Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 1, 173-182
https://doi.org/10.28919/jmcs/5057
ISSN: 1927-5307

COINCIDENCE AND FIXED POINTS OF NONEXPANSIVE TYPE SINGLE VALUED MAPS

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Abstract. Fixed point theory of non-expansive mapping confers techniques for solving a variety of applied problems in engineering and mathematical sciences. The purpose of this paper is to establish some coincidence and fixed-point theorems for non-expansive type single valued mappings in S-metric space.

2010 Mathematics Subject Classifications: 47H10, 54H25.

Keywords: coincidence and fixed points; non-expansive mappings; compatible mappings.

1. INTRODUCTION AND PRELIMINARIES

The importance of non-expansive mappings was outlined, e.g., in 1980 by Bruck [4]. A non-expansive mapping of a complete metric space need not have a fixed point (e.g. translation operator in a Banach space). A fixed point of a non-expansive mapping need not be unique (e.g. $f = I$). The

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Received September 24, 2020

study of non-expansive mappings has been one of the main features in recent developments of fixed-point theory. For more details, the reader is referred to [1, 2, 3, 5, 7, 8, 9, 10]

The study of metric fixed point has played very vital role with the applications in mathematics and applied science. Later it was found very essential to generalize the notion named metric space by many researchers. With this respect many generalizations have come in the metric space like, D-metric space, 2-metric space, S-metric space, cone metric space, fuzzy metric space and manger space, etc. Particularly, Sedghiet al. [11] introduced a new generalized metric space called an S-metric space.

Definition 1.1 (see [11]) Let X be a non-empty set. An S-metric on X is a function $S: X \times X \times X \rightarrow [0, +\infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,

(S1). $S(x, y, z) \geq 0$,

(S2). $S(x, y, z) = 0$ if and only if $x = y = z$,

(S3). $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

Then S is called an S-metric on X and (X, S) is called an S-metric space.

Lemma 1.2 (see [11]) Let (X, S) be an S-metric space. Then $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$.

Definition 1.3 (see [11]) Let X be an S-metric space.

(1). A sequence $\{x_n\}$ converges to x if and only if $\lim_{n\to\infty} S(x_n, x_n, x) = 0$. That is, for each $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0, S(x_n, x_n, x) < \epsilon$ and we denote this by $\lim_{n\to\infty} x_n = x$.

(2). A sequence $\{x_n\}$ is called a Cauchy if $\lim_{n,m\to\infty} S(x_n, x_n, x_m) = 0$. That is, for each $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for all, $m \geq n_0, S(x_n, x_n, x_m) < \epsilon$.

(3). X is called complete if every Cauchy sequence in X is convergent.

From (see [11], Examples in page 260), we have the following.

Example 1.4

(a) Let $\mathbb{R}$ be the real line. Then $S(x,y,z) = |x-z| + |y-z|$ for all $x,y,z \in \mathbb{R}$, is an S-metric on $\mathbb{R}$. This S-metric is called the usual S-metric on $\mathbb{R}$. Furthermore, the usual S-metric space $\mathbb{R}$ is complete

(b) Let Y be a non-empty set of $\mathbb{R}$. Then $S(x,y,z) = |x-z| + |y-z|$ for all $x,y,z \in Y$, is an S-metric on Y. If Y is a closed subset of the usual metric space $\mathbb{R}$, then the S-metric space Y is complete.

Lemma 1.5 (see [11]) Let (X,S) be an S-metric space. If the sequence $\{x_n\}$ in X converges to x, then x is unique.

Lemma 1.6 (see [11]) Let (X,S) be an S-metric space. If $x_n \longrightarrow x$ and $y_n \longrightarrow y$. Then $S(x_n,x_n,y_n) \longrightarrow S(x,x,y)$.

Definition 1.7 (see [11]) Let (X,S) be an S-metric space. A mapping $f\colon X \to X$ is called Lipschitzian if there exists a number $k \geq 0$ such that $S(fx,fx,fy) \leq kS(x,x,y), \forall\, x,y \in X$.The mapping f is called contractive if $k < 1$.

2. MAIN RESULTS

Now, we introduce the concept of nonexpansive mapping in S-metric spaces.

Definition 2.1 Let (X,S) be an S-metric space. A mapping $f\colon X \to X$ is called nonexpansive if there exists a number $k = 1$ such that $S(fx,fx,fy) \leq kS(x,x,y)$ for all $x,y \in X$.

In this paper we use the following nonexpansive type condition: Let $T,f\colon X \to X$ be two self mappings satisfying the condition,

$$\begin{aligned} S(Tx,Tx,Ty) \leq\ & a(x,y)\, S(fx,fx,fy) + b(x,y) \max\{S(fx,fx,Tx), S(fy,fy,Ty)\} \\ & + c(x,y) \max\{S(fx,fx,fy), S(fx,fx,Tx), S(fy,fy,Ty)\} \\ & + e(x,y) \max\{S(fx,fx,fy), S(fx,fx,Tx), S(fy,fy,Ty), S(fx,fx,Ty)\} \quad (2.1) \end{aligned}$$

where $a(x,y), b(x,y), c(x,y), e(x,y) \geq 0$ and

$$\delta = \inf_{x,y\in X} e(x,y) > 0, \qquad \vartheta = \inf_{x,y\in X}\big(2b(x,y) + 4e(x,y)\big) > 0$$

with $\sup_{x,y\in X}\big(a(x,y)+b(x,y)+c(x,y)+3e(x,y)\big)=1.$

Motivated by [6], we introduce the following.

Definition 2.2 Let f and g be two self maps of a S-metric space X. Then f and g are said to be compatible if $\lim_{n\to\infty} S(fgx_n, fgx_n, gfx_n)=0$, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=t\in X$.

Our first main results as follows.

Theorem 2.1 Let (X,S) be an S-metric space, T,f are self maps of X satisfying (2.1) with $T(X)\subseteq f(X)$ and either (a) X is complete and f is surjective; or (b) X is complete, f is continuous and T,f are compatible; or (c) $f(X)$ is complete ; or (d) $T(X)$ is complete. Then f and T have a coincidence point in X. Further, the coincidence value is unique, i.e. $fp=fq$ whenever $fp=Tp$ and $fq=Tq$ $(p,q\in X)$.

Proof Let $x_0\in X$ be arbitrary. Since $T(X)\subseteq f(X)$, choose x_1 such that $y_1=fx_1=Tx_0$. In general, choose $x_{n+1}=fx_{n+1}=Tx_n$. From (2.1), we have

$$
\begin{aligned}
&S(Tx_n,Tx_n,Tx_{n+1})\\
&\le a\,S(fx_n,fx_n,fx_{n+1})+b\max\{S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+c\max\{S(fx_n,fx_n,fx_{n+1}),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+e\max\left\{\begin{matrix}S(fx_n,fx_n,fx_{n+1}),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1}),\\ S(fx_n,fx_n,Tx_{n+1})\end{matrix}\right\}\\
&\le a\,S(fx_n,fx_n,Tx_n)+b\max\{S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+c\max\{S(fx_n,fx_n,Tx_n),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+e\max\left\{\begin{matrix}S(fx_n,fx_n,Tx_n),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1}),\\ [S(fx_n,fx_n,Tx_n)+S(fx_n,fx_n,Tx_n)+S(Tx_{n+1},Tx_{n+1},Tx_n)]\end{matrix}\right\}\\
&=a\,S(fx_n,fx_n,Tx_n)+b\max\{S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+c\max\{S(fx_n,fx_n,Tx_n),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1})\}\\
&+e\max\left\{\begin{matrix}S(fx_n,fx_n,Tx_n),S(fx_n,fx_n,Tx_n),S(fx_{n+1},fx_{n+1},Tx_{n+1}),\\ [2S(fx_n,fx_n,Tx_n)+S(Tx_n,Tx_n,Tx_{n+1})]\end{matrix}\right\}
\end{aligned}
$$

$$= a\, S(fx_n, fx_n, Tx_n) + b \max\{S(fx_n, fx_n, Tx_n), S(fx_{n+1}, fx_{n+1}, Tx_{n+1})\}$$
$$+c \max\{S(fx_n, fx_n, Tx_n), S(fx_n, fx_n, Tx_n), S(fx_{n+1}, fx_{n+1}, Tx_{n+1})\}$$
$$+e \max\left\{\begin{matrix} S(fx_n, fx_n, Tx_n), S(fx_n, fx_n, Tx_n), S(fx_{n+1}, fx_{n+1}, Tx_{n+1}), \\ [2S(fx_n, fx_n, Tx_n) + S(fx_{n+1}, fx_{n+1}, Tx_{n+1})] \end{matrix}\right\}$$

where a, b, c and e are evaluated at (x_n, x_{n+1}). Suppose that for some n, $S(fx_{n+1}, fx_{n+1}, Tx_{n+1}) > S(fx_n, fx_n, Tx_n)$. Then substituting in the above inequality we have

$$S(fx_{n+1}, fx_{n+1}, Tx_{n+1}) \le (a + b + c + 3e) S(fx_{n+1}, fx_{n+1}, Tx_{n+1})$$

a contradiction. Therefore, for all n we have

$$S(fx_{n+1}, fx_{n+1}, Tx_{n+1}) \le S(fx_n, fx_n, Tx_n). \qquad (2.2)$$

Again $S(y_{n-1}, y_{n-1}, Tx_n) = S(Tx_{n-2}, Tx_{n-2}, Tx_n)$

Using (2.1), we have

$$S(y_{n-1}, y_{n-1}, Tx_n) \le a\, S(fx_{n-2}, fx_{n-2}, fx_n) + b \max\{S(fx_{n-2}, fx_{n-2}, Tx_{n-2}), S(fx_n, fx_n, Tx_n)\}$$
$$+c \max\{S(fx_{n-2}, fx_{n-2}, fx_n), S(fx_{n-2}, fx_{n-2}, Tx_{n-2}), S(fx_n, fx_n, Tx_n)\}$$
$$+e \max\{S(fx_{n-2}, fx_{n-2}, fx_n), S(fx_{n-2}, fx_{n-2}, Tx_{n-2}), S(fx_n, fx_n, Tx_n), S(fx_{n-2}, fx_{n-2}, Tx_n)\}$$

Since triangle inequality and (2.2), we have

$$S(fx_{n-2}, fx_{n-2}, fx_n)$$
$$\le S(fx_{n-2}, fx_{n-2}, fx_{n-1}) + S(fx_{n-2}, fx_{n-2}, fx_{n-1}) + S(fx_n, fx_n, fx_{n-1})$$
$$= 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + S(fx_{n-1}, fx_{n-1}, fx_n)$$
$$= 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + S(fx_{n-1}, fx_{n-1}, Tx_{n-1})$$
$$\le 3S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

and

$$S(fx_{n-2}, fx_{n-2}, Tx_n)$$
$$\le S(fx_{n-2}, fx_{n-2}, fx_{n-1}) + S(fx_{n-2}, fx_{n-2}, fx_{n-1}) + S(Tx_n, Tx_n, fx_{n-1})$$
$$= 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + S(fx_{n-1}, fx_{n-1}, Tx_n)$$
$$\le 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + 2S(fx_{n-1}, fx_{n-1}, Tx_{n-1}) + S(Tx_n, Tx_n, Tx_{n-1})$$
$$= 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + 2S(fx_{n-1}, fx_{n-1}, Tx_{n-1}) + S(Tx_{n-1}, Tx_{n-1}, Tx_n)$$

$$= 2S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + 2S(fx_{n-1}, fx_{n-1}, Tx_{n-1}) + S(fx_n, fx_n, Tx_n)$$

$$\leq 5S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

The last inequality gives

$$S(y_{n-1}, y_{n-1}, Tx_n) \leq (3a + b + 3c + 5e)\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

implies that

$$S(fx_{n-1}, fx_{n-1}, Tx_n) \leq \big(3 - 2(b + 2e)\big)\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) \qquad (2.3)$$

Using (2.1), (2.2) and (2.3), we obtain

$$S(y_n, y_n, Tx_n) = S(Tx_{n-1}, Tx_{n-1}, Tx_n)$$

$$\leq a\, S(fx_{n-1}, fx_{n-1}, fx_n) + b \max\{S(fx_{n-1}, fx_{n-1}, Tx_{n-1}), S(fx_n, fx_n, Tx_n)\}$$

$$+c \max\{S(fx_{n-1}, fx_{n-1}, fx_n), S(fx_{n-1}, fx_{n-1}, Tx_{n-1}), S(fx_n, fx_n, Tx_n)\}$$

$$+e \max\left\{\begin{matrix} S(fx_{n-1}, fx_{n-1}, fx_n), S(fx_{n-1}, fx_{n-1}, Tx_{n-2}), S(fx_n, fx_n, Tx_n), \\ S(fx_{n-1}, fx_{n-1}, Tx_n) \end{matrix}\right\}$$

$$= a\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + b\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$+cS(fx_{n-2}, fx_{n-2}, Tx_{n-2}) + e\big(3 - 2(b - 2e)\big)\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$= \Big(a + b + c + e\big(3 - 2(b + 2e)\big)\Big)\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$\leq \big(a + b + c + 3e - 2e(b + 2e)\big)\, S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$= \big(1 - 2e(b + 2e)\big) S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$\leq (1 - \delta\vartheta) S(fx_{n-2}, fx_{n-2}, Tx_{n-2})$$

$$\leq (1 - \delta\vartheta)^{\frac{n}{2}} S(y_0, y_0, y_1)$$

where $\delta = \inf_{x,y \in X} e(x, y) > 0$, $\vartheta = \inf_{x,y \in X}\big(2b(x, y) + 4e(x, y)\big) > 0$ and $\{y_n\}$ is Cauchy, hence converges to a point p in X.

Case (a): Suppose that f is surjective. Then there exists a point z in X such that $p = f\,z$. From (2.1), we have

$$S(fz, fz, Tz) \leq S(fz, fz, y_{n+1}) + S(fz, fz, y_{n+1}) + S(Tz, Tz, y_{n+1})$$

$$= S(fz, fz, y_{n+1}) + S(fz, fz, y_{n+1}) + S(Tx_n, Tx_n, Tz)$$

$$\leq S(fz, fz, y_{n+1}) + S(fz, fz, y_{n+1}) + a\,S(fx_n, fx_n, fz)$$

$$+b\max\{S(fx_n, fx_n, Tx_n), S(fz, fz, Tz)\}$$

$$+c\max\{S(fx_n, fx_n, fz), S(fx_n, fx_n, Tx_n), S(fz, fz, Tz)\}$$

$$+e\ \max\{S(fx_n, fx_n, fz), S(fx_n, fx_n, Tx_n), S(fz, fz, Tz), S(fx_n, fx_n, Tz)\}$$

$$\leq 2S(fz, fz, y_{n+1}) + \sup_{x,y\in X}(b+c+e)\max\left\{\max\begin{Bmatrix} S(fx_n, fx_n, Tx_n), \\ S(fz, fz, Tz) \end{Bmatrix},\right.$$

$$\max\begin{Bmatrix} S(fx_n, fx_n, fz), S(fx_n, fx_n, Tx_n), \\ S(fz, fz, Tz) \end{Bmatrix},$$

$$\left.\max\begin{Bmatrix} S(fx_n, fx_n, fz), S(fx_n, fx_n, Tx_n), \\ S(fz, fz, Tz), S(fx_n, fx_n, Tz) \end{Bmatrix}\right\} + \sup_{x,y\in X} a\ S(fx_n, fx_n, fz)$$

Taking limit as $n \to \infty$, we get $S(fz, fz, Tz) \leq \sup_{x,y\in X}(b+c+e)$ implies that $fz = Tz$.

Case (b): Suppose f is continuous and f and T are compatible. Then since $\lim_{n\to\infty} y_p = p$, we have $\lim_{n\to\infty} fy_p = fp$. Now,

$$S(fp, fp, Tp) \leq S(fp, fp, fy_{n+1}) + S(fp, fp, fy_{n+1}) + S(Tp, Tp, fy_{n+1})$$

$$= 2S(fp, fp, fy_{n+1}) + S(Tp, Tp, fTx_n)$$

$$= 2S(fp, fp, fy_{n+1}) + S(fTx_n, fTx_n, Tp)$$

$$\leq 2S(fp, fp, fy_{n+1}) + 2S(fTx_n, fTx_n, Tfx_n) +$$

$$S(Tp, Tp, Tfx_n) = 2S(fp, fp, fy_{n+1}) + 2S(fTx_n, fTx_n, Tfx_n) + S(Tfx_n, Tfx_n, Tp)$$

Note that since $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Tx_n$ and f, T are compatible, $\lim_{n\to\infty} S(Tfx_n, fTx_n, fTx_n) = 0$. From (2.1), we have

$$S(Tfx_n, Tfx_n, Tp) \leq a\,S(ffx_n, ffx_n, fp) + b\max\{S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp)\}$$

$$+c\max\{S(ffx_n, ffx_n, fp), S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp)\}$$

$$+e\max\{S(ffx_n, ffx_n, fp), S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp), S(ffx_n, ffx_n, Tp)\}$$

$$\leq \sup_{x,y\in X} a\,S(ffx_n, ffx_n, fp) + \sup_{x,y\in X}(b+c+e)\max\{\max\{S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp)\},$$

$$\max\{S(ffx_n, ffx_n, fp), S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp)\},$$

$$\max\{S(ffx_n, ffx_n, fp), S(ffx_n, ffx_n, Tfx_n), S(fp, fp, Tp), S(ffx_n, ffx_n, Tp)\}\}$$

Note that

$$S(ffx_n, ffx_n, Tfx_n) \leq S(ffx_n, ffx_n, fTx_n) + S(ffx_n, ffx_n, fTx_n) + S(Tfx_n, Tfx_n, fTx_n)$$

Using the continuity of f and compatibility of f and T, it follows that $\lim_{n\to\infty} S(Tfx_n, fTx_n, fTx_n) = 0$. Since $\lim_{n\to\infty} ffx_n = fp$, it follows that $\lim_{n\to\infty} Tfx_n = fp$.

Substituting into the above inequality and taking limit as $n \to \infty$, we get

$$S(fp, fp, Tp) \leq \sup_{x,y\in X} (b + c + e)\, S(fp, fp, Tp)$$

implies that $fp = Tp$.

Case (c): In this case $p \in f(X)$. Let $z \in f^{-1}\, p$. Then $p = fz$ and the proof is complete by case (a).

Case (d): In this case $p \in T(X) \subseteq f(X)$ and the proof is complete by case (c).

Uniqueness: Let q be another coincidence point of f and T, then from (2.1) with a, b, c and d evaluated at (p, q),

$$\begin{aligned} S(Tp, Tp, Tq) &\leq a\, S(fp, fp, fq) + b \max\{S(fp, fp, Tp), S(fq, fq, Tq)\} \\ &\quad + c \max\{S(fp, fp, fq), S(fp, fp, Tp), S(fq, fq, Tq)\} \\ &\quad + e \max\{S(fp, fp, fq), S(fp, fp, Tp), S(fq, fq, Tq), S(fp, fp, Tq)\} \\ &\leq (a + c + e\,) S(Tp, Tp, Tq) \end{aligned}$$

This implies that $Tp = Tq$ and hence $fp = fq$.

Corollary 2.2 Let (X, S) be a complete S-metric space and T a self mapping of X satisfying (2.1) with $f = I$, the identity map on X and $\sup_{x,y\in X} \big(a(x,y) + 5b(x,y) + 3c(x,y) + 3e(x,y)\big) = 1$. Then T has a unique fixed point and at this fixed point, T is continuous.

Proof The existence and uniqueness of the fixed point comes from Theorem 2.1 by setting $f = I$. To prove continuity , let $\{y_n\} \subset X$ with $\lim_{n\to\infty} y_n = p$, p the unique fixed point of T. Using (2.1), we have

$$S(Ty_n, Ty_n, Tp) \leq a\, S(y_n, y_n, p) + b \max\{S(y_n, y_n, Ty_n), S(p, p, Tp)\}$$

$$+c\max\{S(y_n,y_n,p),S(y_n,y_n,Ty_n),S(p,p,Tp)\}$$

$$+e\max\{S(y_n,y_n,p),S(y_n,y_n,Ty_n),S(p,p,Tp),S(y_n,y_n,Tp)\}$$

$$\leq a\,S(y_n,y_n,p)+b\max\{S(y_n,y_n,Ty_n),S(p,p,p)\}$$

$$+c\max\{S(y_n,y_n,p),S(y_n,y_n,Ty_n),S(p,p,p)\}$$

$$+e\max\{S(y_n,y_n,p),S(y_n,y_n,Ty_n),S(p,p,p),S(y_n,y_n,p)\}$$

$$\leq a\,S(y_n,y_n,p)+b[S(y_n,y_n,p)+S(y_n,y_n,p)+S(Ty_n,Ty_n,p)]$$

$$+c\max\{S(y_n,y_n,p),[S(y_n,y_n,p)+S(y_n,y_n,p)+S(Ty_n,Ty_n,p)]\}$$

$$+e\max\{S(y_n,y_n,p),[S(y_n,y_n,p)+S(y_n,y_n,p)+S(Ty_n,Ty_n,p)]\}$$

$$\leq \sup_{x,y\in X}(a+2b)\,S(y_n,y_n,p)+\sup_{x,y\in X}(b+c+e)\max\{S(Ty_n,Ty_n,p),$$

$$\max\{S(y_n,y_n,p),[S(y_n,y_n,p)+S(y_n,y_n,p)+S(Ty_n,Ty_n,p)]\}$$

$$\leq \sup_{x,y\in X}(a+2b)\,S(y_n,y_n,p)+\sup_{x,y\in X}(b+c+e)\,[2S(y_n,y_n,p)+S(Ty_n,Ty_n,p)]$$

The last inequality gives

$$S(Ty_n,Ty_n,p)\leq \sup_{x,y\in X}\left(\frac{a+4b+2c+2e}{1-b-c-e}\right)S(y_n,y_n,p)$$

Taking limit as $n\to\infty$ we get $\lim_{n\to\infty}Ty_n=p=Tp$

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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