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(3,1) – TOTAL LABELING OF CYCLE, PATH GRAPHS AND SOME RELATED GRAPHS

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Abstract: The (3,1)- total labeling of a graph G is an assignment of non- negative integers to vertices and edges of G such that no two adjacent vertices or edges have the same label and the difference of labels between a vertex and its incident edges is at least 3. In this paper, we study the (3,1)- total labeling for some types of graphs and derive their (3,1)- total labeling number.

Keywords: graph; graph labeling; total graph; regular graph and cycle.

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1. INTRODUCTION

The frequency assignment problem is to assign a frequency (non- negative integers) to a group of radio transmitters such that assign frequency with at least minimum separation allowed for interfering transmitters. Motivated from this problem, the distance labeling problem of graphs is defined so that finds a proper assignment of channels in a wireless network.

Motivated from this point of study, the frequency assignment problem was defined as a vertex

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coloring problem of graph by Hale [1], Robert [1988] proposed a variation of this problem such that “very closed” transmitters must receive a frequency of at least two apart and “closed” transmitters must receive different frequency.

In graphs, two distinct vertices x and y are said to be “very closed” if they are of distance 1or there is an edge between them and to be “closed” if the distance between them is 2 or there is two edges between them.

Several types of labels are defined on graphs, Griggs and Yeh [2] defined the $L(2,1)$- labeling of graphs as a function f from vertex set $V(G)$ to the set {0, 1, 2, 3, ………, λ} such that $|f(x)-f(y)| \geq 2$ if $d(x,y)=1$ and $|f(x)-f(y)| \geq 1$ if $d(x,y)=2$ where $d(x,y)$ represents the minimum number of edges between x and y. The difference between the highest and lowest label used is the $L(2,1)$- labeling number and denoted by $\lambda_{2,1}$. By the same way the $L(3,2,1)$- labeling is defined as a function f from the set vertices to the set of non- negative integers such that $|f(x)-f(y)| \geq 3$ if $d(x,y)=1$, $|f(x)-f(y)| \geq 2$ if $d(x,y)=2$ and $|f(x)-f(y)| \geq 1$ if $d(x,y)=3$. Amanathulla and Pal [3] defined L(3,2,1)- and $L(4,3,2,1)$-labeling for interval graphs and derive an upper bound for the labeling number for each case; Shao, Yeh and Zhang [4] considered the total graph and defined an upper bound for the labeling number $\lambda(G)$, Ma and Wang [5] defined the (2,1)- total labeling of double graphs for some graphs as star, path and cycle and derived the value of (2,1)-total labeling numbers for these types of graphs. Motived from all these points, we derive $(3,1)-$total labeling number for some graphs such as even cycle and the total graph for odd cycle.

2. PRELIMINARIES

Lemma 2.1 F. Havet and M. Yu [6]: Let G be a graph of maximum degree Δ, then:

(a) $\lambda_d^T(G) \geq \Delta + d - 1$

(b) If G is $\Delta-$ regular, then $\lambda_d^T(G) \geq \Delta + d$

Definition 2.1 Let G be a graph, an assignment of integers to the vertices of G or edges or both of them under certain types of conditions is called graph labeling.

Definition 2.2 A path of vertices of G with vertex sequence $\{v_1, v_2, \dots \dots \dots \dots \dots, v_1\}, v_i \neq v_j, i \neq j$ is called a circuit and a graph consist of a single circuit is called cycle. If cycle with odd numbers of vertices then it called odd cycle, otherwise it called even cycle.

Definition 2.3 The total graph $T(G)$ of a graph $G = (V, E)$ is a graph obtained from G by considering its vertex set $V\big(T(G)\big) = V(G) \cup E(G)$ and its corresponding edge set $E(T(G))$ be a set of edges of two adjacent vertices of G and a vertex with its incident edges from G.

Definition 2.4 A graph G for which all vertices have the same degree, Δ then G is called Δ-regular graph or simply regular graph.

Definition 2.5 (3, 1) - Total labeling of a graph G is considered as the width of smallest range of integers that suffices to label vertices and edges of G such that no two adjacent vertices have the same label, no two adjacent edges have the same label and the difference between the labels of a vertex and incident edges at least 3

Definition 2.6 Split for a graph G denoted by $spl(G)$ is obtained by adding to each vertex v a new vertex $\grave{v}$ such that $\grave{v}$ is adjacent to every vertex that adjacent to v.

Definition 2.7 Poulomi, Sachehida and Anita [7] The shadow graph $D_2(G)$ of a connected graph is constructed by taking two copies of G say G_1 and G_2. Join each vertex u_1 in $G_1$to the neighbors of the corresponding vertex u_2 in G_2.

3. MAIN RESULTS

Theorem 3.1 $\lambda_3^T\big(T(G)\big)$ for the total graph of odd cycle with $n = 3$ is 7 and for $n > 3$ is greater than or equal 7.

Proof: Since the total graph for odd cycle is a regular graph of maximum degree $\Delta = 4$, then from lemma 2.1 $\lambda_d^T(T(G)) \geq \Delta + d$ and this imply that $\lambda_d^T(T(G)) \geq \Delta + 3 = 7$. As $n = 3$ the $(3,1)$ $-$total labeling number for the total graph of odd cycle is 7

Lemma 3.2 Let G be a cycle with even number of vertices, then $\lambda_3^T(G) = 5$.

Proof: The cycle is considered as a regular graph with maximum vertex degree $\Delta = 2$ and from $\lambda_d^T(G) \geq \Delta + d = 5$ therefore $\lambda_3^T(G) = 5$ if G is an even cycle.

Theorem 3.2 $\lambda_3^T(T(G))$ for total graph of $G = P_2$ is 6.

Proof: Consider a path P_2 its total graph is a cycle of order 3. Since C_3 is a 2- regular graph; its (3, 1) – total labeling number is $\lambda_3^T(G) \geq \Delta + 3$ then $\lambda_3^T(T(P_2)) = 6$ by observation.

Theorem 3.3 $\lambda_3^T(D_2(P_2)) = 5$ and $\lambda_3^T(D_2(P_n)) = 7$ for $n \geq 3$

Proof: Consider a path with two vertices; its shadow graph is a 2 – regular graph as shown in figure 1, from lemma 2.1 we have $\lambda_d^T(G) \geq 5$. Consider a path graph with $n \geq 3,$ the shadow graphs for P_n, with $n = 3, 4, 5$ are shown in figures 2, 3 with (3,1) – total labeling and $\lambda_3^T(D_2(P_n)) = 7$, this proof the theorem.

$$\lambda_3^T(D_2(P_n)) = \begin{cases} 5 & n = 2 \\ 7 & n \geq 3 \end{cases}$$

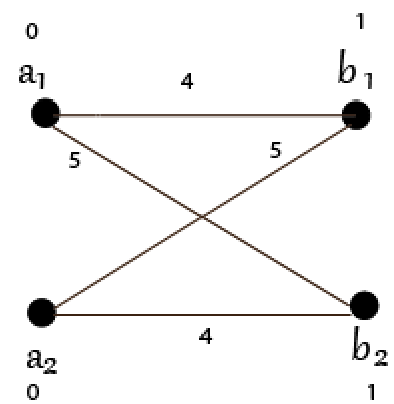

Figure 1: A shadow graph for a path with two vertices

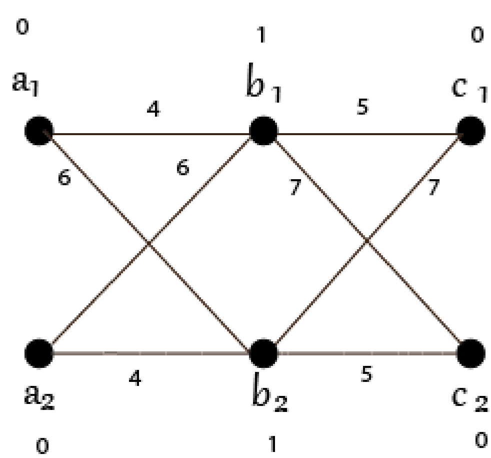

Figure 2: A shadow graph for a path with three vertices

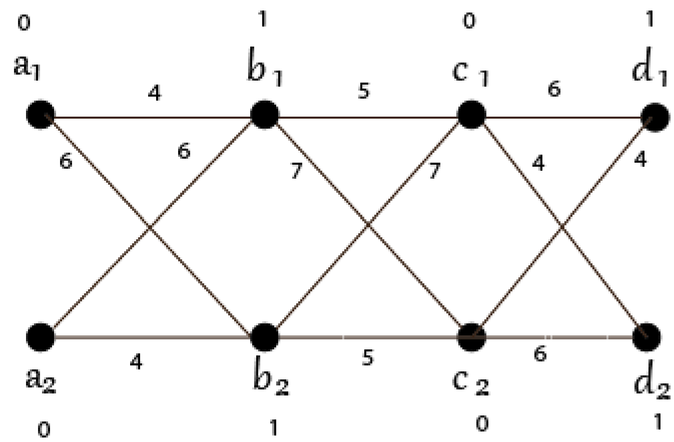

Figure 3: A shadow graph for a path with four vertices

Theorem 3.4 $\lambda_3^T\big(spl(P_2)\big) = 5$ and $\lambda_3^T\big(spl(P_n)\big) = 7$ for $n \geq 3$

Proof: Consider split of a path with two vertices, we can label left most vertices with 0 and right most vertices with 1, according to figure 4 we have $\lambda_3^T\big(spl(P_2)\big) = 5$. Consider a split of a path with three vertices, we can labeling the left most and right most vertices with 0 and middle vertices by 1, also according to figure 5 we have $\lambda_3^T\big(spl(P_3)\big) = 7$, and the same result with a split of a path with four vertices as shown in figure 6. We can obtain the following result:

$$\lambda_3^T\big(spl(P_n)\big) = \begin{cases} 5 & n = 2 \\ 7 & n \geq 3 \end{cases}$$

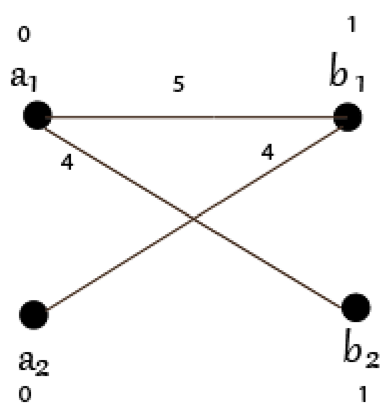

Figure 4: A split graph for a path with two vertices

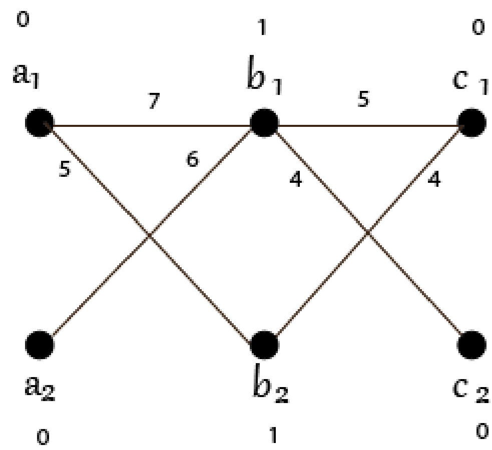

Figure 5: A split graph for a path with three vertices

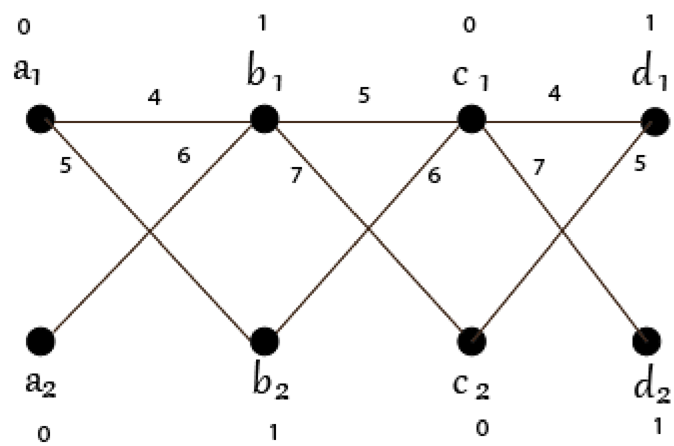

Figure 6: A split graph for a path with four vertices

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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