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THE NUCLEAR EFFECT ANALYSIS IN RELATIVISTIC HEAVY ION COLLISIONS AT BNL ENERGIES

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Abstract In this article, an attempt has been made to understand the behavior of the secondary charged particles multiplicity distributions produced during the heavy ion collisions at ultra-relativistic energies by using the Hurst exponent of the two dimensional (2D) factorial moments, F_q . For this purpose the experimental data have been analyzed by using the “Hurst exponent” in the original intermittency formula by considering different values of Hurst exponent ($H = 1.0, 1.5, 2.0, 2.5$). The investigations reveal the power law behavior, exhibited in self-affine or nuclear effect analysis, better than that in self-similar analysis. Finally, the described works were found very much significant and also it was within good agreement with some other works.

Keywords: multiplicity distribution; cumulant factorial moments; QGP; simulations, high energy density.

2010 AMS Subject Classification: 82D10.

1. INTRODUCTION

The primary objective of particle physics is to discover the fundamental forces and symmetries, and the elementary particles in Nature. A hierarchy of constituents of matter has been observed: macroscopic matter consists of molecules and atoms, the atoms consist of nucleons which in turn are formed of quarks, anti-quarks and gluons (partons). These results have been obtained by

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scattering experiments at higher and higher energies, as required to achieve information on smaller and smaller objects. At the moment the hierarchy ends at quarks: no substructure has been observed for them, so they are regarded as point like particles. Isolated single free quarks have never been observed, and therefore it is conjectured that quarks are confined together with other quarks to form hadrons.

The study of multiplicity correlations and fluctuations of produced charged particles in high energy ion collisions has been well known for few decades. This has never been more apparent than in recent years where these measurements helped to mark the discovery of new state of matter, so called Quark-Gluon Plasma (QGP) [1-5]. Various types of correlations and fluctuations present in heavy ion collisions at relativistic energies can provide us with valuable knowledge regarding the microscopic interactions inside the high density medium. In particular, the non-perturbative aspects of the strong interaction are difficult to study experimentally and probing the hot and dense QGP is one of the few avenues we have available.

The main aim of scaled factorial moments $F_q(M)$ is to look for the possible existence of the dynamical fluctuations so called “Intermittency” in high energy heavy ion collisions [6-8 and references therein]. While studying the behavior of factorial moments in h-h and A-A collisions, two different phenomenon have been observed.

(i) In one dimension (1D) phase space of pseudorapidity, the rise of $\ln \langle F_q \rangle$ with the increasing phase space partition number M is much weaker for A-A collisions than for h-h collisions and the heavier the colliding nuclei are, the weaker is the rising of $\ln \langle F_q \rangle$.

(ii) In 2D or 3D, the $\ln \langle F_q \rangle$ vs. $\ln M$ plot for A-A collisions is bending upwards strongly, much stronger than for h-h collisions and the heavier the colliding nuclei are, the stronger is the upward bending of $\ln \langle F_q \rangle$ vs. $\ln M$ plot.

Here, we illustrate various steps needed to obtain a reliable measurement of the “Hurst exponent” so called H_q moments. This includes the evaluation of both statistical and systematic errors, followed by a short study of the truncation in the tail of the charged-particle multiplicity

distribution. The outcomes were compared to the numerous analytical QCD predictions which exist up to the Next to Next to Leading Logarithm Approximation (NNLLA) based on the study of various Monte Carlo models.

2. EXPERIMENTAL DETAILS

The experiment data has been collected in the present work by FUJI type nuclear emulsion stacks, those were irradiated horizontally with a beam of ^{28}Si nuclei (like projectile) and hit the heterogeneous mixture of nuclear emulsion (fixed target) at 14.6A GeV at Alternating Gradient Synchrotron (AGS) of Brookhaven National Laboratory (BNL), New York, USA. The scanning of the exposed emulsion stacks was performed with the help of Leica DM2500M microscope with a 10 \times objective and 10 \times ocular lens provided with semi-automatic scanning stages. The method of line scanning was used to collect the inelastic ^{28}Si -Em interactions. The interactions collected from line scanning were scrutinized under an optical microscope (Semi-Automatic Computerized, Leica DM6000M) with a total magnification of 10 * 100 using 10 \times eyepiece and 100 \times oil immersion objective. The measuring system associated with it has 1 μm resolution along X and Y axes and 0.5 μm resolution along the Z-axis. The detailed discussion about the present experiment can be found in our earlier publications [9-13].

3. MATHEMATICALLY TOOLS

It has been found that the above two apparently contradictory observations are due to the superposition effect of the contribution from the large number of elementary collisions in a nuclear collision process.

To characterize the phase space partition in 2D, a quantity known as ‘‘Hurst exponent’’ [14-15] is used and is defined such as:

$$H = \frac{\ln M_1}{\ln M_\phi} \quad (1)$$

where M_η and M_ϕ are the number of partitions along the two perpendicular directions. The behavior of factorial moments F_q depends on the value of “H”. Therefore, to study the possible abnormal behavior of F_q on the bin size, analysis should be performed with a suitable value of H. Otherwise, the observed trend of the calculated factorial moments will always be bending upwards, even if there is no fluctuation in the multi-particle production. The upward bending of the F_q moments can be weakened or totally removed if the exponents, H are given a proper value. For the present investigations of nuclear effect in ^{28}Si -emulsion collision at total energy ≈ 409 GeV, we have used different values of Hurst exponent (H = 1.0, 1.5, 2.0, 2.5). The intervals $\Delta\eta$ and $\Delta\phi$ have been divided into sub-cells with the widths such as:

$$\left. \begin{aligned} \delta_\eta &= \Delta_\eta / M_\eta \\ \delta_\phi &= \Delta_\phi / M_\phi \end{aligned} \right\} \quad (2)$$

The scale factors of M_η and M_ϕ are connected to each other by the relation:

$$M_\phi = \{M_\eta\}^{1/\eta} \quad (3)$$

It is clear from eqn. (3) that M_η and M_ϕ cannot be integers simultaneously. Therefore the size of elementary phase space cell cannot take continuously varying values. This problem can be solved by applying the method of L. Lianshou et. al., such as given:

$$M_\phi = N + \alpha \quad (4)$$

where N is the integer part and $0 \leq \alpha < 1$ represents the fractional part.

4. RESULTS AND DISCUSSIONS

In the present work, we used $\Delta\eta = -2 \leq \eta_{\max} \leq +2$ and $\Delta\phi = 0-2\pi$. The M_η was varied from 2-30. Further, to reduce the effect of non flat particle density distributions, the cumulative variables X_η and X_ϕ were used to make it in the corresponding regions 0-1. By using the above partition scheme, the values of $\ln \langle F_2 \rangle$ were calculated with the help of general adaptation of Intermittency / scaled factorial moments $F_q(M)$.

The behavior of $\ln \langle F_2 \rangle$ vs. $\ln M$ have been shown in Fig. 1(a-d) for the collisions of ^{28}Si with emulsion nuclei at an energy 409 GeV for different values of exponent H. From this figure it has been observed that there is strong upward curve bend in Fig. 1 (a). However, when H increases, the upward bending is found weakened in Fig. 1(b-d).

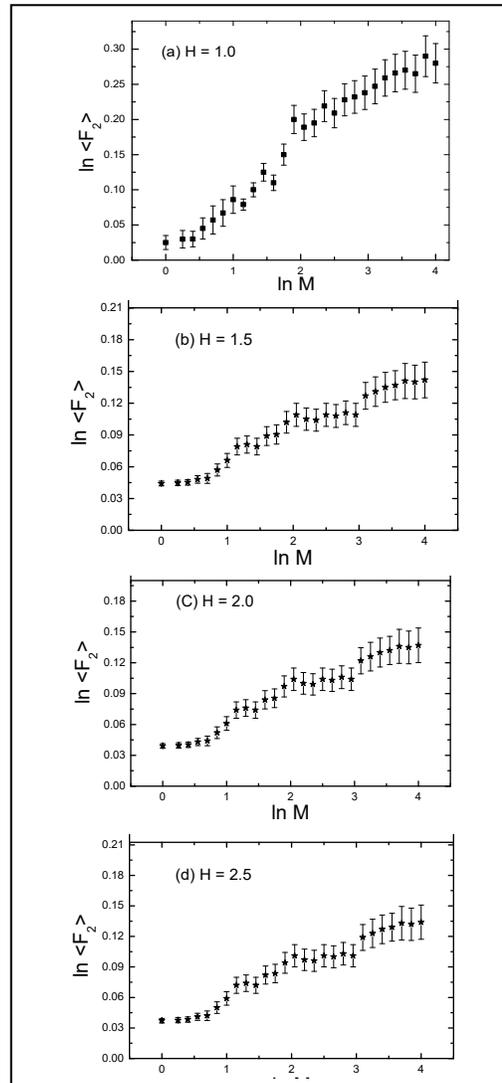


Figure 1(a-d): The dependence of $\ln \langle F_2 \rangle$ on $\ln M$ at energy $\approx 409\text{GeV}$.

We observe that the two dimensional second order factorial moment exhibits an upward bending as a function of partition of space, which is in turn means the superposition of contributions from the elementary collisions in the nucleus-nucleus collisions. This upward bending could, however, be removed by choosing proper partition along the longitudinal and perpendicular directions, that is, the right value of Hurst exponent “H”. Moreover, it has been observed that heavier the colliding are, the strong the upward bending is. It is consistence with the fact that the number of elementary collisions is more for heavier nuclei.

5. CONCLUSION AND FINAL REMARKS

It is worth mentioning that if QGP is formed, then there will be no elementary collisions. This in turn will lead to vanishing of the superposition effect due to the contribution of elementary collisions in nucleus- nucleus (A-A) collisions. Under such conditions, the upward bending in the two dimensional second factorial moment plots is not likely to be seen. Hence the study of the nuclear effect in nucleus- nucleus (A-A) collisions could be used as another indirect test of QGP formation.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

- [1] J.D. Bjorken, Highly relativistic nucleus-nucleus collisions: The central rapidity region, *Phys. Rev. D.* 27 (1983), 140–151.
- [2] M. Gyulassy, Signatures of new phenomena in ultrarelativistic nuclear collisions, *Nuclear Phys. A.* 418 (1984), 59–85.
- [3] M. Stephanov, K. Rajagopal, E. Shuryak, Event-by-event fluctuations in heavy ion collisions and the QCD critical point, *Phys. Rev. D.* 60 (1999), 114028.
- [4] R.C. Hwa, Fractal measures in multiparticle production, *Phys. Rev. D.* 41 (1990), 1456–1462.
- [5] R.C. Hwa, Criticality, Erraticity and Chaoticity in Strong Interaction, *Proc. 7th Int. Workshop on Multiparticle Production, Correlation and Fluctuations, Nijmegen, Netherlands, (1996), 303-312.*
- [6] M.A. Ahmad, A Study of Intermittency and Multifractality in ^{28}Si -emulsion Collisions at 14.6A GeV, Ph.D. thesis, Aligarh Muslim University, Aligarh, India, 2010.
- [7] A. Bialas, R. Peschanski, Intermittency in multiparticle production at high energy, *Nuclear Phys. B.* 308 (1988), 857–867.
- [8] S. Ahmad, M. Ayaz Ahmad, Some observations related to intermittency and multifractality in ^{28}Si and ^{12}C -nucleus collisions at 4.5A GeV, *Nuclear Phys. A.*, 780 (2006), 06-221.
- [9] S. Ahmad, M. Ayaz Ahmad, M. Irfan, M. Zafar, Study of non-statistical fluctuations in relativistic nuclear collisions, *J. Phys. Soc. Japan*, 75(6) (2006), 064604.
- [10] S. Ahmad, M. Ayaz Ahmad, A comparative study of multifractal moments in heavy ion collisions at high energies, *J. Phys. G: Nuclear Part. Phys.* 32 (2006), 1279-1293.
- [11] M.A. Ahmad, S. Ahmad, Study of Angular Distribution and KNO Scaling in the Collisions of ^{28}Si with Emulsion Nuclei at 14.6A GeV, *Ukrainian J. Phys.* 57(12) (2012), 1205-1213.
- [12] M.A. Ahmad, M.H. Rasool, S. Ahmad, Scaling nature of target fragments in the ^{28}Si -emulsion interactions at an energy 14.6A GeV, *Ukrainian J. Phys.* 58(10) (2013), 944-955.
- [13] M.A. Ahmad, J.H. Baker, M.H. Rasool, S. Ahmad, R. Dobra, D. Pasculescu, C.R. Telles, Fluctuations in produced charged particle multiplicities in relativistic nuclear collisions for simulated events, *J. Phys.: Conf. Ser.* 1258 (2019), 012010.
- [14] M.E. Gaddis, M.J. Zyda, *The Fractal Geometry of Nature: Its Mathematical Basis and Application to Computer Graphics*, Naval Postgraduate School, California, 1986.
- [15] B.B. Mandelbrot, *The fractal geometry of nature*, W.H. Freeman, San Francisco, 1982.

- [16] M. Tariq, M.A. Ahmad, S. Ahmad, M. Zafar, Analysis of high N_S -multiplicity events produced in relativistic heavy ion collisions at 4.5A GeV/c, *Romanian Rep. Phys.* 59 (3) (2007), 773-790.
- [17] M.A. Ahmad, S. Ahmad, Study of non-thermal phase transition in ^{28}Si - nucleus collisions at 14.6A GeV, *Int. J. Mod. Phys. E*, 7(8) (2007), 2241-2247.
- [18] S. Ahmad, M.A. Ahmad, M. Tariq, M. Zafar, Charged multiplicity distribution of relativistic charged particles in heavy ion collisions, *Int. J. Mod. Phys. E*, 18(9) (2009), 1929-1944.
- [19] M.A. Ahmad, S. Ahmad, M. Zafar, Intermittent and scaling behaviour of shower particles produced in the collisions of ^{28}Si - Em at 14.6A GeV", *Indian J. Phys.* 84 (12) (2010), 1675-1681.
- [20] M.H. Rasool, M.A. Ahmad, S. Ahmad, Signal of Unusual Large Fluctuations in ^{32}S -Em Interactions at SPS Energies, *J. Korean Phys. Soc.* 67 (2015), 448-457.
- [21] M.H. Rasool, M.A. Ahmad, S. Ahmad, Multifractal study and multifractal specific heat of singly charged particles produced in ^{32}S -Em interactions at 200A GeV, *Chaos Solitons Fractals*, 81 (2015), 197-202.
- [22] M.H. Rasool, M.A. Ahmad, O.V. Singh, S. Ahmad, Multiplicities of Forward-Backward Relativistic Charged Particles Produced in ^{32}S -Emulsion Interactions at 200 AGeV/c, *Chinese J. Phys.* 53 (2015), 100302.
- [23] M. Rîșteiu, R. Dobra, D. Pasculescu, A.A. Mohammad, Quality Engineering Tools Focused on Designing Remote Temperature Measurements for Inaccessible Locations by Using Light Components Parameterization of the Heated Materials, *IOP Conf. Ser.: Mater. Sci. Eng.* 133 (2016), 012059.
- [24] M.L. Boca, R. Dobra, D. Pasculescu, M. A. Ahmad, Analysis and simulation of industrial distillation processes using a graphical system design model, *Proc. SPIE*, VIII (10010), (2016), 100102U.
- [25] M.H. Rasool, M.A. Ahmad, M. Bhat, O.V. Singh, S. Ahmad, Multiplicity Characteristics of Forward-Backward Emitted Particles in Heavy-Ion Interactions at SPS Energies, in: B. Bhuyan (Ed.), *XXI DAE-BRNS High Energy Physics Symposium*, Springer International Publishing, Cham, 2016: pp. 61–66.
- [26] M.J. Mbunwe, M.A. Ahmad, S.K. Mustafa, An effective energy saving design strategy to maximize the use of electricity, *J. Math. Comput. Sci.* 10 (2020), 1808-1833.
- [27] I. Nevliudov, V. Yevsieiev, J.H. Baker, M.A. Ahmad, V. Lyashenko, Development of a cyber design modeling declarative Language for cyber physical production systems, *J. Math. Comput. Sci.* 11 (2021), 520-542.
- [28] S.S. Safaai, S.L. Yap, S.V. Muniday, M.A. Ahmad, Some Aspects of Fluctuations Dynamics of Particles in Dusty Plasma, *Commun. Math. Biol. Neurosci.* in Press.
- [29] V. Lyashenko, S.K. Mustafa, I. Tvoroshenko, M.A. Ahmad, Methods of Using Fuzzy Interval Logic During Processing of Space States of Complex Biophysical Objects, *Int. J. Emerg. Trends Eng. Res.* 8(2), (2020), 372-377.

- [30] S.K. Mustafa, M.A. Ahmad, S. Sotnik, O. Zeleniy, V. Lyashenko, O. Alzahrani, Brief review of the Mathematical Models for Analyzing and Forecasting Transmission of COVID-19, *J. Critical Rev.* 7(19) (2020), 4206-4210.
- [31] S.K. Mustafa, M.A. Ahmad, et. al. COVID-19 and Immune Function – “A Significant” Zinc, *Oriental J. Chem.* 36(6) (2020), 1026-1036.
- [32] V. Pavlova, B.B. Popov, V.K. Hristova et al. Chemical characterization of VARUMIN (1 and 2) oral solutions as potential therapeutic beverages, *Int. J. Pharm. Res.* 12 (2020), 4454-4460.
- [33] S.K. Mustafa, M.A. Ahmad, V. Baranova, et al. Using Wavelet Analysis to Assess the Impact of COVID-19 on Changes in the Price of Basic Energy Resources, *Int. J. Emerg. Trends Eng. Res.* 8(7) (2020), 2907-2912.