow(ii)$

Let λ be a fuzzy closed set of Y. Then $S\lambda$ is a fuzzy open set of Y. By Definition 5, $f^{-1}(S\lambda)$ is fuzzy $b^{\#}$-$\mathscr{C}$-open in $X\Rightarrow Sf^{-1}(\lambda)$ is fuzzy $b^{\#}$-$\mathscr{C}$-open in X. Since fuzzy $b^{\#}$-$\mathscr{C}$-open and fuzzy $b^{\#}$-open coincides when complement function is standard complement, $Sf^{-1}(\lambda)$ is fuzzy $b^{\#}$-open in X. Thus $f^{-1}(\lambda)$ is a fuzzy $b^{\#}$-closed set of X.

$(ii)\Rightarrow(iii)$

Let μ be any fuzzy subset of Y. Since $cl(\mu)$ is closed in Y, then $f^{-1}(cl(\mu))$ is fuzzy$b^{\#}$-$\mathscr{C}$-closed in X. Therefore $b^{\#}_{\mathscr{C}}cl(f^{-1}(\mu))\leq b^{\#}_{\mathscr{C}}cl(f^{-1}(cl(\mu)))=f^{-1}(cl(\mu))$. Thus $b^{\#}_{\mathscr{C}}cl(f^{-1}(\mu))\leq f^{-1}(cl(\mu))$.

$(iii)\Rightarrow(iv)$

Let λ be any fuzzy subset of X.By (iii), $f^{-1}(cl(f(\lambda)))\geq b^{\#}_{\mathscr{C}}cl(f^{-1}(f(\lambda)))\geq b^{\#}_{\mathscr{C}}cl(\lambda)$. Thus $f(b^{\#}_{\mathscr{C}}cl(\lambda))=clf(\lambda)$.

$(iv)\Rightarrow(v)$

Let μ be any fuzzy subset of Y. By (iv), $f(b^{\#}_{\mathscr{C}}cl(\mathscr{C}(f^{-1}(f(\mu)))\leq cl(f(\mathscr{C}(f^{-1}(\mu))))$.

$\Rightarrow f^{-1}(int_{\mathscr{C}}\mu)\leq b^{\#}_{\mathscr{C}}intf^{-1}(\mu)$.

$(v)\Rightarrow(i)$

Let μ be any fuzzy open subset of Y. Then $f^{-1}(int(\mu))\leq b^{\#}_{\mathscr{C}}int(f^{-1}(\mu))$.

$\Rightarrow f^{-1}(\mu) \leq b^{\#}_{\mathscr{C}}int(f^{-1}(\mu))$

But $b^{\#}_{\mathscr{C}}int(f^{-1}(\mu)) \leq f^{-1}(\mu)$

$\Rightarrow f^{-1}(\mu) = b^{\#}_{\mathscr{C}}int(f^{-1}(\mu))$ Therefore $f^{-1}(\mu)$ is fuzzy $b^{\#}$-$\mathscr{C}$-open in X. Hence f is fuzzy $b^{\#}$-$\mathscr{C}$-continuous function. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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