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FREE CONVECTIVE MHD FLOW PAST A VERTICAL PLATE THROUGH A POROUS MEDIUM WITH RAMPED WALL TEMPERATURE AND CONCENTRATION

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Abstract: This study attempts to examine the ramped wall temperature and concentration of an unsteady free MHD, viscous, incompressible, electrically conducting fluid past a vertical plate through a medium of porous nature under thermal radiation. Sets of dimensional governing equations are considered by taking suitable assumptions, which are then transformed into non-dimensional forms. Non-dimensional equations are being solved analytically with the help of Laplace Transform method. Resultant effects of some parameters concerning the problem on temperature, velocity, concentration, coefficient of skin-friction, Nusselt number and Sherwood number are discussed through different graphs and are physically interpreted. From the graphs, results are found out and related interpretations are made.

Keywords: MHD; porous medium; concentration; Laplace transform method.

2010 AMS Subject Classification: 76W05.

1. INTRODUCTION

Magnetohydrodynamics (MHD) is the science dealing with analysis of interaction between magnetic fields and electrically conducting fluids in motion. MHD principles are applied in engineering, plasma-physics, biotechnology, biomedical science, astrophysics, geophysics,

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electronics etc. So, many engineers and scientists are interested in its applications in their respective fields. MHD flows are important in many fields and due to its importance several researchers have put their attention in doing their research works on MHD field. Some notable among them are Babu et al. [1], Panezai et al. [2], Rajesh et al. [3], Basha et al. [4], Raju et al. [5] etc.

Convection and radiation are the two important ways of heat transfer. In the process of convection heat transfer takes through actual motion of matter, while in radiation transfer of heat occurs through electromagnetic waves. The processes of radiation and convections associated with fluid flows characterise the radiative convection flows which find suitable applications in various energy plants. Effects of radiation on MHD free fluid have also become more important in varied industrial activities and research. Due to its ever-growing importance, many researchers have carried out their research works on free convective incompressible viscous fluid flow taking thermal radiation into account. In this context, significant works are done by Lavanya[6], Ali Shah et al.[7], Chiranjibi et al. [8], Sambath et al. [9], Cogley et al. [10], Makinde et al. [11], Vasu et al. [12], etc.

Flow can easily be transmitted through the pores of a medium. Properties and behaviours of flow passing through a porous medium have become an important theme used in different fields of applied engineering and sciences. Considering the practical utility of research on fluid mechanics involving porous medium, volume of works done in this line has been increasing. The notable research works relating to this field include those conducted by Mishra et al. [13], Siddabasappa[14], Mehta et al.[15], Prasad and Reddy [16], Krishna et al.[17]. It needs to be mentioned here that Sinha et al. [18] studied MHD free convective flow through a porous medium past a vertical plate with ramped wall temperature. Importantly, the model of Cogley's et al. [10] has been applied by them to analyse radiative heat flux.

This work is an extension of the investigation carried out by Sinha et al. [18], where the effects of concentration have been examined in addition to temperature parameter. Because, it is essential to know the effects of concentration level of fluids on the rate of mass transfer.

2. FORMULATION OF THE PROBLEM

The present study is to investigate an unsteady MHD free convective radiative fluid flow of an optically thin viscous incompressible fluid past an infinite vertical plate through a porous medium in presence of temperature and concentration. A coordinate system is introduced, where X-axis is considered along vertical direction of the wall and Y-axis is considered along the normal to the wall as shown in Figure 1.



Figure 1: The problem represented in physical model

Let velocity components u and v be put respectively along X-axis and Y-axis. The flow is set in the region y > 0 and the plate merges at y=0. Here B_o is the uniform strength of magnetic field applied normal to the plate. At time $t' \le 0$, surrounding fluid and the plate remains at same constant temperature T_w and concentration C_w . But, at $o < t' \le t_o$, the temperature and concentration of the wall get reduced to $T_{\infty} + (T_W - T_{\infty})\frac{t'}{t_o}$ and $C_{\infty} + (C_W - C_{\infty})\frac{t'}{t_o}$ respectively. Here, fluid is considered as optically thin gray gas with free convection and radiation.

Following assumptions are taken into consideration in the investigation:

- i. Only fluid density varies, but other fluid properties are kept constant.
- ii. As compared to Y direction, radiative heat flux in X direction is minimal.

- iii. Viscous dissipation of energy is also very minimum.
- iv. In the case of plate being infinite in X-direction, all the physical variables become functions of y and t.

Considering above mentioned assumptions, dimensional governing equations are as follows:

1. Momentum Equation:

$$\frac{\partial u}{\partial t'} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta^{\bullet} \left(T - T_{\infty} \right) - \frac{\upsilon}{K^{\bullet}} u - \frac{\sigma B_o^2 u}{\rho}$$
(1)

2. Energy Equation:

$$\frac{\partial T}{\partial t'} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y}$$
(2)

3. Concentration Equation:

$$\frac{\partial C}{\partial t'} = D \frac{\partial^2 C}{\partial y^2}$$
(3)

Initial and boundary conditions as regards to velocity, temperature and concentration fields are:

$$y \ge 0: \quad u = 0, \ T = T_{\infty}, \ C = C_{\infty} \quad \text{for } t' \le 0$$

$$y = 0: \quad u = U_{o}, \quad \text{for } t' > 0$$

$$T = T_{\infty} + \left(T_{W} - T_{\infty}\right) \frac{t'}{t_{o}}, \ C = C_{\infty} + \left(C_{W} - C_{\infty}\right) \quad \text{for } o < t' \le t_{o}$$

$$T = T_{W}, \ C = C_{W} \quad \text{for } t' > t_{o}$$

$$(4.1)$$

$$y \to \infty: u \to o, T \to T_{\infty}, C = C_{\infty} \text{ for } t' > o$$

$$(4.3)$$

Following Cogley's model [10], the radiative heat flux rate for a non gray gas in an optically thin fluid is as follows:

$$\frac{\partial q_{r}}{\partial y} = 4 \left(T - T_{\infty} \right) \int K_{\lambda_{o}} \left(\frac{\partial e_{\lambda h}}{\partial T} \right)_{o} d\lambda$$
(5)

Here, K_{λ_o} denotes absorption co-efficient, λ means wavelength, $e_{\lambda h}$ represents Planck's function. Again subscript 0 means that all physical quantities are found out at temperature T_{∞} . Applying equation (5) in equation (2), the equation (6) is obtained as:

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_{p}} \frac{\partial^{2} T}{\partial y^{2}} - \frac{4}{\rho C_{p}} \left(T - T_{\infty} \right) I$$
(6)
Where, $I = \int_{0}^{\infty} K_{\lambda_{0}} \left(\frac{\partial e_{\lambda h}}{\lambda T} \right)_{0} d\lambda$

To normalize dimensional governing equations, following non-dimensional variables and parameters are introduced:

$$\eta = \frac{y}{U_{o}t_{o}}, t = \frac{t'}{t_{o}}, u_{i} = \frac{u}{U_{o}}, \theta = \frac{T - T_{\infty}}{T_{W} - T_{\infty}}, \phi = \frac{C - C_{\infty}}{C_{W} - C_{\infty}},$$

$$M = \frac{\sigma B_{o}^{2} t_{o}}{\rho}, Gr = \frac{g\beta^{\bullet} \upsilon (T - T_{\infty})}{U_{o}^{3}}, Gm = \frac{g\beta' \upsilon (C - C_{\infty})}{U_{o}^{3}}, Pr = \frac{\upsilon \rho C_{p}}{\kappa},$$

$$K = \frac{K^{\bullet} U_{o}^{2}}{\upsilon^{2}}, t_{o} = \frac{\upsilon}{U_{o}^{2}}, Sc = \frac{\upsilon}{D}$$
(7)

Using equation (7) in equations (1), (3) and (6), the following non- dimensional equations are derived:

$$\frac{\partial u_{i}}{\partial t} = \frac{\partial^{2} u_{i}}{\partial \eta^{2}} + Gr \theta + Gm \phi - \frac{u_{i}}{K} - M u_{i}$$
(8)
$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^{2} \theta}{\partial \eta^{2}} - Ra \theta ,$$
Where,
(9)
$$Ra = \frac{4 \upsilon I}{\rho C p U_{o}^{2}}, I = \int_{o}^{\infty} K_{\lambda_{o}} \left(\frac{\partial e_{\lambda h}}{\partial T} \right)_{o} d\lambda$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^{2} \theta}{\partial \eta^{2}}$$
(10)

Again, using equation (7) in the boundary conditions defined by equations (4.1), (4.2) and (4.3), the following non-dimensional equations of boundary conditions are found out:

$$\eta = o: \quad u_1 = o, \ \theta = o, \ \phi = o, \ for \ t \le o$$

$$\eta = o: \quad u_1 = 1, \quad for \ t > o$$
(11.1)

$$\begin{aligned} \theta = t, \ \phi = t, & \text{for } 0 < t \le 1, \\ \theta = 1, \ \phi = 1, & \text{for } t > 1, \end{aligned}$$
 (11.2)

$$\eta \to \infty$$
: $u_1 \to 0, \ \theta \to 0, \ \phi \to 0 \text{ for } t > 0$ (11.3)

3. METHOD OF SOLUTION

Applying Laplace Transform technique in the equations (8) to (10), solutions can be written in following way:

$$\frac{d^{2} \overline{u}_{_{1}}}{d\eta^{^{2}}} - (s + M_{_{1}})\overline{u}_{_{1}} = -Gr \overline{\theta} - Gm \overline{\phi}, \text{ Where } M_{_{1}} = \frac{1 + MK}{K}$$
(12)
$$\frac{d^{2} \overline{\theta}}{d\eta^{^{2}}} - Pr \overline{\theta} (s + M_{_{1}}) = 0$$
(13)
$$\frac{d^{2} \overline{\phi}}{d\eta^{^{2}}} - Sc. s \overline{\phi} = 0$$
(14)

The boundary condition equations (11.1) to (11.3) are also transformed to equation (15) by using Laplace Transform technique as:

$$\overline{u}_{1} = \frac{1}{s}, \quad \overline{\theta} = \frac{1}{s^{2}} \left(1 - e^{-s} \right), \quad \overline{\phi} = \frac{1}{s^{2}} \left(1 - e^{-s} \right) \quad \text{at} \quad \eta = 0$$

$$\overline{u}_{1} \to 0, \quad \overline{\theta} \to 0, \quad \overline{\phi} \to 0 \quad \text{as} \quad \eta \to \infty$$
(15)

Applying Laplace Transform technique in the equations (12) to (14) and considering the boundary conditions defined in equation (15), the solutions of the problem are obtained. Solutions are written using error function (erf) and complementary error function (erfc) as:

$$\phi(\eta, t) = \phi_{1}(\eta, t) - \phi_{1}(\eta, t-1)H(t-1)$$

$$\phi_{1}(\eta, t) = t \left[\left(1 + \frac{Sc \eta^{2}}{2t} \right) erfc \left(\frac{\eta \sqrt{Sc}}{2\sqrt{t}} \right) - \frac{\eta \sqrt{Sc}}{\sqrt{\pi t}} e^{-\frac{Sc \eta^{2}}{4t}} \right]$$

$$= \beta \left(Sc, \eta, t \right)$$
(16)

$$\theta(\eta, t) = \theta_{1}(\eta, t) - \theta_{1}(\eta, t-1) H(t-1)$$
⁽¹⁷⁾

Where,

$$\theta_{1}(\eta,t) = \frac{1}{2} \begin{bmatrix} \left(t + \frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{\eta\sqrt{PrRa}} \operatorname{erfc}\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}} + \sqrt{tPr}\right) + \\ \left(t - \frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{-\eta\sqrt{PrRa}} \operatorname{erfc}\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}} - \sqrt{tPr}\right) \end{bmatrix}$$
$$= \frac{1}{2} \begin{bmatrix} L(Pr,Ra,\eta,t) + \overline{L}(Pr,Ra,\eta,t) \end{bmatrix}$$
$$L(Pr,Ra,\eta,t) = \left(t + \frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{\eta\sqrt{PrRa}} \operatorname{erfc}\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}} + \sqrt{tPr}\right)$$
$$u_{1}(n,t) = h_{1} + p_{1}h_{35} + p_{2}h_{36} \tag{18}$$

Where,
$$p_1 = \frac{Gr}{A}$$
, $p_2 = \frac{Gm}{G}$, $A = Pr - 1$, $G = Sc - 1$, $h_{35} = h_9 - h_{25}$, $h_{36} = h_{16} - h_{34}$,
 $h_1 = \frac{1}{2} \begin{bmatrix} e^{\eta \sqrt{M_1}} erfc \left(\frac{\eta}{2\sqrt{t}} + \sqrt{M_1 t} \right) + \\ e^{\eta \sqrt{M_1}} erfc \left(\frac{\eta}{2\sqrt{t}} - \sqrt{M_1 t} \right) \end{bmatrix} = f(M_1, \eta, t),$
 $h_2 = e^{-Q_1 t} f(\lambda_1, \eta, t), h_3 = Q_1 h(M_1, \eta, t), h_4 = \frac{1}{Q_1^2} [h_2 - h_1 + h_3], h_5 = e^{-Q_1 t} f(\lambda_1, \eta, t - 1) H(t - 1),$

$$\begin{split} h_{_{6}} &= f\left(M_{_{1}},\eta,t-1\right)H\left(t-1\right), h_{_{7}} = Q_{_{1}}h\left(M_{_{1}},\eta,t-1\right)H\left(t-1\right), h_{_{8}} = \frac{1}{Q_{_{1}}^{^{2}}}\left[h_{_{5}}-h_{_{6}}+h_{_{7}}\right], h_{_{9}} = h_{_{4}}-h_{_{8}}, \\ h_{_{10}} &= e^{Q_{_{2}}t}f\left(\lambda_{_{2}},\eta,t\right), h_{_{11}} = Q_{_{2}}h\left(M_{_{1}},\eta,t\right), h_{_{12}} = h_{_{10}}-h_{_{1}}-h_{_{11}}, h_{_{13}} = e^{Q_{_{2}}t}f\left(\lambda_{_{2}},\eta,t-1\right)H\left(t-1\right), \\ h_{_{14}} &= Q_{_{2}}h\left(M_{_{1}},\eta,t-1\right)H\left(t-1\right), h_{_{15}} = h_{_{13}}-h_{_{6}}-h_{_{14}}, h_{_{16}} = \frac{1}{Q_{_{2}}^{^{2}}}\left[h_{_{12}}-h_{_{15}}\right], h_{_{17}} = e^{-Q_{_{1}}t}f\left(Pr,\lambda_{_{3}},\eta,t\right), \\ h_{_{18}} &= f\left(Pr,Ra,\eta,t\right), h_{_{19}} = Q_{_{1}}h\left(Pr,Ra,\eta,t\right), h_{_{20}} = \frac{1}{Q_{_{1}}^{^{2}}}\left[h_{_{17}}-h_{_{18}}+h_{_{19}}\right], h_{_{21}} = e^{-Q_{_{1}}t}f\left(Pr,\lambda_{_{3}},\eta,t-1\right)H\left(t-1\right), \end{split}$$

$$\begin{split} h_{22} &= f \left(Pr, Ra, \eta, t-1 \right) H \left(t-1 \right), h_{23} = Q_1 h \left(Pr, Ra, \eta, t-1 \right) H \left(t-1 \right), h_{24} = \frac{1}{Q_1^2} \Big[h_{21} - h_{22} + h_{23} \Big], h_{25} = h_{20} - h_{24}, \\ h_{26} &= e^{Q_2 t} f \left(Sc, Q_2, \eta, t \right), h_{27} = f \left(Sc, \eta, t \right), h_{28} = Q_2 h \left(Sc, \eta, t \right), h_{29} = \frac{1}{Q_2^2} \Big[h_{26} - h_{27} - h_{28} \Big], \\ h_{30} &= e^{Q_2 t} f \left(Sc, Q_2, \eta, t-1 \right) H \left(t-1 \right), h_{31} = f \left(Sc, \eta, t-1 \right) H \left(t-1 \right), h_{32} = Q_2 h \left(Sc, \eta, t-1 \right) H \left(t-1 \right) \right) \\ h_{33} &= \frac{1}{Q_2^2} \Big[h_{30} - h_{31} - h_{32} \Big], h_{34} = h_{29} - h_{33}, Q_1 = \frac{B}{A}, B = Pr Ra - M_1, Q_2 = \frac{M_1}{G}, M_1 = \frac{1 + MK}{K}, \\ \lambda_1 &= M_1 - Q_1, \lambda_2 = M_1 + Q_2, \lambda_3 = Ra - Q_1, Ra = \frac{4 \upsilon I}{\rho C p U_0^2}, I = \int_0^\infty K_{\lambda_0} \left(\frac{\partial e_{\lambda h}}{\partial T} \right)_0 d\lambda, \end{split}$$

Nusselt Number

Co-efficient of heat transfer rate considering Nusselt number (Nu) is as follows:

$$Nu = \left[\frac{\partial \theta}{\partial \eta}\right]_{\eta=0}$$

= $\theta_{2}(\eta, t) - \theta_{2}(\eta, t-1)H(t-1)$ (19)
 $\theta_{2}(\eta, t) = -\left[\left(\frac{1}{2}\sqrt{\frac{Pr}{Ra}} + t\sqrt{PrRa}\right)erf(\sqrt{tRa}) + \sqrt{\frac{Prt}{\pi}}e^{-tRa}\right]$

Sherwood Number

Now, co-efficient of mass transfer rate considering Sherwood number (Sh) is as follows:

$$Sh = \left[\frac{\partial \phi}{\partial \eta}\right]_{\eta=0}$$
(20)
$$= \phi_{2}(\eta, t) - \phi_{2}(\eta, t-1) H(t-1)$$

$$\phi_{2}(\eta, t) = -2\sqrt{\frac{Sct}{\pi}}$$

Skin-friction co-efficient

Co-efficient of Skin-friction (τ) is:

$$\tau = \left[\frac{\partial u_{1}}{\partial \eta}\right]_{\eta=0}$$

$$\tau = \psi_{1} + p_{1}\psi_{19} + p_{2}\psi_{33}$$
(21)

Where,

$$\begin{split} \psi_{1} &= \psi\left(M_{1}, t\right), M_{1} = \frac{1+MK}{K}, p_{1} = \frac{Gr}{A}, p_{2} = \frac{Gm}{G}, G = Sc - 1, \\ A &= Pr - 1, \psi_{19} = \psi_{9} - \psi_{18}, \psi_{9} = \psi_{4} - \psi_{8}, \psi_{18} = \psi_{13} - \psi_{17}, \\ \psi_{33} &= \psi_{35} - \psi_{32}, \psi_{35} = \frac{1}{Q_{2}^{3}} \Big[\psi_{20} - \psi_{1} - \psi_{21} - \psi_{32} + \psi_{33} + \psi_{34} \Big], \\ \psi_{34} &= \frac{1}{Q_{2}^{3}} \Big[\psi_{26} - \psi_{27} - \psi_{28} - \psi_{29} + \psi_{30} + \psi_{31} \Big], \\ \psi_{1} &= \psi\left(M_{1}, t\right) \\ &= - \Big[\frac{1}{\sqrt{\pi t}} e^{-M_{1}t} + \sqrt{M_{1}} \operatorname{erf}\left(\sqrt{M_{1}t}\right) \Big] \\ \psi_{2} &= e^{-Q_{1}t} \psi\left(\lambda_{1}, t\right), \psi_{3} &= Q_{1}\beta\left(M_{1}, t\right), \lambda_{1} = M_{1} - Q_{1}, \\ \beta\left(M_{1}, t\right) &= - \left[\left[\frac{1}{2\sqrt{M_{1}}} + t\sqrt{M_{1}} \right] \operatorname{erf}\sqrt{M_{1}t} + \sqrt{\frac{t}{\pi}} e^{-M_{1}t} \right], \\ \psi_{4} &= \frac{1}{Q_{1}^{2}} \Big[\psi_{2} - \psi_{1} + \psi_{3} \Big], \psi_{5} &= e^{-Q_{1}t}\psi\left(\lambda_{1}, t - 1\right), \psi_{6} &= \psi\left(M_{1}, t - 1\right), \\ \psi_{7} &= Q_{1}\beta\left(M_{1}, t - 1\right), \psi_{8} &= \frac{1}{Q_{1}^{2}} \Big(\psi_{5} - \psi_{6} + \psi_{7}\right), \psi_{10} &= e^{-Q_{1}t}\psi\left(Pr, \lambda_{3}, t\right), \\ \lambda_{3} &= Ra - Q_{1}, \psi_{11} &= \psi\left(Pr, Ra, t\right), \psi_{12} &= Q_{1}\beta\left(Pr, Ra, t\right), \\ \psi_{11} &= \psi\left(Pr, Ra, t\right) \\ &= - \left[\frac{\sqrt{Pr}}{\sqrt{\pi t}} e^{-Rat} - \sqrt{Pr} Ra \right], \end{split}$$

$$\begin{split} \beta \left(\Pr, \operatorname{Ra}, t \right) &= - \left[\left(\frac{1}{2} \sqrt{\frac{\Pr}{\operatorname{Ra}}} + t \sqrt{\Pr\operatorname{Ra}} \right) \operatorname{erf} \sqrt{\operatorname{Ra} t} + \sqrt{\frac{\Pr t}{\pi}} e^{-\operatorname{Ra} t} \right], \\ \psi_{13} &= \frac{1}{Q_{1}^{2}} \left[\psi_{10} - \psi_{11} + \psi_{12} \right], \psi_{14} = e^{-Q_{1} t} \psi \left(\Pr, \lambda_{3}, t - 1 \right), \\ \psi_{15} &= \psi \left(\Pr, \operatorname{Ra}, t - 1 \right), \\ \lambda_{3} &= \operatorname{Ra} - Q_{1}, \psi_{16} = Q_{1} \beta \left(\Pr, \operatorname{Ra}, t - 1 \right), \\ \psi_{17} &= \frac{1}{Q_{1}^{2}} \left[\psi_{14} - \psi_{15} + \psi_{16} \right], \psi_{18} = \psi_{13} - \psi_{17}, \\ \psi_{19} &= \psi_{9} - \psi_{18}, \psi_{20} = e^{Q_{2} t} \psi \left(\lambda_{2}, t \right), \psi_{21} = Q_{2} \beta \left(M_{1}, t \right), \lambda_{2} = M_{1} + Q_{1}, \\ \psi_{22} &= e^{Q_{2} t} \psi \left(\lambda_{2}, t - 1 \right), \psi_{23} = \psi \left(M_{1}, t - 1 \right), \psi_{24} = Q_{2} \beta \left(M_{1}, t - 1 \right), \\ \psi_{26} &= e^{Q_{2} t} \psi \left(\operatorname{Sc}, \operatorname{Q}_{2}, t - 1 \right), \psi_{27} = \psi \left(\operatorname{Sc}, t \right), \psi_{28} = Q_{2} \beta \left(\operatorname{Sc}, t - 1 \right), \\ \lambda_{3} &= \operatorname{Ra} - Q_{1}, \psi_{32} = \frac{1}{Q_{2}^{2}} \left[\psi_{26} - \psi_{27} - \psi_{28} - \psi_{29} + \psi_{30} + \psi_{31} \right], \\ \psi_{33} &= \psi_{25} - \psi_{32}, \\ \psi_{25} &= \frac{1}{Q_{2}^{2}} \left[\psi_{20} - \psi_{1} - \psi_{21} - \psi_{22} + \psi_{23} + \psi_{24} \right], \end{split}$$

4. RESULTS

For obtaining physical situations of the problem, necessary calculations of non-dimensional velocity, temperature, concentration, co-efficient of skin-friction, co-efficient of rate of heat transfer in terms of Nusselt number Nu and co-efficient of mass transfer in terms of Sherwood number Sh by assigning some arbitrary values of different parameters, viz. magnetic parameter M, Schmidt number Sc, Prandtl number Pr, radiation parameter Ra and time t have been carried out. The effects of these values of parameters on flow are examined and illustrated using graphs. From the graphs, the results are interpreted physically. Values of Prandtl number Pr are considered constant at 0.71 which indicates both air and the values of Schmidt number Sc are arbitrarily chosen.

Attempt has been made to describe velocity u_1 versus η under the influence of magnetic

parameter M and Schmidt number Sc as shown in **figures 2-3**. In both graphs, it is found that the increasing value of magnetic parameter M and Schmidt number Sc tends to reduce the fluid velocity. From this phenomenon, it becomes clear that high magnetic intensity M and high value of mass diffusivity Sc are compelled the fluid motion to slow down. So, applying high magnetic intensity and high mass diffusivity, it is possible to control the fluid motion. High magnetic intensity offers resistance to fluid motion.

Figures 4-5 depict temperature θ versus η under the action of Prandtl number Pr and radiation parameter Ra. As seen from the two figures, the fluid temperature drops down under the influence of Pr and Ra. Rising values of Pr and Ra are seen to reduce the fluid temperature. In figure 5, it appears that using high radiation, the fluid temperature can be controlled.

The effects of Schmidt number Sc and time t on fluid concentration are demonstrated in figures 6-7. Figure 6 indicates that high mass diffusivity, i.e. high value of Sc has forced the concentration of fluid to fall down. But in figure 7, an opposite phenomenon has been observed. In this case, concentration level of fluid goes up with an increase in the value of time t. When time t increases the fluid concentration also increases.

In figures 8-9, the co-efficient of Skin-friction τ against t under the action of magnetic parameter M and Schmidt number Sc is depicted. Both the figures show that rising of M and Sc diminishes the co-efficient of Skin-friction. So, high magnetic intensity and high mass diffusivity tend to minimize the co-efficient of Skin-friction.

The effect of Prandtl number Pr and radiation parameter Ra against the co-efficient of rate of heat transfer in terms of Nusselt number Nu from the plate to the fluid is presented in figures 10-11. As evident from the figure 10, the Nusselt number Nu increases for increasing value of Pr. But, figure 11 shows opposite behaviour, i.e. Nusselt number Nu tends to fall down under the influence of radiation parameter Ra.

The co-efficient of rate of mass transfer in terms of Sherwood number Sh from the plate to the fluid under the effect of Schmidt number Sc has been illustrated in figure 12. This figure demonstrates that high mass diffusivity Sc has made an increase in Sherwood number Sh. It suggests that the Sherwood number Sh accelerates for any increasing value of Sc, i.e. mass flux from the plate to the fluid gets accelerated under the influence of mass diffusivity.



Figure 2: Velocity u versus η under K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=25, Sc=0.60, t=0.5



Figure 4: Temperature θ versus η under Ra=2, t=0.5



Figure 6: Concentration ϕ versus η under t=0.5



Figure 3: Velocity u versus η under K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=25, M=5, t=0.5



Figure 5: Temperature θ versus $~\eta~$ under Pr=0.71, t=0.5



Figure 7: Concentration $\phi~$ versus $~\eta~$ under \$\$ Sc=0.60 \$\$



Figure 8: Skin friction τ versus t under K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=25, Sc=0.60



Figure 9: Skin friction τ versus t under K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=25, M=5









Figure 12: Sherwood number Sh versus t

5. COMPARISON OF RESULTS

Work of Sinha et.al. [18] is considered for comparing the results of the present paper. Comparing figure 13 with figure 2 of the work done by Sinha et.al. [18], we observe the same kind of behaviour due to the implementation of magnetic intensity in velocity profile. i.e. there is a significant effect of magnetic parameter on this profile. Thus, there is an excellent agreement between the results obtained by Sinha et.al. [18] and those arrived at by the present authors.



Figure 2. Velocity profile for variations in M when Ra=2, Pr=0.71, Gr=25, K=0.04, t=1.0



Figure 13: Velocity u versus η under K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=0, Sc=0, t=1

6. CONCLUSIONS

- 1. Fluid motion gets reduced due to action of magnetic parameter M and Schmidt number.
- 2. Temperature of fluid falls down for high values of Prandtl number Pr and radiation parameter Ra. With high radiation, temperature of fluid can be controlled.
- 3. Concentration level of fluid leads to decrease for increasing value of Schmidt number Sc, but concentration level of fluid rises for increasing value of time t.
- 4. The Skin-friction co-efficient tends to diminish for high value of magnetic intensity M and high mass diffusivity Sc.

- 5. Nusselt number goes on rising for increasing value of Pr, but it tends to fall down under the influence of radiation parameter Ra.
- 6. High mass diffusivity Sc is found to enhance the mass flux.

7. NOMENCLATURE

u: Velocity of fluid, (X, Y): Cartesian co-ordinates, t': Dimensional time, g: Acceleration due to gravity, T: Dimensional temperature, T_w : Dimensional temperature at the plate, T_∞ : Dimensional temperature far away from the plate, C: Dimensional concentration, C_w : Dimensional concentration at the plate, C_∞ : Dimensional concentration far away from the plate, K[•]: Dimensional porosity number, B_o : Strength of applied magnetic force, C_p : Specific heat at constant pressure, D: Molecular mass diffusivity, U_o :

Scale of free stream velocity, M: Magnetic parameter, K: Non-dimensional porosity number, G_r : Thermal Grashof number, G_m : Mass Grashof number, P_r : Prandtl number, S_c : Schmidt number, R_a : Radiation parameter, erf: Error function, erfc: Complement of error function, H(t-1): Unit step function, u_1 : Non-dimensional velocity at the plate, t_o : Characteristic time.

Greek Symbol: U: Kinematic viscosity, β^{\bullet} : Co-efficient of thermal expansion, σ : Electrical conductivity, ρ : Density of fluid, κ : Thermal conductivity, θ : Non-dimensional temperature, ϕ : Non-dimensional concentration.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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