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APPLICATION OF GENERALIZED INTUITIONISTIC FUZZY MATRIX IN MULTI-CRITERIA DECISION MAKING PROBLEM

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Abstract. The aim of this paper is to investigate the multiple attribute decision making problems to a selected projects with generalized intuitionistic fuzzy information in which the information about weights is completely known and the attributes values are taken from the generalized intuitionistic fuzzydata. Also, extend the technique for order performance by similarity to ideal solution (TOPSIS) for the generalized intuitionistic fuzzy data. In addition, obtained the concept of possibility degree of generalized intuitionistic fuzzy numbers and used to solve ranking alternative in multi-attribute decision making problems.

Keywords: intuitionistic fuzzy set, generalized intuitionistic fuzzy set, generalized intuitionistic fuzzy matrix, generalized intuitionistic fuzzy preference relation, generalized intuitionistic fuzzy TOPSIS method, possibility degree of generalized intuitionistic fuzzy data.

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1. Introduction

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A lot of multiple criteria decision making (MCDM) approaches have been developed and applied to diverse fields, like engineering, management, economics etc. As one of the known classical MCDM approaches, TOPSIS (technique for the order preference by similarity to ideal solution) was first developed by Hwang and Yoon [7]. The primary concept of TOPSIS approach is that the most preferred alternative should not only have the shortest distance from the positive ideal solution but also have the farthest distance from the negative ideal solution. The advantages for the TOPSIS include (a) simple, rationally, comprehensive concept, (b) good computational efficient (c) ability to measure the relative performance for each alternative in a simple mathematical form.

In 1965 Zadeh [16] introduced first the theory of fuzzy sets. Later on, many researchers have been working on the process of dealing with fuzzy decision making problems by applying fuzzy set theory. Atanassov[3, 4] introduced the concept of intuitionistic fuzzy sets (IFSs). IFSs are proposed using two characteristic functions expressing the degree of membership and the degree of non-membership of elements of the universal set to the IFS.

The structure of this paper is organized as follows. In Section 2, introduce the preliminaries and some definition related to intuitionistic fuzzy sets and generalized intuitionistic fuzzy sets. In Section 3, introduce the TOPSIS method MADM problems to a selected projects with generalized intuitionistic fuzzy information. In Section 4, illustrate proposed method with an example. Finally, at the end of this paper a conclusion is given.

2. Preliminaries

Here we recall some preliminaries, definitions of IFSs and GIFSs. Also define Hamming distance of two GIFSs.

0.1. **Fuzzy set and Instuitionistic fuzzy set.**

Definition 1. ***(Fuzzy set (FS))*** *A fuzzy set A in a universal set X is defined as* $A = \{\langle x, \mu_A(x)\rangle | x \in X\}$ *where* $\mu_A : X \to [0, 1]$ *is a mapping called the membership function of the fuzzy set A.*

Definition 2. ***(Instuitionistic fuzzy set (IFS))*** *An instuitionistic fuzzy set (IFS) A over X is an object having the form $A = \langle x, \mu(x), \nu(x) \,|x \in X\rangle$; where $\mu(x) : X \to [0,1]$ and $\nu(x) : X \to [0,1]$. Where $\mu(x)$ and $\nu(x)$ are called the membership and non-membership value of x in A satisfying the condition $0 \leq \mu(x) + \nu(x) \leq 1$.*

An element x of X is called significant with respect to a fuzzy subset A of X if the of membership $\mu_A(x) > 0.5$, otherwise insignificant and non-membership $\nu_A(x) = 1 - \mu_A(x)$ can not be significant. Further, for an IFS $A = \langle x, \mu_A(x), \nu_A(x) \,|x \in X\rangle$ it is observe that $0 \leq \mu(x) + \nu(x) \leq 1$, for all $x \in X$ and hence it is observe that $\mu_A(x) \wedge \nu_A(x) \leq 0.5$, for all $x \in X$.

Definition 3. [8] *A generalized intuitionistic fuzzy set A of X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and degree of nonmembership of the element $x \in X$ to the set A, which is a subset of E and for every $x \in X$ satisfy the condition*

$$\mu_A(x) \wedge \nu_A(x) \leq 0.5, \text{ for all } x \in X.$$

This condition is called generalized intuitionistic fuzzy condition (GIFC).

The maximum value of $\mu_A(x)$ and $\nu_A(x)$ is 1.0, therefore GIFC imply that

$$0 \leq \mu_A(x) \wedge \nu_A(x) \leq 0.5, \text{ for all } x \in X.$$

In GIFSs A there is another parameter is:

$$\pi_A(x) = 1.5 - \mu_A(x) - \nu_A(x)$$

which is known as generalized intuitionistic fuzzy index or hesitation degree of whether x belongs to A or not.

It is obviously seen that for every $x \in X$, $0 \leq \pi_A(x) \leq 1$.

If the value of $\pi_A(x)$ is very small, then knowledge about x is certain; if $\pi_A(x)$ is great, then the knowledge is more uncertain. Obviously, when $\mu_A(x) = 1 - \nu_A(x)$, for all elements of the universe, the traditional fuzzy set concept is recovered.

It may be noted that all GIFs are IFSs but the converse is not true.

Definition 4. *Let A and B be two GIFSs on X, where*

$$A = \{x, \langle \mu_A(x), \nu_A(x) \rangle : x \in X\} \text{ and}$$

$$B = \{x, \langle \mu_B(x), \nu_B(x) \rangle : x \in X\}. \text{ Then,}$$

Hamming distance between GIFSs A and B is given as follows:

$$d(A, B) = \frac{1}{2}[|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|]$$

Definition 5. ***(Generalized intuitionistic fuzzy matrix (GIFM))*** Generalized intuitionistic fuzzy matrix (GIFM) of order $m \times n$ is defined as $A = \langle a_{ij\mu}, a_{ij\nu} \rangle$ where $a_{ij\mu}$ and $a_{ij\nu}$ are the membership value and non-membership value of the ij-th element in A satisfying the condition $0 \leq \mu_A(x) \wedge \nu_A(x) \leq 0.5$, and the maximum value of $\mu_A(x)$ and $\nu_A(x)$ is 1.0,for all i, j.

3. An integrated generalized intuitionistic fuzzy multi criteria decision making method

In this section the TOPSIS method is extended to generalized intuitionistic fuzzy environment, which is a very suitable for solving decision making problem.

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a set of alternatives $C = \{C_1, C_2, \cdots, C_m\}$ be a set of criteria. Generalized intuitionistic fuzzy TOPSIS method consists of the following steps which are given as follows:

(1) Construct an generalized intuitionistic fuzzy preference relation matrix:

Let $B = (b_{ij})_{n \times n}$ be an generalized intuitionistic preference matrix of criteria as follows:

$$B = \begin{bmatrix} \widetilde{b}_{11} & \widetilde{b}_{12} & \widetilde{b}_{13} & \cdots & \widetilde{b}_{1n} \\ \widetilde{b}_{21} & \widetilde{b}_{22} & \widetilde{b}_{23} & \cdots & \widetilde{b}_{2n} \\ \widetilde{b}_{31} & \widetilde{b}_{32} & \widetilde{b}_{33} & \cdots & \widetilde{b}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \cdots \\ \widetilde{b}_{m1} & \widetilde{b}_{m2} & \widetilde{b}_{m3} & \cdots & \widetilde{b}_{mn} \end{bmatrix}$$

where $\widetilde{b}_{ij}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ and satisfies the following condition:

$$(\mu_{ij})^* = \max\left\{\mu_{ij}, \max_p\left\{\frac{\mu_{ip}\mu_{pj}}{\mu_{ip}\mu_{pj} + (1.5 - \mu_{ip})(1.5 - \mu_{pj})}\right\}\right\} \tag{1}$$

$$(\nu_{ij})^* = \max\left\{\nu_{ij}, \max_p\left\{\frac{\nu_{ip}\nu_{pj}}{\nu_{ip}\nu_{pj} + (1.5 - \nu_{ip})(1.5 - \nu_{pj})}\right\}\right\} \tag{2}$$

where $(\mu_{ij})^*$ and $(\nu_{ij})^*$, the element of $(B)^*$ matrix, are the membership and non-membership degree of the alternative x_i over x_j respectively and $0 \le (\mu_{ij})^* + (\nu_{ij})^* \le 1.5$ for all i, j, $k = 1, 2, \cdots, n$, then we call $(B)^*$ a multiplicative consistent generalized intuitionistic fuzzy preference relation. If $(B)^*$ does not satisfy the condition of $0 \le (\mu_{ij})^* + (\nu_{ij})^* \le 1.5$ for all i, j, $k = 1, 2, \cdots, n$, then we call $(B)^*$ an inconsistent multiplicative generalized intuitionistic fuzzy preference relation.

(2) Obtain the priority vector of criteria.

After obtained aggregated generalized intuitionistic fuzzy preference matrix, the priority vector of criteria $w = (w_1, w_2, \cdots, w_n)^T$ can be estimated with the following equation

$$w_j = [w_j^L, w_j^U] = \left(\frac{1}{\sum_{j=1}^n \left(\frac{(1.5-\widetilde{\mu}_{ij}^*)}{\mu_{ij}^*}\right)}, \frac{1}{\sum_{j=1}^n \left(\frac{\nu_{ij}^*}{(1.5-\widetilde{\nu}_{ij}^*)}\right)}\right) \tag{3}$$

(3) Construct a generalized intuitionistic fuzzy decision matrix:

$\widetilde{R} = (\widetilde{r}_{ij})_{m\times n}$ is an generalized intuitionistic fuzzy decision matrix such that Let $B = (b_{ij})_{n\times n}$ be an generalized intuitionistic preference matrix of criteria as follows:

$$R = \begin{bmatrix} \widetilde{r}_{11} & \widetilde{r}_{12} & \widetilde{r}_{13} & \cdots & \widetilde{r}_{1n} \\ \widetilde{r}_{21} & \widetilde{r}_{22} & \widetilde{r}_{23} & \cdots & \widetilde{r}_{2n} \\ \widetilde{r}_{31} & \widetilde{r}_{32} & \widetilde{r}_{33} & \cdots & \widetilde{r}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{m1} & \widetilde{r}_{m2} & \widetilde{r}_{m3} & \cdots & \widetilde{r}_{mn} \end{bmatrix}$$

where $r_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$, $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$, which contained in an generalized intuitionistic fuzzy decision matrix.

(4) Determine the generalized intuitionistic fuzzy positive ideal solution and the generalized intuitionistic fuzzy negative ideal solution:

Let J_1 be the set of benefit criteria, J_2 be the set of cost criteria, A^* be the generalized intuitionistic fuzzy positive ideal solution and A^- be the generalized intuitionistic fuzzy negative ideal solution, then A^* and A^- can be determined respectively as:

$$A^* = (r_1^*, r_2^*, \cdots, r_n^*), r_j^* = (\mu_j^*, \nu_j^*, \pi_j^*), j = 1, 2, \cdots, n \quad (4)$$

$$A^- = (r_1^-, r_2^-, \cdots, r_n^-), r_j^- = (\mu_j^*, \nu_j^-, \pi_j^-), j = 1, 2, \cdots, n \quad (5)$$

$$\mu_{ij}^* = \left\{ \left(\max_i \{\mu_{ij}\} | j \in J_1 \right), \left(\min_i \{\mu_{ij}\} | j \in J_2 \right) \right\} \quad (6)$$

$$\nu_{ij}^* = \left\{ \left(\min_i \{\nu_{ij}\} | j \in J_1 \right), \left(\max_i \{\nu_{ij}\} | j \in J_2 \right) \right\} \quad (7)$$

$$\mu_{ij}^- = \left\{ \left(\min_i \{\mu_{ij}\} | j \in J_1 \right), \left(\max_i \{\mu_{ij}\} | j \in J_2 \right) \right\} \quad (8)$$

$$\nu_{ij}^- = \left\{ \left(\max_i \{\nu_{ij}\} | j \in J_1 \right), \left(\min_i \{\nu_{ij}\} | j \in J_2 \right) \right\} \quad (9)$$

$$\begin{aligned} \pi_{ij}^* = \Big\{ & \Big(1.5 - \max_i \{\mu_{ij}\} - \min_i \{\nu_{ij}\} | j \in J_1 \Big), \\ & \Big(1.5 - \min_i \{\mu_{ij}\} - \max_i \{\nu_{ij}\} | j \in J_2 \Big) \Big\} \quad (10) \\ \pi_{ij}^- = \Big\{ & \Big(1.5 - \min_i \{\mu_{ij}\} - \max_i \{\nu_{ij}\} | j \in J_1 \Big), \\ & \Big(1.5 - \max_i \{\mu_{ij}\} - \min_i \{\nu_{ij}\} | j \in J_2 \Big) \Big\} \quad (11) \end{aligned}$$

(5) Calculate the weighted separation measure:

The weighted Hammining distance is used to obtain separation measures. The weighted lower and upper separation measures $(S_i^*)^L$, $(S_i^*)^U$ and $(S_i^-)^L$, $(S_i^-)^U$ of each alternative from the generalized intuitionistic fuzzy positive ideal solution and the generalized intuitionistic fuzzy negative ideal solution are respectively calculated as follows:

$$(S_i^*)^L = \frac{1}{2}\sum_{j=1}^{n} w_j^L[|\mu_{ij} - \mu_j^*| + |\nu_{ij} - \nu_j^*| + |\pi_{ij} - \pi_j^*|] \tag{12}$$

$$(S_i^*)^U = \frac{1}{2}\sum_{j=1}^{n} w_j^U[|\mu_{ij} - \mu_j^*| + |\nu_{ij} - \nu_j^*| + |\pi_{ij} - \pi_j^*|] \tag{13}$$

$$(S_i^-)^L = \frac{1}{2}\sum_{j=1}^{n} w_j^L[|\mu_{ij} - \mu_j^-| + |\nu_{ij} - \nu_j^-| + |\pi_{ij} - \pi_j^-|] \tag{14}$$

$$(S_i^-)^U = \frac{1}{2}\sum_{j=1}^{n} w_j^U[|\mu_{ij} - \mu_j^-| + |\nu_{ij} - \nu_j^-| + |\pi_{ij} - \pi_j^-|] \tag{15}$$

(6) Calculate the relative closeness coefficient of each alternative to the generalized intuitionistic fuzzy positive and negative ideal solutions:

The relative closeness coefficient of an alternative A_i with respect to the generalized intuitionistic fuzzy positive ideal solution A^* and the generalized intuitionistic fuzzy negative ideal solution A^- is defined as follows:

$$((C_i^*)^L, (C_i^*)^U) = \left\{\Big(\frac{(S_i^-)^L}{(S_i^*)^U + (S_i^-)^U}\Big), \Big(\frac{(S_i^-)^U}{(S_i^*)^L + (S_i^-)^L}\Big)\right\} \tag{16}$$

(7) Rank the alternative according to the descending order of the relative closeness coefficients $C_i^* = ((C_i^*)^L, (C_i^*)^U)$.

In order to rank alternatives the possibility degree formula is used.

Definition 6. *Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$ be two interval numbers where is $0 \leq a^L \leq a^U \leq 1$ and $0 \leq b^L \leq b^U \leq 1$ then the possibility degree of $a \geq b$ is defined as:*

$$p(a \geq b) = \max\left\{1 - \max\Big(\frac{b^U - a^L}{b^U - b^L + a^U - a^L}, 0\Big), 0\right\} \tag{17}$$

that is a superior to b to degree of, denoted by $a \succ^{p(a\geq b)} b$.

Similarly, the degree of possibility $b \geq a$ is defined as:

$$p(b \geq a) = \max\left\{1 - \max\Big(\frac{a^U - b^L}{b^U - b^L + a^U - a^L}, 0\Big), 0\right\} \tag{18}$$

that is b superior to a to degree of, denoted by $b_{\succ}^{p(b\geq a)} a$.

Let $P = p_{ij} = p(a_i \geq a_j)$ be the complementary generalized intuitionistic fuzzy matrix and given as follows:

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n12} & p_{n3} & \cdots & p_{nn} \end{pmatrix}.$$

where $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = .5$ and i, $j = 1, 2, \cdots, n$.

Summing all elements in each line of matrix P, then:

$$p_i = \sum_{j=1}^{n} p_{ij} \quad i, j = 1, 2, \cdots, n.$$

Alternatives are ranked according to descending order of p_i.

4. Illustrate Example

A manufacturing company is select to location for building new plant. There are four candidates place A_1, A_2, A_3 and A_4 are chosen for further evaluation. In order to evaluate candidate locations, expansion possibility (C_1), availability of acquirement material (C_2), distance to market (C_3) and labour cost (C_4) are considered as evaluation factor.

(1) Construct an generalized intuitionistic fuzzy preference matrix.

Let $B = (b_{ij})_{4\times 4}$ generalized intuitionistic fuzzy preference relation matrix

$$B = \begin{pmatrix} \langle 0.80, 0.40\rangle & \langle 0.70, 0.50\rangle & \langle 0.80, 0.20\rangle & \langle 0.60, 0.50\rangle \\ \langle 0.70, 0.40\rangle & \langle 0.80, 0.50\rangle & \langle 0.70, 0.30\rangle & \langle 0.60, 0.50\rangle \\ \langle 0.60, 0.40\rangle & \langle 0.90, 0.30\rangle & \langle 0.60, 0.30\rangle & \langle 0.50, 0.50\rangle \\ \langle 0.40, 0.60\rangle & \langle 0.50, 0.40\rangle & \langle 0.70, 0.30\rangle & \langle 0.80, 0.30\rangle \end{pmatrix}$$

Here $B = (b_{ij})_{4\times 4}$ has been consistent generalized intuitionistic fuzzy preference relation matrix due to satisfying the condition 1-2 and the matrix existing as follows:

$$B^* = \begin{pmatrix} \langle 0.80, 0.40\rangle & \langle 0.70, 0.50\rangle & \langle 0.80, 0.20\rangle & \langle 0.60, 0.50\rangle \\ \langle 0.70, 0.40\rangle & \langle 0.80, 0.50\rangle & \langle 0.70, 0.30\rangle & \langle 0.60, 0.50\rangle \\ \langle 0.60, 0.40\rangle & \langle 0.90, 0.30\rangle & \langle 0.60, 0.30\rangle & \langle 0.50, 0.50\rangle \\ \langle 0.40, 0.60\rangle & \langle 0.50, 0.40\rangle & \langle 0.70, 0.30\rangle & \langle 0.80, 0.30\rangle \end{pmatrix}.$$

(2) Obtain the priority vector of criteria.

The priority vector of criteria has been estimated by utilizing Eq.3 as follows: $w_1 = \langle 0.228, 0.659\rangle$, $w_2 = \langle 0.215, 0.620\rangle$, $w_3 = \langle 0.208, 0.733\rangle$ and $w_4 = \langle 0.148, 0.653\rangle$

(3) Construct the generalized intuitionistic fuzzy decision matrix.

The generalized intuitionistic fuzzy decision matrix has been constructed as follows:

Candidates	*Criteria*			
	C_1	C_2	C_3	C_4
A_1	$\langle 0.74, 0.44, 0.32\rangle$	$\langle 0.72, 0.38, 0.40\rangle$	$\langle 0.64, 0.44, 0.62\rangle$	$\langle 0.69, 0.45, 0.36\rangle$
A_2	$\langle 0.85, 0.42, 0.23\rangle$	$\langle 0.78, 0.45, 0.27\rangle$	$\langle 0.71, 0.36, 0.43\rangle$	$\langle 0.82, 042, 0.26\rangle$
A_3	$\langle 0.76, 0.36, 0.38\rangle$	$\langle 0.66, 0.38, 0.46\rangle$	$\langle 0.48, 0.65, 0.37\rangle$	$\langle 0.70, 0.40, 0.40\rangle$
A_4	$\langle 0.78, 0.46, 0.26\rangle$	$\langle 0.75, 0.42, 0.30\rangle$	$\langle 0.68, 0.48, 0.34\rangle$	$\langle 0.75, 0.35, 0.40\rangle$

(4) Determine the generalized intuitionistic fuzzy positive ideal solution and the generalized intuitionistic fuzzy negative ideal solution.

Considering that expansion possibility, availability of acquirement material are the benefit criteria $J_1 = \{C_1, C_2\}$ and distance to the market and labour cost are the cost criterion $J_2 = \{C_3, C_4\}$. Then the generalized intuitionistic fuzzy positive ideal solutions and generalized intuitionistic fuzzy positive ideal solutions have been obtained by employing Eq.4- Eq.11 as follows:

$$A^* = \begin{cases} \langle 0.85, 0.23, 0.42\rangle, & \langle 0.78, 0.38, 0.34\rangle; \\ \langle 0.48, 0.65, 0.37\rangle, & \langle 0.69, 0.45, 0.36\rangle. \end{cases}$$

$$\text{and} \quad A^- = \begin{cases} \langle 0.74, 0.46, 0.30\rangle, & \langle 0.66, 0.45, 0.39\rangle; \\ \langle 0.71, 0.36, 0.43\rangle, & \langle 0.82, 0.35, 0.33\rangle. \end{cases}$$

(5) Calculate the weighted separation measures.

Generalized intuitionistic fuzzy negative and positive separation measures based on the weighted lower and upper Hamming distance for each candidate have been calculated by utilizing Eq.12-Eq.15 and given in

Candidates	$(S^*)^L$	$(S^*)^U$	$(S^-)^L$	$(S^-)^U$
A_1	0.176	0.598	0.089	0.315
A_2	0.155	0.514	0.090	0.317
A_3	0.191	0.641	0.154	0.449
A_4	0.254	0.812	0.146	0.452

(6) Calculate the relative closeness coefficient of each candidate to the generalized intuitionistic fuzzy positive ideal solution and the generalized intuitionistic fuzzy negative ideal solutions. The relative closeness coefficients of each candidate to the generalized intuitionistic fuzzy positive ideal solutions and generalized intuitionistic fuzzy positive ideal solutions have been calculated by using Eq.16 as follows:

$$\left((C_1^*)^L, (C_1^*)^U\right) = \left\{(\frac{0.089}{0.598 + 0.315}), (\frac{0.315}{0.176 + 0.089})\right\} = (0.097, 1.189)$$

$$\left((C_2^*)^L, (C_2^*)^U\right) = \left\{(\frac{0.090}{0.514 + 0.317}), (\frac{0.317}{0.155 + 0.090})\right\} = (0.108, 1.294)$$

$$\left((C_3^*)^L, (C_3^*)^U\right) = \left\{(\frac{0.154}{0.641 + 0.449}), (\frac{0.449}{0.191 + 0.154})\right\} = (0.141, 1.301)$$

$$\left((C_4^*)^L, (C_4^*)^U\right) = \left\{(\frac{0.146}{0.812 + 0.452}), (\frac{0.452}{0.146 + 0.254})\right\} = (0.088, 1.130)$$

(7) Rank the candidates according to the descending order of the relative closeness coefficients. Four candidate locations have been ranked according to the descending order of relative closeness coefficients. The candidates have been ranked by using the possibility degree formula and the following matrix has been constructed as

follows :

$$P = \begin{pmatrix} 0.50 & 0.47 & 0.47 & 0.52 \\ 0.53 & 0.50 & 0.49 & 0.54 \\ 0.53 & 0.51 & 0.50 & 0.55 \\ 0.48 & 0.47 & 0.45 & 0.50 \end{pmatrix}$$

Summing all elements in each line of matrix P, then:

$$p_1 = 1.96, \quad p_2 = 2.06, \quad p_3 = 2.09 \quad \text{and} \quad p_4 = 1.90.$$

The candidates have been ranked as A_3, A_2, A_1 and A_4 according to the descending order p_i, $i = 1, 2, 3, 4$. Thus A_3 has been selected s the most desirable facility location among the candidates.

5. Conclusion

The success of companies depends on their capability on making right strategic decisions. Facility location selection is one of these strategic decisions, which it is a costly and difficult to reverse activity for companies. Therefore, this has presented the integration of generalized intuitionistic fuzzy preference relation and generalized fuzzy intuitionistic TOPSIS method for selecting the most desirable facility location. the generalized intuitionistic fuzzy preference relation has been applied to derive the weights of criteria and generalized intuitionistic fuzzy TOPSIS method has been used to rank alternative. The intregrated generalized intuitionistic fuzzy multi-criteria decision making method has enormous chances of success for multi-criteria decision making problems due to great superiority on dealing with uncertainty information. In future, the proposed method can be used for dealing with uncertainty in a variety of multi-criteria decision making problems.

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