DEGREE OF INTUITIONISTIC L-FUZZY GRAPH

V.S. SREEDEVİ*, BLOOMY JOSEPH

Department of Mathematics, Maharajas College Ernakulam, Kerala, India

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Abstract. In this paper we continue the studies related to Intuitionistic L Fuzzy Graph which is a generalisation of Intuitionistic Fuzzy Graph. We try to define the connectivity of vertices and edges in Intuitionistic L Fuzzy Graph. We also try to define the degree of a vertex in an Intuitionistic L Fuzzy Graph and its properties.

Keywords: intuitionistic L-fuzzy graph; degree of a vertex in an ILFG; degree matrix of an intuitionistic L-fuzzy graph.

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1. INTRODUCTION

There has been an unprecedented progress in the study of Graph Theory in the twentieth century. Real world problems have often been analysed and studied successfully using Graphs. These problems and other famous puzzles have resulted in development in various topics in Graph theory. Eulerian graph theory is inspired from the famous Konigsberg bridge problem.

Rosenfeld in his classical paper introduced the concept of fuzzy graphs as a means to model various real life situations. An L-fuzzy set is a set in which the range[0,1] is replaced by a lattice, according to Klir and Yuan. Pramada Ramachandranand K V Thomas introduced the

*Corresponding author
E-mail address: sreedevis92@gmail.com
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concept of L-Fuzzy graph. Isomorphism and associated matrices of L-fuzzy graph were studied by them.

Intuitionistic fuzzy sets were introduced as a generalisation of fuzzy sets by Atanassov [3] in 1983 along with the concept of intuitionistic fuzzy graph. M G Karunambigai and R Parvathi [4][5] introduced the concept of fuzzy graph elaborately and analysed its components. Akram et al described the properties of strong intuitionistic fuzzy graphs, intuitionistic fuzzy cycle and intuitionistic fuzzy trees [6][7]. A Nagoor Gani and S Shajitha Begum examined the properties of various types of degrees, order and size of IFG.

In this paper we studied the degree and other properties of Intuitionistic L fuzzy graphs. We have continued on our work detailed in our paper titled 'Intuitionistic L-fuzzy graph’

2. PRELIMINARIES

2.1. Definition. An Intuitionistic Fuzzy Graph is of the form G=(V,E) where V = {v1,v2,v3,...vn} such that

(i) \( \mu_1 : V \longrightarrow [0,1] \) and \( \gamma_1 : V \longrightarrow [0,1] \) of the element \( v_i \) in V respectively and denote the degree of membership and non membership of the element \( v_i \) in V respectively and

\[ 0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \] for every \( v_i \) in V \( (i = 1,2,3,...n) \)

(ii) \( E \subseteq V \times V \) where \( \mu_2 : V \times V \longrightarrow [0,1] \) and \( \gamma_2 : V \times V \longrightarrow [0,1] \) are such that

\[ \mu_2(v_i,v_j) \leq \min(\mu_1(v_i),\mu_1(v_j)) \], \[ \gamma_2(v_i,v_j) \leq \max(\gamma_1(v_i),\gamma_1(v_j)) \]

and \( 0 \leq \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) \leq 1 \) for every \( (v_i,v_j) \) in E

2.2. Definition. Let G=(V,E) be an IFG. Then the degree of a vertex \( v \) is defined by

\[ d(v) = (d\mu(v),d\gamma(v)) \] where \( d\mu(v) = \sum_{u \neq v} \mu_2(v,u) \) and \( d\gamma(v) = \sum_{u \neq v} \gamma_2(v,u) \)

2.3. Definition. An Intuitionistic fuzzy graph G = (V,E) is said to be complete Intuitionistic fuzzy graph if \( \mu_{2ij} = \min(\mu_{1i},\mu_{1j}) \) and \( \gamma_{2ij} = \max(\gamma_{1i},\gamma_{1j}) \) for every \( v_i,v_j \) in V

The triple \( < v_i,\mu_{1i},\gamma_{1i} > \) denote the degree of membership and non membership of the vertex \( v_i \). The triple \( < e_{ij},\mu_{2ij},\gamma_{2ij} > \) denote the degree of membership and non membership of the edge relation \( e_{ij} = (v_i,v_j) \) on V
2.4. Definition. Let \((L, \leq)\) be a complete lattice with an Involutive order reversing operation \(N: L \rightarrow L\). Let a set \(E\) be fixed. An Intuitionistic L-fuzzy set \(A^*\) in \(E\) is defined as an object having the form \(A^* = \{ < x, \mu_A(x), \nu_A(x) | x \in E \}\) where the function \(\mu_A : E \rightarrow L\) and \(\nu_A : E \rightarrow L\) define the degree of membership and degree of non membership respectively of the elements \(x \in E\) and for every \(x \in E\), \(\mu_A(x) \leq N(\nu_A(x))\).

3. Degree of Intuitionistic L-Fuzzy Graph

3.1. Definition. An Intuitionistic L-Fuzzy graph is of the form \(G_L = (V_L, E_L)\) where \(V_L = \{v_1, v_2, v_3, ..., v_n\}\) such that

1) \(\mu_1 : V \rightarrow L\) and \(\gamma_1 : V \rightarrow L\) denote the degree of membership and non membership grade of the element \(v_i\) in \(V\) respectively and
\[ \mu_1(v) \leq N(\gamma_1(v)) \] for all \(v \in V\)

where \(N(v)\) is an involutive order reversing operation.

2) \(E \subseteq V \times V\) where \(\mu_2 : V \times V \rightarrow L\) and \(\nu_2 : V \times V \rightarrow L\) such that
\[ \mu_2(v_i, v_j) \leq \mu_1(v_i) \land \mu_1(v_j) \] and \[ \nu_2(v_i, v_j) \leq \gamma_1(v_i) \lor \gamma_1(v_j) \] \(\mu_2(v_i, v_j) \leq N(\nu_2(v_i, v_j))\)
denote the membership and non membership of an edge \((v_i, v_j)\) in \(E\) respectively.

3.2. Definition. The degree of a vertex \(w\) of an ILFG \(G_L = (V_L, E_L)\) is defined by \(d(w) = (d(\mu(w), d(\gamma(w)))\) where \(d(\mu(v)) = \bigvee_{u \neq w} \mu_2(w, u)\) and \(d(\gamma(w)) = \bigwedge_{u \neq w} \gamma_2(w, u)\)

3.3. Example. Consider the lattice

Here \(d(v_1) = (G_1, G_8)\), \(d(v_2) = (G_8, G_1)\), \(d(v_3) = (G_7, G_8)\)
3.4. Definition. Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph is defined by $d(G) = (\bigvee d\mu(v_i), \bigwedge d\gamma(w))$ for all $v_i$ in $V$.

3.5. Theorem. Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph is equal to $d(G) = (\bigvee \mu_2(u, v), \bigwedge \gamma_2(u, v))$ for all $(u, v)$ in $E$.

Proof: Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph.

Then the degree of the graph $d(G) = (\bigvee d\mu(v_i), \bigwedge d\gamma(v_i))$ for all $v_i$ in $V$.

$$= (\bigvee_{u \neq v_i} \mu_2(v_i, u), \bigwedge_{u \neq v_i} \gamma_2(v_i, u))$$

$$= (\bigvee \mu_2(u, v), \bigwedge \gamma_2(u, v))$$

for all $(u, v)$ in $E$.

3.6. Remark. In normal Graph the removal of an edge reduces the degree of its end vertices by one. But in Intuitionistic L Fuzzy Graph the removal of an edge need not reduce the degree of its end vertices.

In Example 3.3, the removal of the edge $(v_2, v_3)$ does not affect the degree of the vertex $v_2$.

3.7. Remark. In Intuitionistic L Fuzzy Graph the addition of an edge need not increase the degree of its end vertices.

In Example 3.3, adding the edge $((v_1, u), G_1, G_{10})$ does not increase the degree of the vertex $v_1$ in $G_L$.

3.8. Remark. If $((u, v), 1, 0)$ is an edge in an Intuitionistic L Fuzzy Graph $G_L$ then both of its end vertices has degree $(1, 0)$.

Proof

Let $((u, v), 1, 0)$ is an edge in an Intuitionistic L Fuzzy Graph $G_L$ then

$$d(u) = (d\mu(u), d\gamma(u)) = (\bigvee_{w \neq u} \mu_2(w, u), \bigwedge_{w \neq u} \gamma_2(w, u))$$

$$= (\mu_2(u, v), \gamma_2(u, v))$$

$$= (1, 0)$$

Similarly,

$$d(v) = (d\mu(v), d\gamma(v))$$
\begin{align*}
&= (\bigvee_{w \neq v} \mu_2(w, v), \bigwedge_{w \neq v} \gamma_2(w, v)) \\
&= (\mu_2(u, v), \gamma_2(u, v)) \\
&= (1, 0)
\end{align*}

3.9. Definition. A Complete Intuitionistic L Fuzzy Graph is an Intuitionistic L Fuzzy Graph such that \( \mu_2(u, v) = \mu_1(u) \land \mu_1(v) \) and \( \gamma_2(u, v) = \gamma_1(u) \lor \gamma_1(v) \)

3.10. Theorem. The degree of all the vertices of a complete Intuitionistic L Fuzzy Graph is same and it is given by \( d(v) = (\bigvee [\bigwedge \mu_1(v)], \bigvee [\bigwedge \gamma_1(v)]) \)

Proof: Let \( G_L = (V_L, E_L) \) be a complete Intuitionistic L Fuzzy Graph Then
\( \mu_2(u, v) = \mu_1(u) \land \mu_1(v) \) and \( \gamma_2(u, v) = \gamma_1(u) \lor \gamma_1(v) \) for all \((u,v)\) in \( E \)

Let \( v \) be an arbitrary vertex in \( G_L \),
\( d(v) = (\bigvee \mu_2(u, v), \bigwedge \gamma_2(u, v)) \)
\( = (\bigvee \mu_2(u, v), \bigwedge \gamma_2(u, v)) \)
\( = (\bigvee [\mu_1(u) \land \mu_1(v)], \bigwedge [\gamma_1(u) \lor \gamma_1(v)]) \)
\( = (\bigvee [\mu_1(v)], \bigwedge [\gamma_1(v)]) \)

3.11. Theorem. Let \( G_{1L} \) and \( G_{2L} \) be two Intuitionistic L Fuzzy Graphs and \( G_L \) be the union of \( G_{1L} \) and \( G_{2L} \). Let \( d_1(v) = (d_{1\mu}(v), d_{1\gamma}(v)) \), \( d_2(v) = (d_{2\mu}(v), d_{2\gamma}(v)) \), \( d(v) = (d_{\mu}(v), d_{\gamma}(v)) \)

be the degree of vertex \( v \) in \( G_{1L}, G_{2L} \) and \( G_L \) respectively. Then \( d(v) = (d_{\mu}(v), d_{\gamma}(v)) = (d_{1 \mu}(v) \lor d_{2 \mu}(v), d_{1 \gamma}(v) \land d_{2 \gamma}(v)) \) for all \( v \) in \( V_1 \cup V_2 \)

Proof:

Let \( G_L \) be the union of \( G_{1L} \) and \( G_{2L} \).

Let \( v \) be an arbitrary vertex in \( V = V_1 \cup V_2 \)

Then the degree of \( v \) in \( G_L \) is
\( d(v) = (\bigvee d_{\mu}(v), \bigwedge d_{\gamma}(v)) = (\bigvee_{u \neq v} \mu_2(u, v), \bigwedge_{u \neq v} \gamma_2(u, v)) \)
\( = (\bigvee_{u \neq v} \mu_2(u, v), \bigwedge_{u \neq v} \gamma_2(u, v)) \)
\( = (\bigvee_{u \neq v} \mu_2(u, v), \bigwedge_{u \neq v} \gamma_2(u, v)) \)
\( = (d_{1 \mu}(v) \lor d_{2 \mu}(v), d_{1 \gamma}(v) \land d_{2 \gamma}(v)) \) for all \( v \) in \( V_1 \cup V_2 \)
4. **Degree Matrix of an Intuitionistic L Fuzzy Graph**

4.1. **Definition.** Let $G_L = (V_L, E_L)$ be an Intuitionistic L Fuzzy Graph with $n$ vertices. Then the degree matrix $D_L$ of $G_L$ is an $n \times n$ diagonal matrix defined as

$$d(v) = \begin{cases} 
    d(v_i) & \text{if } i=j \\
    0 & \text{if otherwise}
\end{cases}$$

where $d(v_i)$ is the degree of the vertex $v_i$ in $G_L$.

4.2. **Theorem.** There exist a one one correspondance between every ILFG $G_L$ and degree matrix $D_L$

**Explanations:**

Let $\{G_L\}$, $\{S_L\}$, and $\{D_L\}$ be the collection of ILFGs, Degree sequences and Degree matrices respectively. We can define bijective functions as below

$\phi : \{G_L\} \rightarrow \{S_L\}$ and $\psi : \{S_L\} \rightarrow \{D_L\}$.

Then composition of $\phi$ and $\psi$ is a bijective function from $\{G_L\}$ to $\{D_L\}$.

4.3. **Remark.** We cannot find an ILFG for every diagonal matrix.

**Example:**

Consider lattice in Example3.3

$$\begin{bmatrix} 
(G_1, G_{10}) & (G_{10}, G_{10}) \\
(G_{10}, G_{10}) & (G_{10}, G_1) 
\end{bmatrix}$$

Here $G_L$ contains two vertices say $v_1$ and $v_2$.

$$d(v1) = (d\mu(v1), d\gamma(v1)) = (\bigvee_{u \neq v1} \mu_2(v1, u), \bigwedge_{u \neq v1} \gamma_2(v1, u))$$

$$= (\mu_2(v1, v2), \gamma_2(v1, v2))$$

$$= (G_{10}, G_{10})$$

$$d(v2) = (d\mu(v2), d\gamma(v2)) = (\bigvee_{u \neq v2} \mu_2(v2, u), \bigwedge_{u \neq v2} \gamma_2(v2, u))$$

$$= (\mu_2(v1, v2), \gamma_2(v1, v2))$$

$$= (G_{10}, G_1)$$
\( \mu_2(v_1, v_2) \) and \( \gamma_2(v_1, v_2) \) have two different values which is a contradiction.

5. Conclusion

In this paper we defined Intuitionistic L-Fuzzy Graph. Then we defined degree of a vertex in Intuitionistic L-fuzzy graphs. We proved some properties related to degree of vertex in Intuitionistic L-fuzzy graph. We defined the degree of an Intuitionistic L-Fuzzy graph. We have also discussed matrices associated with degree of an Intuitionistic L-Fuzzy graph. There is a scope to introduce more concepts related to degree matrix of an Intuitionistic L-Fuzzy Graph.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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