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## A NEW COMPUTATIONAL APPROACH ON SIGNAL PROPAGATION INSIDE THE COCHLEA

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**Abstract:** Signal propagation inside the cochlea is analyzed from an approximate point of view. This concept has been established by enabling assumptions regarding the feasibility of modeling experiments on the structure of the cochlea in line with the expected results of mathematical models. This article sheds light on reviewing the hearing system from a mathematical, mechanical, electrical, and chemical point of view. This is not to determine which model is better, but to analyze the advancement of the work on cochlear modeling. This paper attempts to present which side of the ear and how the Fourier transformation actually performs its function (separating complex waves at many frequencies of the sine waves). It provides several improved observations that integrate the effects of the observed studies in such a way that certain characteristics are captured in different types of linear & non-linear modeling in Cochlea, which is considered to be a frequency analyzer existing in the inner ear. Our proposed approach has been developed following advanced computational language-python.

**Keywords:** Fourier transform; Cochlea; amplitude spectrum; stereocilia (hair cells); basilar membrane (BM); organ of corti; python (programming language).

**2010 AMS Subject Classification:** 65T50.

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## 1. INTRODUCTION

### 1.1. Anatomy of the Hearing System

Among the sensory organs, it is only the ear that informs us of what is going on in the surrounding. Therefore, it is an efficient transducer. The brainstem contains various neural centers. These neural centers also exist in the brain that processes information from the auditory nerves. Auditory nerves, together with the inner, middle, and outer ear [Figure 1], forms the peripheral auditory system. Alternatively, the brain and brainstem form parts of the central nervous hearing organ. The two systems work hand in hand and are responsible for auditory and hearing perception [1].

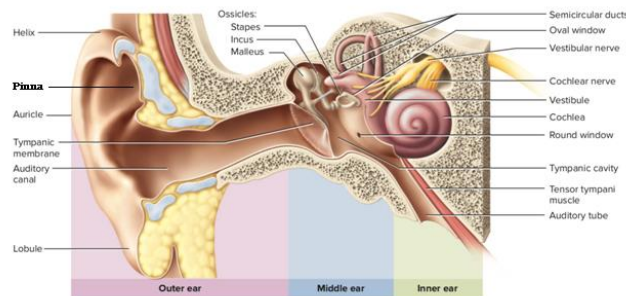


Figure 1

### 1.2. The External Ear

The ear is made up of both a pinna [Figure 1] as well as an external auditory duct. The protruding visible part located beside the head is referred to as pinna. It serves an important role in the hearing process by collecting sound waves from the environment, amplifying the sound, and funneling it down the canal. The tympanic membrane vibrates if the waves hit the external canal [2].

### 1.3. Middle Ear

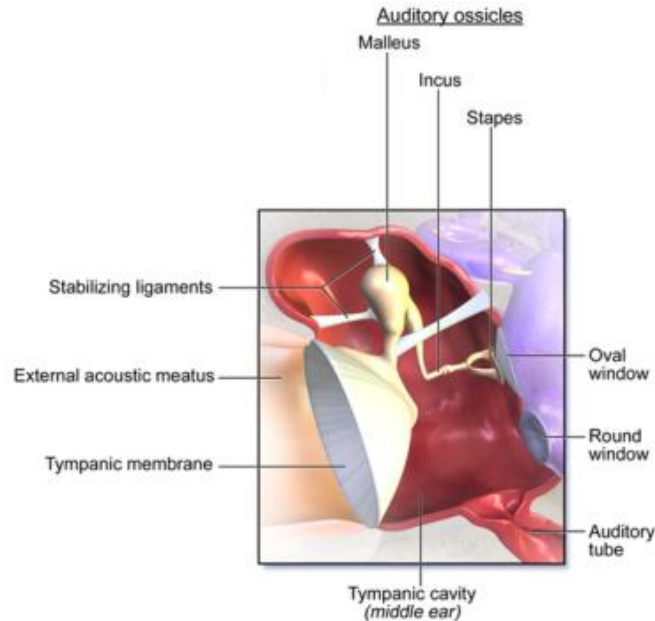


Figure 2

It is in this region where the tympanic membrane is located. It is composed of auditory ossicles (incus, Malleus, and stapes), the Eustachian tube, and lastly, the round and oval window [Figure 2]. The middle ear bones are responsible for sound conduction from the eardrum to interior ear fluids. The ear in the middle serves the mechanical lever purpose as well as increasing the sound pressure at the cochlea entrance. The oval window is typically 15 to 30 times lower than that of the eardrum [3].

Mathematically, pressure increases when the surface area is decreased. Therefore, there is a decreased area between the two membranes is responsible for the high increase in pressure. Therefore, it is the imperative to maximize sound quality that comes channeled to the interior ear fluids. It is also important to regulate pressure inside the ear. For this reason, the ear has got an Eustachian tube [4] that links the nose and the middle ear. The purpose of the Eustachian tube to allow equilibrium to be achieved between the middle ear air pressure and that of the atmospheric air pressure. Equilibrium air pressure is crucial since it allows the eardrum to vibrate freely efficiently.

### 1.4. Inner Ear

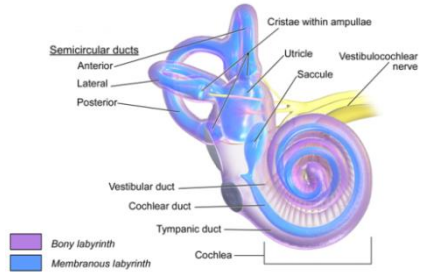


Figure 3

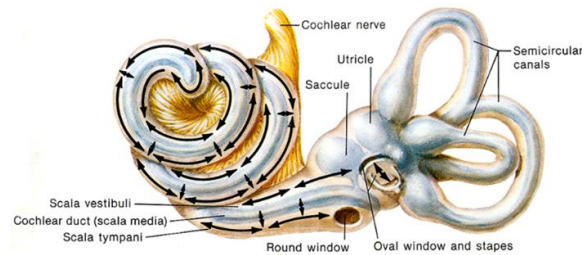


Figure 4

The inner part [Figure 3, 4] of the ear comprises of three parts and is enclosed in a temporal bone. The three parts of the inner ear are the vestibular apparatus, Cochlea, and semicircular canals (ducts). The Cochlea is a Greek word meaning "snail" due to its spiral shape in appearance. There are two openings at the base end known as windows. The middle ear is connected to the oval window while an elastic membrane covers the round window. The Cochlea consists of 3-coiled canals namely; scala tympani, media (Cochlea duct), and vestibule. An organ is known as Corti separate scala tympani and media. Corti is built on a flexible basilar membrane. Additionally, Reissner's membrane is thin and modeled as one chamber, separates the scala vestibule and scala media [5].

### 1.5. Functions of Cochlear

Mapping sounds of various frequencies to their respective characteristics positions on the basilar membrane (BM) is the fundamental function of Cochlea. The fluid-filled Cochlea [Figure 4] vibrates to respond to oval window motion. The basilar membrane (BM) is moved as it vibrates. The basilar membrane has varying stiffness and width along its length in a manner that different parts of the membrane resonate at different frequencies. The thin stiff ends are only moved with high-frequency vibration while the low frequencies vibrations are responsible for moving the thick compliant end. The entire transformation can be perceived as an acoustic signal real-time spectral decomposition procedure that produces spatial frequency in Cochlea. The human ear has the capability of responding to various frequencies between 20 Hz and 20 KHz as well as various intensity range up to 120 decibels [6].

### 1.6. The Organ of Corti (Spiral Organ)

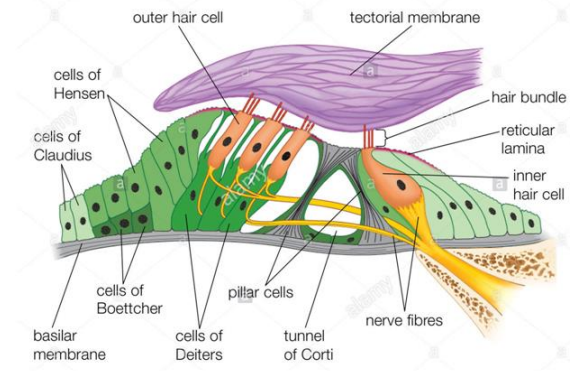


Figure 5

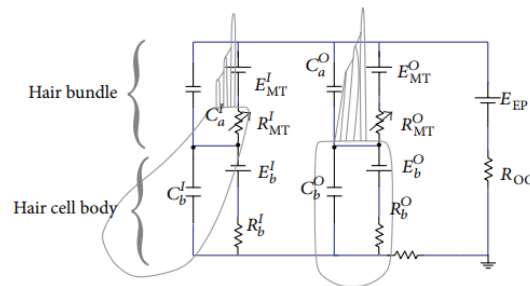
This is a tiny organ (organ of Corti) spirally shaped. Corti has got hair cells accountable for mechanical energy conversion of resulting from basilar membrane vibration into electrical impulses. The resulting electrical impulses are received by the vestibulocochlear nerve (auditory vestibular nerve/ auditory nerve), which in turn facilitates the transmission of the information to the brainstem as well as to the auditory cortex. Corti's organ has around 16,000 audible sensory hair cells named inner hair cells (IHC) & outer hair cells (OHC), [7] every one having a bundle of stereocilia (hair bundle), The tips are near to or attached with the tectorial membrane [Figure 5]. The brain stem function is tested several times to see if someone's brain is alive. OHCs are used to amplify the mechanical signal (i.e. sound) that enter the inner ear, IHCs are the right receptor cells for hearing and are associated with the nerves [8].

### 1.7. Cochlea's Electrical Property

The cochlea's functions to transform the movement of hair cells into electrical signals. The motion is the result of incoming sound waves. Once they are converted into electrical signals, the signals move to the brain through the auditory path neural for processing. Within the Cochlea are three endolymph and perilymph fluids compartments. The endolymph fluid, contained in the scala media, is more electrically positive compared to other fluids due to its unique ion content. The stria vascularis keeps supplying potassium ions [9] into the Cochlea through electromagnetic pumping to maintain the ionic concentration. The difference in electrical potential creates the movement of ions through the structures with stable current potentials within the Cochlea.

As the basilar membrane vibrates, the stereocilia deflect, and the ions flow is modulated. The stereocilia deflection regulates Mechano-Electrical Transduction channels (MET) [10] opening and closing. This is largely facilitated by the potential differences that lie between intercellular potential, perilymph, and endolymph. With the shutting and opening of the MET, the flow of ions changes, thereby activating the hair cells. It is possible to model the Cochlea into an electrical model, which is a biological network capacitance, resistance, current, and voltage sources, by the properties and effects investigation process of the flow of ions, both standing and alternation.

The Inner hair cells (IHC) transduce mechanical vibrations to stimulate the neural. The neural is then transmitted to the brain for interpretation, while nonlinear amplification of the BM is the function of the OHC to make weak stimuli more stable and compress high-level stimuli. It is reported that the basilar membrane's functioning is associated with both OHC and IHC because the absence of the OHC lowers the sharpness of tuning and limits the sensitivity to soft sounds. There is an increase in the dynamic range of hearing associated with the operations of the OHCs. The electrical lumped model consists of both IHCs and OHCs. Both IHCs and OHCs can be sub-categorized into basolateral and apical parts [11]. Membrane Voltage, capacitance, resistance sources can model these subcategories. The Corti organ exhibits similar characteristics to other biological tissues in the absence of hair cells. In such cases, they can act as a passive electrical grid. In the presence of hair cells, the sensory epithelium exhibits special electrical properties that affect its mechanical behaviors. When both electrical and mechanical parts of the Corti organ interact together, they bring a mutual effect that mediates the cochlea functions. [12]. A realistic cochlear model should incorporate such interaction.



**Figure 6**

Figure 6 : A diagram representation of the inner as well as OHC.  $I$  denote the inner hair cells, and  $O$  denotes the outer hair cell.  $MT$ ,  $a$  and  $b$  represent the MET channel, basolateral hair cells parts, and apical [13].

### 1.8. Chemical Properties of the Hearing System

The three compartments of the Cochlea contain Perilymph and Endolymph fluids [Figure 7]. Within the scala tympani and scala vestibule is the perilymph fluid. The fluid is negatively charged [14] This has a high sodium content (140mM) and poor calcium content (1,2mM) and potassium (**5mM**). The scala media contain endolymph liquid, which is positively charged. This has a high potassium content (150mM), low in sodium (1mM), and nearly deficient in calcium (20-30  $\mu$ M). The lining of the chambers is made up of specialized cells that keep pumping potassium into the chambers to maintain the concentration difference. Sensory cells use the chemical energy (like a battery) generated by the difference in the chemical composition of the fluid to conduct their activities. The membrane of the chambers is made up of tightly-compacted cells that prevent the fluids from mixing. [15]

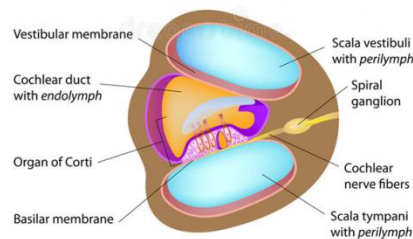


Figure 7

Composition	Perilymph	Endolymph
Na (mM)	140	1
K (mM)	4-5	150
Cl (mM)	110	130
Ca (mM)	1,2	0,02
Proteins (g/l)	1	0,15
Glucose (mM)	4	0,5
pH	7,4	7,4
Osmolarity (mosm/l)	290	315
Potential (mV)	0	80

Figure 8: Composition of the two cochlear fluids [16]

## 2. LITERATURE REVIEW

The Cochlea is a major sensory component in the auditory system. It is impossible to explicitly visualize cochlear movements. Therefore, Cochlear Modeling is a significant approach to realize the efficacy of cochlear function. The model of the inner ear as a one-dimensional structure is a traditional technique [17,18,19,20,21,22].

Cochlea's model occurs in various forms and formulations. These structures are made up of beams or plates and fluids [23] which constitute electrical properties [24] such as amplifiers, diodes, capacitors, resistance, and inductors. Mathematical construction and calculation of the structures are possible. The newly developed finite-element approach employed in several 3-dimensional concepts [Figure 12 ] in the middle ear [25,26,27,28,29,30,31]. The earlier model of one-dimension [32,33] held that the fluid pressure of the cochlea channel is constant. Ranke and Zwislocki also made similar assumptions in their Two-dimensional model. Ranke [34] used the deep water approximation in his model, while Zwislocki [35] used the shallow water theory. However, measurement results [36] have found some discrepancies with these assertions. As a result, Beyer [37] developed a 2-dimensional model of computation which holds that the Cochlea is flat with rectangular strips. The strips are equally split into 2-sections by a line referred the basilar membrane [Figure 11].

There are also many cochlea models, such as the Cochlear model of the nonlinear time domain [13,14,15,16], accompanied by an auditory nerve (AN) response model. An outer hair cell model [17,18] computed mechanical models [38,39,40,41,42]. and physical models. These models are made from metal and plastic material or electrical network [43,44,45]. Every slice of the model in which the Cochlea is split longitudinally into finite segments has about 1 to over 1000 degrees of freedom [46,47]. Before, the cochlea models were meant for simulating the phase and magnitude of the passive, linear responses into one stimulation tone [48,49,50,51,52]. The subsequent development of the models' incorporated nonlinearity [53,54,55,56] and active processes. The nonlinear models used either the perturbation [57,58,59] or iterative techniques to solve the frequency domain or in the time domain [60,61,62,63,64,65]. The first discovery of the



nonlinearity of the Cochlea was made in 1971 by Rhode [66], who reported that higher-level stimuli have less frequency selectivity in BM's response to sinusoidal stimuli. The advancement of measurement technologies further affirmed that the Cochlear is both functional and nonlinear.

There is the simulation of neural-like tuning in basilar membrane displacements by the responsive prototypes of cochlear mechanics. Passive models can simulate postmortem measurements of the basilar membrane displacements. The prevailing advancement in computational resources that provide numerical solutions has enhanced the rapid development of mathematical models of cochlea mechanics.

The processing of signal through the cochlear can be represented by a wavelet transform [67,68].

$$Wf(s, u) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t-u}{s} \right) dt \text{----- (i)}$$

The wavelet work is designed to find some form of equilibrium between the time domain and the frequency domain. One can see extremely low-frequency sections everywhere when it is extended and interpreted the parent wavelet, whereas a high-frequency section can be decisively located at little  $s$ . At the heart of this representation, there are two basic rules: linearity and scaling. It is understood from their research that the Cochlea conducts a wavelet transformation. The cochlear channel characterizes the wavelet. The fact that a wavelet transformation is played out by the Cochlea - in a first guess - seen in the above writing in 1992.

Wavelets refer to the linear transformation process. The wavelets are reliant on parameters and this method is consistent with the statement of signal processing by wavelet transformations process in the cochlea.

To date, the investigator has proposed a variety of potential alternatives to seeking a suitable technique for a successful solution to the hearing system. Here, we proposed a new approach to the hearing system to achieve a simple, workable solution.

In our newly proposed method, we have also followed these typical methods to present the cochlear function. However, the main difference of our method with the available literature is that it has been developed based on the latest computational scheme which makes the calculations very convenient. Due to the advancement of computational language and associated

coding, this method provides the desired results immediately following a simple input (with some basic data). In fact; the proposed method shows very user-friendly and contains a two-step simple process.

### 3. THE THEORETICAL CONCEPT OF THE HEARING SYSTEM

Sensory organs such as skin, tongue, ear, and eye are specifically sensitive to certain forms of energy. For instance, chemical energy is detected by the nose and tongue. On the other hand, light energy is detected by the eye, while mechanical energy is detected by the skin. The existence of specialized sensory cells makes it possible for all the sensory organs to convert the signals from the environment into electrical energy. The electrical energy contains charges and is converted into numeral code, which is then passed to the brain. The cochlea in the inner ear is a complex three-dimensional system in which the sensory hair cells [Figure 9] codify sound into electrical impulses that pass to the brain through the auditory nerve.

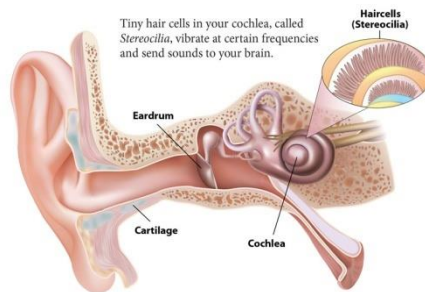


Figure 9

The Cochlea is made up of a spiral of tissue filled with fluid in our inner ears and thousands of tiny hairs [Figure 9] that progressively get smaller from the outside of the spiral to the inside. Every hair is attached to the nerve. The longer hair resonates at lower frequencies, and the smaller hair resonates at higher frequencies [Figure 10]. Sounds is complex wave composed of various signals having distinct amplitudes and frequencies

**How does sound travel through the ear to the brain:**

- i. The pinna captures sound waves and directs them down the outer auditory channel, where the tympanic membrane is struck and vibrated.
- ii. Eardrum vibrations force a series of three tiny bones toward each other: the malleus, the incus, and the stapes, pushing them back and forth.
- iii. Inside the cochlea, the movement of the stapes against the cochlea transmits pressure waves into the fluid, triggering the vibration of the fluid.
- iv. The fluid movements trigger tiny hair cells that move softly back and forth inside the cochlea. As the hair cells move, they release chemical signals that activate nerve fibers near the cochlea.
- v. The signals are sent to the auditory nerve and to the brain by the nerve fibers

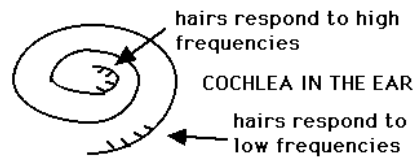


Figure 10

**3.1. Mathematical Modeling**

**3.1.1. Linear Model:** An auditory signal  $f(t)$  in the form of a pressure vacillation causes the motion  $u(x, t)$  of the basilar membrane [Figure 11] in the  $x$ -direction in the cochlea. At a specified level of acoustic pressure, the interaction between the input signal and the activity of the basilar membrane is remarkably linear. Nevertheless, the process is extremely impressive with regards to sound intensity, and therefore, it can not be linear. The "quasilinear pattern" is taken care of in the present environment. It is a model that relies on the factors, e.g. the sound pressure level in the present scenario. The model is a linear one for set parameters. The values of these set parameters are represented as a linear expressions of the procedure. An example of a 2-dimensional frame with a fluid thickness study [69,70]. In this concept, a plane depicts the outline of the cochlear and the BM by an endless line separating the plane into two sections.

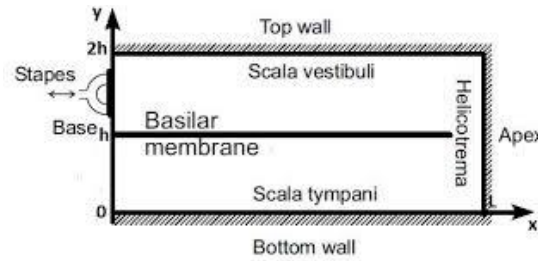


Figure 11: A 2-dimensional model of the cochlea

**3.1.2. Non-linear Modeling**

To display the non-linear characteristics of the Cochlea, To understand the context we need to explain the non-linear function. Usually, the independent variable is  $u$  and the dependent variable is  $V$

$$V = H(u) \text{ -----(ii)}$$

If any constant factor-like  $\beta$  multiplies  $u$  then the relation is

$$\beta V = H(\beta u) \text{ -----(iii)}$$

For multiple variables of input data  $u_1, u_2, \dots, u_n$ , then the relationship is

$$H(u_1 + u_2 + \dots + u_n) = H(u_1) + H(u_2) + \dots + H(u_n) \text{ -----(iv)}$$

The linear method can not produce signal components that are absent in the stimulation range, nevertheless, every non-linear method can generate Harmonic distorting goods in reaction to different tonal stimulation. So more complicated stimuli generate more diverse distortion spectrums of the substance. A time-domain analysis is typically appropriate to analyze cochlear nonlinear behavior.

In order to continue the time domain study, all applicable machine equations should be placed through differentiation and integration where applicable.

Typically, the association between pressure differential,  $p$ , and acceleration of BM,  $\ddot{\omega}$ , at various locations,  $x$ , around the cochlea, may be represented as follows

$$\frac{\delta^2 p(x)}{\delta x^2} = -\frac{2\rho}{h} \ddot{\omega}(x) \text{ ----- (v)}$$

The behavior of BM relates to the differentiation in pressure as

$$p(x) = m(x) \ddot{\omega} + r(x, \dot{w}) \dot{w} + s(x) w, \quad \text{-----} \quad \text{(vi)}$$

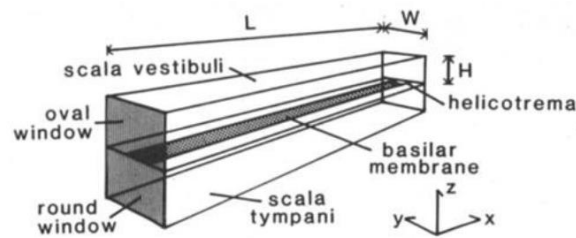
where the assumption is that mass is a stable

From the equations (v) & (vi),

$$[r(x, \dot{w}) \dot{w} + m(x) \ddot{\omega} + s(x) w]_{xx} = 2\rho/h \dot{w}(x), \quad \text{-----} \quad \text{(vii)}$$

where  $[ ]_{xx}$  means 2<sup>nd</sup> times derivatives with respect to x.

The differential equations of 2<sup>nd</sup> order can also be modified as sets of differential equations of 1<sup>st</sup> order, that can be formulated using method Runge-Kutta.



**Figure 12: A 3-dimensional model of the cochlea.**

The "shell" is separated into two part and packed with incompressible inviscid liquid (Fig. 11, 12 & 13). The top section is scala vestibule and the lower section is scala tympani, the division is geometrically symmetrical. Here, L indicates the Box length, W means width and H means height.

The pressure Dispersion shall differ seamlessly at the canal boundaries

$$0 < x < L ; 0 < y < W ; 0 < z < H$$

Say, p denotes the gap between the top portion channel  $p_{uc}$  and bottom portion channel  $p_{lc}$  in pressure

$$p(x, y, z) = p_{lc}(x, y, z) - p_{uc}(x, y, z) \quad \text{-----} \quad \text{(viii)}$$

P needs to fulfill Laplace's theorem with relevant boundary limits and the fundamental calculation may be formulated for cochlear macro mechanics as follows.

$$\nabla^2 p \equiv \frac{\delta^2 p}{\delta x^2} + \frac{\delta^2 p}{\delta y^2} + \frac{\delta^2 p}{\delta z^2} = 0 \quad \text{-----} \quad \text{(ix)}$$

Then p is a harmonic function.

At the border restriction, the fluid passes straight to the neighboring surface with the same flow effect.

and the acceleration at a basal wall is  $\ddot{\xi}_s \eta(y, z)$

$$\frac{\delta}{\delta x} p(0, y, z) = 2\rho \ddot{\xi}_s \eta(y, z) \text{-----(x)}$$

and the apical wall is stationary:

$$\frac{\delta}{\delta x} p(L, y, z) = 0 \text{-----(xi)}$$

Both immobile at the side walls are

$$\frac{\delta}{\delta y} p(x, 0, z) = 0 \text{-----(xii)}$$

And

$$\frac{\delta}{\delta y} p(x, W, z) = 0 \text{-----(xiii)}$$

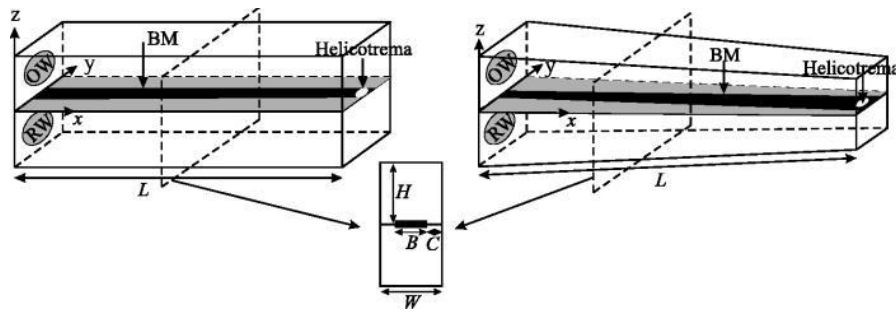


Figure 13

The acceleration in divider border is  $\ddot{\xi}_1(x)\xi(y)$  :

$$\frac{\delta}{\delta z} p(x, y, 0) = 2\rho \ddot{\xi}_1(x)\xi(y) \text{-----(xiv)}$$

and the top surface does not shift:

$$\frac{\delta}{\delta z} p(x, y, H) = 0 \text{-----(xv)}$$

Usually, a cochlear container layout is a 3-dimensional depiction of the cochlea, As the liquid has the potential to flow throughout all directions. The cochlear box is considered to be symmetrical. Above and below the BM are two liquid chambers of the same size [46]. Therefore, the pressure levels of both chambers are similar and opposite. and it is easier to operate using a

single distribution (x , y , z) , similar to the gap in pressure. which doubles the pressure for every chamber.

#### 4. METHODOLOGY

In the past, when researchers created different methods, programming language and skills did not improve so much at that time. Recent advances in rapid computing and the development of user-friendly methods have led to more practical approaches. In this case, the advanced computational language-Python has been followed in our proposed method and a programming platform-MATLAB. A necessary mathematical tool used in signal analysis is termed as Fourier Transform. The sound wave is represented in the time domain as a series of pressure changes (motions/oscillations) that occur over time.

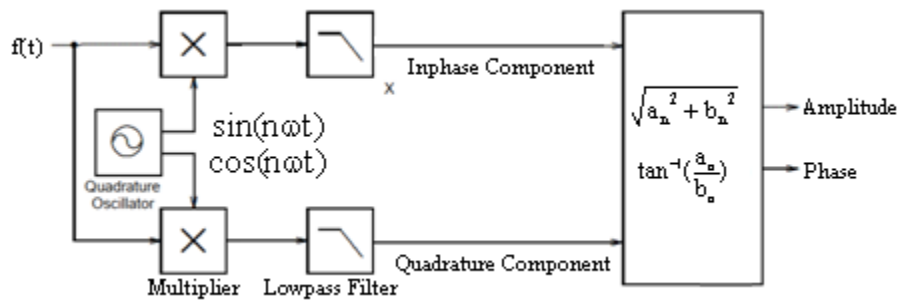


Figure 14

Fourier Transform changes the domain of the signal, time domain to frequency domain. A sinusoidal signal in the time-domain is the relating energy (amplitude) at that particular time. The spectrum thus represents the respective energy (amplitude) at that specific frequency when a Fourier transform is introduced. [Figure 14]

We have

$$g(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt \text{ ----- (xvi) } \quad [:\omega = 2\pi f]$$

$$g(\omega) = F[f(t)] \text{ -----(xvii)}$$

$$f(t) = F^{-1}[g(\omega)] \text{ -----(xviii)}$$

The equation (xvii) and (xviii) are named a Fourier transform pair;  $\mathbf{g}(\omega)$  is considered the Fourier transform of  $f(t)$  and conversely,  $f(t)$  is regarded as the inverse Fourier transform of  $\mathbf{g}(\omega)$ .

From the linear algebra level, the signal may be decomposed into a linear base combination if the signal is in a base spanned space, it is,

$$\mathbf{f}(t) = \sum_k \mathbf{a}_k \phi_k(t) \text{-----(xix)}$$

Here,  $k$  represents a pointer,  $\mathbf{a}_k$  is the coefficients of  $\phi_k(t)$  and  $\phi_k(t)$  are complex functions,

Again,  $\mathbf{f}(t) = \sum_{k=-\infty}^{\infty} \mathbf{c}_k e^{jk\omega t}$  ----- (xx) This is known as the complex form of the Fourier series.

From (xvii) & (xviii),  $\phi_k(t) = e^{jk\omega t}$

The Real part of the complex function  $e^{jk\omega t}$  is  $\cos \omega t$ , that is,  $\mathbf{R}(e^{j\omega t}) = \cos \omega t$  [

$$\because e^{j\omega t} = \cos \omega t + i \sin \omega t ]$$

## 5. PROPOSED METHOD

The stages of the proposed new approach are listed below

Phase 1: The ear receives sound waves which are broken down into its constituent sinusoids with different frequencies and their corresponding amplitudes, which is mentioned frequency spectrum (line/amplitude spectrum) of the wave.

Phase 2: the conic shape of the cochlea makes a certain frequency resonate at a certain point inside the cochlea.

Phase 3: The signal can be synthesized in an inverse Fourier transformation process by adding its constituent frequencies to the brain.

### 5.1. Algorithm of the Proposed Approach

Following the aforementioned three phases, an algorithm is developed and presented in Table 1.

Infact, this algorithm forms the key workflow or the computational instruction to execute the



calculation. Based on this algorithm, the necessary computational code (program code) has been developed using the python programming language. The developed computer program or code is shown in Table 2. With the help of this algorithm, one can easily understand the computer coding language.

**Table 1:** Algorithm for matching line spectrum with the Hair cells.

1.	PROGRAM Handshaking with Line spectrums of a complex wave by Haircells (Stereocilia) in Cochlea
2.	INPUT: The equation of $a_n$
3.	INPUT: The equation of $b_n$
4.	INPUT: The value of frequency $nw$
5.	INPUT: The value $n$ (No of line spectrum)
6.	
7.	FOR $i:=1$ to $n$
8.	CALCULATE: The value of $a_i, b_i$
9.	CALCULATE: The value of amplitude $R_i = \sqrt{a_i^2 + b_i^2}$
10.	CALCULATE: The value of frequency $iw$
11.	DRAW: line spectrum with respect to frequency= $iw$ and amplitude= $R_i$
12.	ENDFOR
13.	END
14.	
15.	PROGRAM match line spectrum with the haircells
16.	FOR $i:=1$ to $n$
17.	SET: Flag:= 0
18.	FOR $j:=1$ to the all thehaircells
19.	IF line spectrum frequency( $iw$ ) equal to haircells_frequency of( $j$ ) and line spectrum
20.	amplitude( $R_i$ ) equal to haircells_height of ( $j$ )
21.	THEN line spectrum( $i$ ) match to the haircells( $j$ )
22.	PRINT: Haircell( $j$ ) receives and Handshakes with Line spectrum( $i$ )
23.	SET: Flag:= 1
24.	
25.	ENDIF
26.	ENDFOR
27.	IF Flag equal to 0 (Line spectrum not match with any haircells)
28.	PRINT: Line spectrum( $i$ ) not receive by any Haircell
29.	ENDFOR
30.	END

**Table 2:** Code for the above algorithm:

```

import math
import matplotlib.animation as ani
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D

def fun_an(an, n):
    return eval(an)
def fun_bn(bn, n):
    return eval(bn)
def fun_nw(nw, n):
    return eval(nw)
input_an = input('Enter an equation in n: ')
input_bn = input('Enter bn equation in n: ')
input_nw = input('Enter nw equation in n: ')
input_n = input('Enter the value of n: ')
n=eval(input_n)
fig=plt.figure(figsize=(6,5))
ax1=fig.add_subplot(111,projection="3d")
frequency=[]
amplitude=[]
colors=["r","m","g","y","c","orange"]
for i in range(1,n+1):
    ai=fun_an(input_an,i)
    bi=fun_bn(input_bn, i)
    ri=math.sqrt(ai*ai+bi*bi)
    amplitude.append(ri)
    iw=fun_nw(input_nw, i)
    frequency.append(iw)
ax1.bar(frequency,amplitude,zs=1,zdir="y",color=colors,alpha=1)
frequency_hair=[]
height_hair=[]
for i in np.arange(1.0,n+1,0.3):
    ai=fun_an(input_an,i)
    bi=fun_bn(input_bn, i)
    ri=math.sqrt(ai*ai+bi*bi)
    height_hair.append(ri)
    iw=fun_nw(input_nw, i)

```

```

frequency_hair.append(iw)
ax1.bar(frequency_hair,height_hair,zs=10,zdir="y",color="k",alpha=1)
ax1.set_xlabel('frequency domain (nw)')
ax1.set_ylabel('Amplitude Spectrum ')
plt.show()

def isclose(a,b):
    astr=str(a)
    aprec=len(astr.split('.')[1]) if '.' in astr else 0
    bstr=str(b)
    bprec=len(bstr.split('.')[1]) if '.' in bstr else 0
    prec=min(aprec,bprec)
    return round(a,prec)==round(b,prec)
for i in range(0,n):
    flag=0
    for j in range(len(frequency_hair)):
        if(isclose(frequency[i],frequency_hair[j]) and isclose(amplitude[i],height_hair[j])):
            flag=1
            print("haircell(",j,") will receive line spectrum(",i,") and handshake")
    if flag==0:
        print("any haircell will not receive line spectrum(",i,")")

```

## 6. IMPLEMENTATION

### Example 01:

An audio signal 'example\_WAV\_1MG.wav' file was read and sampled at 22 kHz. We know the frequency at which the audio was sampled in order to continue with the analysis. Sounds that we hear are complex sounds, meaning that they are composed of several frequencies. These frequencies can be isolated by the cochlea. The simplest way to break down an audio signal into its frequency components using MATLAB.

*MATLAB Code:*

```

>> [y, Fs] = audioread('example_WAV_1MG.wav') % write a matrix of audio data, y, with
sample rate Fs
>> sound(y,Fs) % listen to sound of the above wave.
>> z=length(y)
>> T = length(y)/22000

```

```

>> f(length(y)+1) = 0
>> t = T*[0:z-1]/z % Create a vector t the same length as y, that represents elapsed time.
>> plot(t,y)
>> xlabel('Time')
>> ylabel('Audio Signal')

```

Now plotting the energy spectrum after taking this signal's FFT on both axes semilog axes and regular axes

```

>> w = fft(y);

```

### Result:

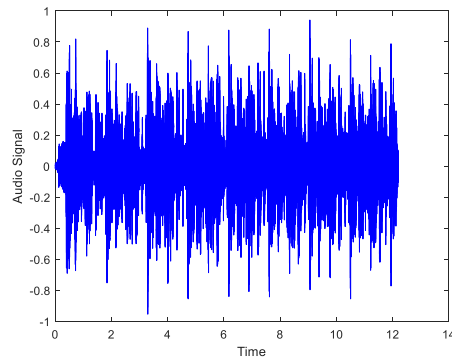


Figure 15: time domain

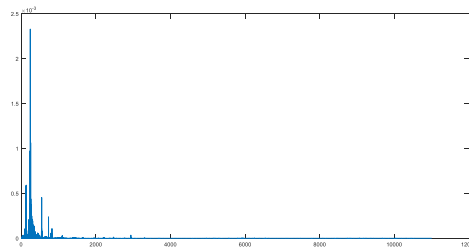


Figure 16: Frequency domain

Here it is seen that almost all of the energy is below 1 kHz. and no power over 4 kHz.

Considering clip out all frequencies greater than about 2 kHz. This refers to approximately the 30,000<sup>th</sup> frequency in the FFT (15 sec \* 2000 Hz) = 30, 000.

**Example 02:** Let the equation of a complex wave  $f(t) = \pi - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi t$

Here,

$$a_n = 0; b_n = \frac{2}{n}; n\omega = n\pi$$

The output of the above complex wave and its amplitude spectrum done by MATLAB shown below:

### Result

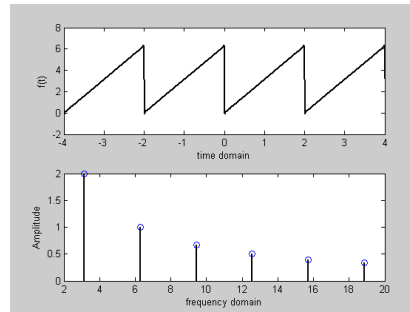


Figure 17

**Example 03:** The Code for computational approach on signal propagation inside the Cochlea is shown below using python.

**Table 3:** Python code for the figure 18

```
import matplotlib.animation as ani
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
from mpl_toolkits.mplot3d import Axes3D
fig=plt.figure(figsize=(5,5))
ax1=fig.add_subplot(111,projection="3d")
#ax1.view_init(30, 30)
#ax1.view_init(25,-135)
ax1.view_init(25,-45)
plt.xlim([3, 19])
plt.xticks(np.arange(3.1416, 19, 3.1416))
colors=["r","m","g","y","c","orange"]
ax1.margins(y=.1, x=.1,z=.1)
def draw_barchart(z_axis):
    ax1.clear()
    ax1.set_xlim(3.1416, 19)
    ax1.set_xticks(np.arange(3.1416, 20, 3.1416))
    n=np.arange(start=1, stop=7, step=1)
    n1=np.arange(start=1, stop=6.3, step=0.2)
    x2=np.multiply(n1,np.pi)
    y2=np.divide(2, n1)
    x1=np.multiply(n,np.pi)
    y1=np.divide(2, n)
    plt.subplots_adjust(wspace=0.5, hspace=0.5, left=0.2, bottom=0.2, right=0.9, top=0.9)
    ax1.bar(x1,y1,zs=1,zdir="y",alpha=0)
```

```

ax1.bar(x1,y1,zs=z_axis,zdir="y",color=colors,alpha=1)
ax1.bar(x1,y1,zs=10,zdir="y",color="royalblue",alpha=1,label='Haircells(Stereocilia)')
ax1.bar(x2,y2,zs=10,zdir="y",color="k",alpha=0.4,label='Haircells(Stereocilia)')
ax1.set_xlabel('frequency domain (nw)')
ax1.set_ylabel('Amplitude Spectrum ')
#ax1.set_zlabel('ZZZZZZZZZZZZ(z)', linespacing=3.4)
plt.tight_layout()
ax1.legend()
plt.box(False)
draw_barchart(1)
plt.show()
import matplotlib.ticker as ticker
import matplotlib.animation as animation
from IPython.display import HTML
animator = animation.FuncAnimation(fig, draw_barchart,frames=range(1,11),interval=500)
animator.save('code.gif', writer='pillow')
HTML(animator.to_jshtml())

```

## Result:

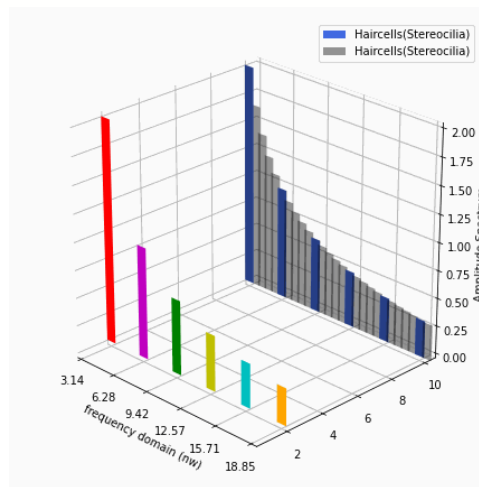


Figure 18 [Live: <https://bit.ly/3n00Vjm>]

For the above figure 18, the following link is shown for Live Signal Propagation within the Cochlea. <https://bit.ly/3n00Vjm>

## 7. DISCUSSION

This paper examines current knowledge and reports new findings on some of the nonlinear processes underlying the mammalian cochlea's work. These processes take place within mechano-sensory hair cells that form part of the organ of Corti.

In example 1, A complex signal has been taken as a sample. The Matlab code shows the frequency and amplitude of the sine wave inside this signal. Figure 15 shows the complex wave in the time domain and figure 16 shows the frequency spectrum of complex wave in the frequency domain.

In example 2, an equation of a complex signal has been taken as a sample. The Matlab code presents the frequency and amplitude of the sine wave inside this complex signal. Figure 17 shows the complex wave in the time domain and the corresponding frequency spectrum of complex wave in the frequency domain.

Figure 18 shows the amplitude of 6 different frequencies in the frequency domain. Here 6 different (as a sample) frequencies are shown with 6 different colors. These are labeled line spectrum or amplitude spectrum or frequency spectrum. And the hair cells (Stereocilia) of the cochlea inside the ear are characterized by blue and gray color. When a wave enters the ear then it is divided into different frequencies. The small hairs in the cochlea resonate with the high frequency of sine wave and the long hairs in the cochlea resonate with the short frequency of sine wave. It then travels through the nerves to the brain. How this work is done is shown live in a link (<https://bit.ly/3n00Vjm>) made by python language. If the sound or words entered the ear are not fully heard, the speech is impaired, it is important to understand that a few hair cells are not working in the cochlea. Then someone needs to take the help of a doctor. How the frequency spectrum is resonated by the hair-cell is shown three-dimensionally in this paper.

Interestingly, the Cochlea performs the Fourier transform, which is a mathematical tool. In other words, the inner ear can perform complex calculus and has been doing so way before the discovery of Fourier transforms in the 19th century. Additionally, the auditory nerve and the hair cells are responsive to a specific frequency of the sound refers to characteristic frequencies (CF).

With the entry of force generation, it is possible to tap the CF of hearing nerve and hair cells and pinpoint their exact location with the aid of a map of tonotopic.

A complex signal  $f(t)$  may be viewed as a set of sinusoidal waves. Such sinusoidal forms with varying types, wavelengths, and voltage levels. The representation of the amplitude, phase, and frequency of the various signals is referred to as the *spectrum* of the signal or light. Fourier Series is a method of evaluating a time-domain signal in order to identify its frequency. In a nutshell, the time-domain signal is transformed into a frequency-domain through FT. Conversely, a spectrum may be converted back into a time-domain through the inverse FT. Sounds consist of a variety of distinct frequencies. Fundamentally, the basilar membrane behaves like a mechanical FFT computation, and the array of cilia and neurons act as a bank of band-pass channels. This is the moment when we hear the sound, for example, music and speech, our brain is getting a bank of action potentials that relate to the FFTs.

## **8. CONCLUSION**

Our proposed method is designed based on a more proper algorithm together with an advanced computational capability. The existing methods in the literature have been developed long ago but we developed our method through a new computational approach; hence our method shows better result. Due to the advancement of computer programming, the proposed method shows very simple to realize and needs a limited amount of computational time relative to other approaches available in the literature.

This paper explores the complex properties of the mammalian cochlea and provides some preliminary findings to demonstrate its unique non-linear components. The modeling has enabled the understanding of the nature and quantity of knowledge on signal processing and processing systems in Cochlea. Importantly, it has provided knowledge of the delicate balance between the phase and amplitude. Basic research is still needed in the hearing system for more accuracy. It is necessary for cochlear modelers to understand the intended application of models in the clinical sector.



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**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

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