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# MHD HEAT AND MASS FLOW OF NANO-FLUID OVER A NON-LINEAR PERMEABLE STRETCHING SHEET

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**Abstract.** In this study, we examine the Magneto-hydrodynamics (MHD) heat and mass flow of nano-fluid over a non-linearly permeable stretching sheet. The resulting partial differential equations are converted to a system of ordinary differential equations using the similarity transformation and solved numerically using shooting technique with fourth order Runge-Kutta method. The effect of some fluid parameters on the momentum, thermal and nanoparticle volume fraction boundary layers were expatiated for prescribed surface temperature and constant surface temperature through graphs. An excellent agreement was found when the results obtained in this study were compared with results in previous literature.

**Keywords:** nano-fluid; magneto-hydrodynamics; porous medium; non-linear stretching sheet. **2010 AMS Subject Classification:** 76D05, 76W05.

#### **1.** INTRODUCTION

In the past few years, MHD heat and mass flow has gained tremendous ground in engineering and scientific processes. Boundary layer flows due to a stretching sheet played important roles in technological processes such as paper production, hot rolling, glass-fibre production, extraction of polymer sheets etc. Sakiadis [1] did the pioneer work on continuous flow of a boundary layer.

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Crane [2] analyzed boundary layer flow past a stretching plate. A stretching sheet with suction or blowing using the method of similar solution was studied by Gupta and Gupta [3].

Using numerical methods, Vajravelu [4] worked on heat transfer over a non-linear stretching sheet. He showed that the heat flow is always from the stretching sheet to the fluid. Wang [5] researched on suction and surface slip in a viscous flow on a stretching sheet. Cortell [6, 7] investigated stretching surface of a fluid with internal heat generation through a porous medium. Prasad et al [8] analyzed non-linear stretching sheet of mixed convection with variable fluid properties.

Lately, researchers began to show interest in nano-fluid due to its importance in many processes in heat transfer such as petroleum refining processes, pharmaceutical processes, engine cooling/vehicle thermal management, chiller, fuel cells, domestic refrigerator, in grinding etc. Nano-fluid is a fluid containing nano-particles (less than 100nm). Common base fluids include water, ethylene glycol and oil (convectional mineral oils, crude oil and oils refined from crude oil). Khan and Pop [9] studied stretching sheet of a nano-fluid flow. Rana and Bhargava [10] investigated a non-linear stretching sheet of a nano-fluid flow and heat transfer. Ibrahim and Shankar [11] studied a nano-fluid transfer past a permeable stretching sheet with velocity, thermal and solutal slip. They concluded that local Nusselt number decreases with an increase in Brownian motion and thermophoresis parameter. Fauzi et al [12] worked on a non-linear shrinking sheet with slip effects at a stagnation point flow. Das [13] investigated a nano-fluid flow with partial slip over a non-linear permeable stretching sheet. He showed that an increase in the slip parameter and non-linear stretching parameter leads to decrease in the velocity of the nano-fluid. Eid and Mahny [14] researched on non-Newtonian nano-fluid flow of an unsteady MHD transfer over a permeable stretching wall with heat generation/absorption. Jahan et al [15] studied regression and stability analyzed on nano-fluid heat transfer past a convectively heated permeable stretching/shrinking sheet. Senge et al [16] investigated MHD flow over a stretching sheet in a thermally stratified porous medium.

The objective of this research work is to extend the work of Das [13] by examining the megnetohydrodynamics heat and mass flow of a nano-fluid over a non-linearly permeable stretching sheet. The ordinary differential equations are solved numerically using shooting method along 5198 IDONGESIT F. EKANG, ENOBONG E. JOSHUA, ISAAC O. SENGE, OBED O. NWACHUKWU

with fourth order Runge-Kutta method. The effects of the parameters on the fluid velocity, temperature and concentration distributions is discussed and shown graphically.

### 2. MATHEMATICAL MODELING AND FORMULATION

Consider an incompressible, two-dimensional steady boundary layer flow of MHD heat and mass transfer of a nano-fluid over a non-linear permeable stretching sheet coinciding with the plane y = 0. The flow takes place at  $y \ge 0$  (y is the coordinate measured normal to the surface of the stretching sheet in the vertical direction). The flow is generated as a result of a sheet which comes out of a slit at the origin (x = y = 0). The stretching velocity  $u_w = ax^n$  is assumed to vary non-linearly with the distance from the slit, where a > 0, n is a non-linear stretching parameter and x is the coordinate measured along the surface. The surface of the sheet is subjected to a Prescribed Surface Temperature (P.S.T.) and a Constant Surface Temperature (C.S.T.) which are given as:

(1) 
$$T = T_{\infty} + bx^r \qquad \text{at } y = 0$$

(2) 
$$T = T_{\infty} + (T_w - T_{\infty})\theta(\eta) \quad \text{at } y = 0$$

where b > 0, r is the surface temperature parameter in the Prescribed Surface Temperature (P.S.T.) boundary condition,  $T_{\infty}$  is the ambient temperature,  $T_w$  is the temperature at the surface. The surface is also maintained at constant concentration,  $C_w$  and its value is assumed to be greater than the ambient concentration,  $C_{\infty}$  which is given as:

(3) 
$$C = C_{\infty} + (C_w - C_{\infty})\phi(\eta) \quad \text{at } y = 0$$

A uniform magnetic field of strength  $B_o$  is assumed to be applied at y > 0 normal to the stretching sheet. The governing equations under this consideration are as follows:

(4) 
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
  
(5)  $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2}{\rho}u - \frac{v}{K}u$ 

(6) 
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau \left[ D_B \left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_{\infty}}\right) \left\{ \left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 \right\} \right]$$

(7) 
$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_{\infty}}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

The associated boundary conditions are:

(8) 
$$u = u_w + u_s, \quad v = \pm v_w, \quad T = T_w, \quad C = C_w \quad at \ y = 0$$

$$(9) u \to 0, T \to T_{\infty}, C \to C_{\infty} as y \to \infty$$

In equations (4) - (9), *u* and *v* are the velocity components along the *x* - and *y* - axes respectively, *v* is the kinematic viscosity,  $\sigma$  is the electrical conductivity,  $\rho$  is the fluid density, K is the constant permeability of the porous medium,  $\alpha$  is the thermal diffusivity,  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$  is the ratio between the effective heat capacity of the nano-particle material and heat capacity of the fluid, c is the volumetric volume expansion coefficient,  $\rho_f$  is the density of the base fluid,  $D_T$  is the thermophoretic diffusion coefficient,  $D_B$  is the Brownian diffusion coefficient,  $v_w$  is the suction/ injection and  $u_s$  is the slip velocity which is assumed to be proportional to the local wall stress as follows:

(10) 
$$u_s = l \frac{\partial u}{\partial y}|_{y=0}$$

where l is the slip length as a proportional constant of the slip velocity. Equations (5) - (7) are transformed into the non-dimensional ordinary differential equations by the following transformation

$$\begin{split} \Psi &= \left(\frac{2\nu a x^{n+1}}{n+1}\right) f(\eta), \quad \eta = \left(\frac{a(n+1)x^{n-1}}{2\nu}\right)^{\frac{1}{2}} y, \quad u = a x^n f'(\eta) \\ v &= -\left(\frac{\nu a(n+1)x^{n-1}}{2}\right)^{\frac{1}{2}} \left[f + \frac{(n-1)}{(n+1)}\eta f'\right], \quad T = T_{\infty} + b x^r \\ \theta(\eta) &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}} \end{split}$$

where  $\psi$  is the stream function defined as:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

The transformed ordinary differential equations are as follows:

(11)  
$$\left(1 + \frac{(n-1)^2 \eta^2}{2(n+1)Re}\right) f''' + ff'' + \frac{(n-1)(2n-1)\eta}{(n+1)Re} f'' - \frac{2n}{n+1} f'^2 + \frac{2}{n+1} \left[\frac{n(n-1)}{Re} - M - \lambda\right] f' = 0$$

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$$\begin{pmatrix} \frac{(n-1)^2\eta^2 + 2(n+1)Re}{4} \end{pmatrix} \frac{1}{P_r} \theta'' + \frac{(n+1)}{2} Ref \theta' - rRef' \theta + r(r-1) \frac{1}{P_r} \theta \\ + \frac{(n-1)(2r-1)\eta}{2P_r} \theta' + \frac{(n-1)}{2} r\eta Nb \phi' \theta + r^2 Nt \theta^2 \\ + (n-1)r\eta Nt \theta \theta' + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) Nb \phi' \theta' \\ (12) + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) \phi'' - \frac{(n-1)\eta}{2} \phi' + Lef \phi' + r(r-1) \frac{Nt}{Nb} \theta \\ + \frac{(n-1)(2r-1)\eta}{2} \frac{Nt}{Nb} \theta' \\ (13) + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) \frac{1}{P_r} \theta'' + \frac{(n+1)}{2} Ref \theta' + \frac{(n-1)(n-3)\eta}{4P_r} \theta' \\ + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) Nt \theta'^2 = 0 \\ \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) \frac{1}{P_r} \theta'' + \frac{(n+1)}{2} Ref \theta' + \frac{(n-1)(n-3)\eta}{4P_r} \theta' \\ + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) Nt \theta'^2 = 0 \\ \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) \phi'' + \frac{(n+1)}{2} Lef \phi' - \frac{(n-1)}{2} \eta \phi' + \frac{(n-1)(n-3)}{4} \eta \frac{Nt}{Nb} \theta' \\ (15) + \left(\frac{(n-1)^2\eta^2 + 2(n+1)Re}{4}\right) \frac{Nt}{Nb} \theta'' = 0 \end{cases}$$

Subject to the boundary conditions as follows:

(16) 
$$f(\eta) = F_w, \quad f(\eta) = 1 + \beta f''(\eta), \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \text{ at } \eta = 0$$

(17) 
$$f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty$$

where

$$Re = \frac{xu_w}{v} \text{ is the Reynolds number}$$

$$M = \frac{x\sigma B_0^2}{u_w \rho} \text{ is the Magnetic parameter}$$

$$\lambda = \frac{xv}{u_w K} \text{ is the Permeability parameter}$$

$$Pr = \frac{v}{\alpha} \text{ is the Prandtl number}$$

$$Le = \frac{xu_w}{D_B} \text{ is the Lewis number}$$

$$Nb = \frac{\tau D_B(C_w - C_\infty)}{v} \text{ is the Brownian motion parameter}$$

$$Nt = \frac{\tau D_T(T_W - T_\infty)}{vT_\infty} \text{ is the thermophoresis parameter}$$

$$F_w = -\frac{v_w}{\left(\frac{av(n+1)x^{n-1}}{2}\right)^{1/2}} \text{ is the suction/injection parameter}$$

$$\beta = l\left(\frac{a(n+1)x^{(n-1)}}{2v}\right)^{1/2} \text{ is the slip parameter for liquids}$$

The physical quantities of main interest are the skin friction coefficient,  $C_f$ , the local Nusselt number,  $Nu_x$ , and the Sherwood number,  $Sh_x$ , which are defined as follows:

(18) 
$$C_f = \frac{\tau_w}{\rho u_w}, \quad N u_x = \frac{xq_w}{k_1(T_w - T_\infty)}, \quad S h_x = \frac{xJ_w}{D_B(C_w - C_\infty)}$$

where

$$\tau_{w} = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
 is the surface shear stress  
$$q_{w} = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$$
 is the heat flux  
$$J_{w} = -D_{B} \left(\frac{\partial C}{\partial y}\right)_{y=0}$$
 is the wall mass flux

 $\mu$  is the dynamic viscosity and k is the thermal conductivity. By using similarity variables in (18), we have

(19) 
$$Re_x^{\frac{1}{2}}C_f = \left(\frac{n+1}{2}\right)^{\frac{1}{2}}f''(0), \quad Re_x^{-\frac{1}{2}}Nu_x = -\left(\frac{n+1}{2}\right)^{\frac{1}{2}}\theta'(0), \quad Re_x^{-\frac{1}{2}}Sh_x = -\left(\frac{n+1}{2}\right)^{\frac{1}{2}}\phi'(0)$$

## **3.** Method of Solution

We solve the non-linear differential equations (11) - (13) for Prescribed Surfaced Temperature (P.S.T.) and equations (11), (14) and (15) for Constant Surfaced Temperature (C.S.T.) separately subject to the boundary conditions, equations (16) - (17), numerically using the shooting method with Fourth Order Runge-Kutta technique. First, we convert the Prescribed Surfaced Temperature (P.S.T.) coupled system and Constant Surfaced Temperature (C.S.T.) coupled system separately into a system of First Order Differential Equations. The boundary conditions as  $\eta \rightarrow \infty$  is replaced by a finite value in accordance with standard practice in the boundary layer analysis. We guess values for f''(0),  $\theta'(0)$ ,  $\phi'(0)$  so that equation (17) will be satisfied. To validate the numerical method used in this work, the result for the velocity profiles were compared with that of Kalidas Das[13] for the fluid parameter n = 1.0, r = 0.0, M = 0.0,  $\lambda = 0.0$ , Re = 1. The comparison is shown in figure (1).



FIGURE 1. (a) Present work

(b) Kalidas[13]

### 4. **RESULTS AND DISCUSSION**

MHD heat and mass flow of nano-fluid over a non-linearly permeable stretching sheet are studied. The impact of fluid parameters such as non-linear stretching parameter(n), magnetic parameter(M), permeability parameter( $\lambda$ ), Prandtl number(Pr), Brownian motion parameter(Nb), Thermophoresis parameter(Nt) on the velocity profile, temperature profile and nano-particle concentration profile will be discussed for Prescribed Surface Temperature(P.S.T.) and Constant Surface Temperature(C.S.T.) systems.

The effect of the non-linear stretching parameter on the velocity profiles, temperature profiles and nano-particles concentration profiles are illustrated in Figures 2, 3, and 4 for both P.S.T. and C.S.T. respectively. It is observed that the  $f'(\eta)$  and  $\theta(\eta)$  increases with increase in nonlinear stretching parameter for both P.S.T. and C.S.T. As a result, the momentum boundary layer thickness increases with increasing non-linear stretching parameter. Also, the rate of heat transfer decreases with increasing non-linear stretching parameter. Whereas, an increase in non-linear stretching parameter increases the  $\phi(\eta)$  for P.S.T. and decreases the  $\phi(\eta)$  for C.S.T., that is, the rate of mass transfer decreases for P.S.T. and increases for C.S.T. with increasing non-linear stretching parameter.



FIGURE 2. Influence of n on velocity profiles



FIGURE 3. Influence of n on temperature profiles



FIGURE 4. Influence of n on nano-particle concentration profiles

The influence of magnetic parameter and permeability parameter on  $f'(\eta)$ ,  $\theta(\eta)$ , and  $\phi(\eta)$  are shown in Figures 5, 6 and 7 for both P.S.T. and C.S.T. respectively. For P.S.T., an increase in magnetic parameter results in a reduction of the velocity profiles. This leads to the fact that the magnetic field introduces a retarding body force which acts transverse to the direction of the applied magnetic field. This body force, called the Lorentz force, decelerates the boundary layer flow and thickens the momentum boundary layer. For C.S.T., an increase in magnetic parameter increases the velocity profiles. It is observed that the velocity boundary layer thickness of the nano-fluid is insignificantly increased with increasing values of magnetic parameter. The permeability parameter decreases the velocity profiles for P.S.T. and insignificantly increases the momentum boundary layer. The  $\theta(\eta)$  are observed to be significantly enhanced for P.S.T. and insignificantly enhanced for C.S.T. with increasing magnetic parameter. The resistive force opposes the fluid motion thereby producing heat and as a result, the thermal boundary layer thickness becomes thicker. The nano-particles concentration,  $\phi(\eta)$ , are seen to be insignificantly reduced for both P.S.T. and C.S.T. with increasing magnetic parameter and as a result, nano-particle volume fraction boundary layer thickness becomes weaker.



FIGURE 5. Influence of magnetic and permeability parameters on velocity profiles



FIGURE 6. Influence of magnetic and permeability parameters on temperature profiles



FIGURE 7. Influence of magnetic and permeability parameters on nano-particle concentration profiles

Figure 8 represents the temperature profiles for various values of Prandtl number. For P.S.T., an increase in Prandtl number results in a decrease in the temperature distribution because, the thermal boundary layer thickness decreases with increasing Prandtl number. An increase in the Prandtl number slows down the rate of thermal diffusion, whereas for C.S.T., an increase in Prandtl number increases  $\theta(\eta)$ . It is noted that increase in Prandtl number corresponds to a

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weaker thermal diffusivity and a thinner thermal boundary layer. Furthermore, heat transfers from the hot sheet to the ambient fluid.



FIGURE 8. Influence of Pr on temperature profiles

Figures 9 and 10 show the effect of Brownian motion parameter  $\theta(\eta)$  and  $\phi(\eta)$  respectively. From the figures, it can be seen that an increase in Brownian motion parameter increases the temperature profiles for P.S.T. and nano-particle concentration profiles for both P.S.T. and C.S.T. respectively. Increasing the Brownian motion parameter thickens the thermal boundary layer and increases the diffusion of nano-particles due to the Brownian effect. Brownian motion is a random motion of particles suspended in a fluid resulting from their collision with the quick atoms or molecules in the fluid. The collision between the fluid particles and their zigzag motion increases which results in the increase of both  $\theta(\eta)$  and  $\phi(\eta)$ . Also, an increase in Brownian motion parameter decreases the temperature profiles for C.S.T., thus, it weakens the thermal boundary layer.



FIGURE 9. Figure: Influence of Nb on temperature profiles



FIGURE 10. Influence of Nb on nano-particle concentration profiles

Figures 11 and 12 manifest the thermophoresis parameter on  $\theta(\eta)$  and  $\phi(\eta)$  respectively. It is observed that the  $\theta(\eta)$  increases and  $\phi(\eta)$  decreases with increasing values of thermophoresis parameter. The thermophoresis parameter increases the thermal boundary layer thickness and the temperature on the surface of the sheet increases. This is because the thermophoresis parameter is directly proportional to the heat transfer coefficient associated with the fluid. Concentration is decreasing function of thermophoresis parameter. For hot surfaces, thermophoresis tends to blow the nano-particle volume fraction boundary layer away from the surface since hot surface repels the submicron-sized particles from it, thereby forming a relatively particle-free layer near the surface.



FIGURE 11. Influence of Nt on temperature profiles



FIGURE 12. Influence of Nt on nano-particle concentration profiles

#### **5.** CONCLUSION

The effect of Magneto-hydrodynamics (MHD) heat and mass flow of nano-fluid over a nonlinearly permeable stretching sheet is investigated. From the analyses, the following conclusions may be drawn:

- (1) An increase in non-linear stretching parameter enhances velocity and temperature profiles for both P.S.T. and C.S.T. cases and also the nano-particle concentration profiles for P.S.T. case while it decreases for C.S.T. case.
- (2) The temperature profiles increase and nano-particle concentration profiles decrease with increasing values of magnetic and permeability parameters whereas the velocity profiles decreases for P.S.T. case and increase for C.S.T. case.
- (3) An increase in Prandtl number decreases and increases the thermal boundary layers for P.S.T. and C.S.T. cases respectively.
- (4) The nano-particle concentration profiles increase with increasing Brownian motion parameters for both P.S.T. and C.S.T. cases and also the temperature profiles for P.S.T. case while it decreases for C.S.T. case.

(5) An increase in thermophoresis parameter enhances the temperature profiles and decreases the nano-particle concentration profiles.

#### **CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

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