SEGREGATED EXTENSION OF GRAPHS

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Abstract. A graph in which any two adjacent vertices have distinct degrees is totally segregated. In this article segregating sequence, which is a new tool for finding segregated extension of given graph is introduced. If $G$ is an undirected graph which contains a vertex $v$, then the graph $G \circ v$ is obtained from $G$ by adding a new vertex $v'$ which is connected to all the neighbors of $v$. More generally, if $v_1, v_2, \ldots, v_n$ are the vertices of $G$ and $t = (t_1, t_2, \ldots, t_n)$ is a vector of positive integers then $H = G \circ t$ is constructed by substituting for each $v_i$ an independent set of $t_i$ vertices $v_i^1, v_i^2, \ldots, v_i^{t_i}$ and joining $v_i^j$ with $v_j^l$ if and only if $v_i$ and $v_j$ are adjacent in $G$. If $G$ is not totally segregated and $G \circ t$ is totally segregated, then the sequence $t$ is a segregating sequence of $G$. Here it is proved that any graph can be embedded as an induced subgraph in a totally segregated graph. Further, segregating sequence for many classes of graphs are determined.

Keywords: segregated graph; multiplication of vertices; segregated extension of graph; segregating sequence.

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1. INTRODUCTION

It is known that (Konig [11]) any graph $G$ of maximum degree $\Delta(G)$ is an induced subgraph of some $\Delta(G)$-regular graph $H$. Erdos and Kelly [6] determined the minimum number of vertices, the induced regulation number, which is to be added to a graph $G$ to obtain such a $\Delta(G)$-regular supergraph $H$. The latter was also extended to digraphs by Beineke and Pippert [3].

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regulation number of a graph $G$ is the minimum number of vertices which must be added to $G$ to construct a $\Delta(G)$-regular supergraph $H$. In this case, $G$ need not be an induced subgraph of $H$. Regulation number of graphs was introduced by Akiyama, Era and Harary [1] and was further studied by Akiyama and Harary [2] and Harary and Schmidt [9]. Analogous concepts for digraphs and multigraphs were introduced by Harary and Karabed [8] and Chartrand, Harary and Ollermann [5] respectively. In [4] Buckley and Harary studied the problem of embedding a highly irregular graph $G$ as an induced subgraph in a self-centered graph $H$ of smallest possible order so that $H$ is regular with the same maximum degree as $G$.

A connected graph $G$ is totally segregated if $\deg_G u \neq \deg_G v$, for every edge $uv \in E(G)$. The class of totally segregated graphs was studied by Jackson and Entringer [10]. In this paper our attempt is to find segregated extension of some graph.

2. SEGREGATING SEQUENCE

The concept of multiplication of vertices was given by Golumbic [7] as follows. If $G$ is an undirected graph which contains a vertex $v$, then the graph $G \circ v$ is obtained from $G$ by adding a new vertex $v'$ which is connected to all the neighbors of $v$. More generally, if $v_1, v_2, \cdots, v_n$ are the vertices of $G$ and $t = (t_1, t_2, \cdots, t_n)$ is a vector of non negative integers then $H = G \circ t$ is constructed by substituting for each $v_i$ an independent set of $t_i$ vertices $v^1_i, v^2_i, \cdots, v^{t_i}_i$ and joining $v^t_i$ with $v^s_j$ if and only if $v_i$ and $v_j$ are adjacent in $G$. We say that $H$ is obtained from $G$ by multiplication of vertices. This definition allows $t_i = 0$, in which case $H$ includes no copy of $v_i$. Thus every induced subgraph of $G$ can be obtained by multiplication of the appropriate $(0, 1)$-valued vector.

**Definition 2.1.** Let $G = (V, E)$ be a graph where $V = \{v_1, v_2, \cdots, v_n\}$ and $t = (t_1, t_2, \cdots, t_n)$ be a sequence of positive integers. If $G \circ t$ is totally segregated, then the sequence $t$ is a segregating sequence of $G$ and $G \circ t$ is the segregated extension of $G$ which is denoted by $G^S$. The sequence $t = (t_1, t_2, \cdots, t_n)$ is said to be a minimal segregating sequence of the graph $G$ if no sequence $t' = (t'_1, t'_2, \cdots, t'_n)$ with $\sum_{i=1}^n t'_i < \sum_{i=1}^n t_i$ is a segregating sequence of $G$. If $t$ is minimal segregating sequence of $G$, $G \circ t$ is called minimal segregated-extension of $G$ which is denoted by $G^{S^-}$. The sequence $t' = (t'_1, t'_2, \cdots, t'_n)$ is said to be a perfect segregating sequence of the graph $G$ if the
graph $G \circ t'$ is totally segregated with $\Delta(G \circ t') = \Delta(G)$ and $G \circ t'$ is the perfect segregated-extension of $G$ which is denoted by $G^S$. The graph $G$ which can be segregated using a perfect segregating sequence is called perfect segregated-extendable graph.

Remark 2.1.

- To define segregating sequence of the graph $G$ ordering the vertex set $V(G)$ is important.
- For a totally segregated graph $G$, the segregating sequence is $t = (1, 1, \cdots, 1)$. Here $G^S = G$.
- If $G^S = (V^S, E^S)$, $|V^S| = t_1 + t_2 + \cdots + t_n$

Proposition 2.1. Any graph $G$ has segregated extension.

Proof. Suppose $G$ is not totally segregated. If $G$ is $P_2$, $(2, 1)$ is its minimal segregating sequence. Suppose $G \not\cong P_2$. An edge $uv$ of $E(G)$ is said to be balanced if $\deg_G u = \deg_G v$. Since $G$ is not totally segregated, it has at least one balanced edge. Let $uv$ be one of the balanced edges of $G$. Multiply the vertex $u$, $\Delta(G)$ times and let the resultant graph be $G_1$. Then $\deg_{G_1} v > \Delta(G)$. Let $b$ be the number of balanced edges in $G$ and $b_1$ be the number of balanced edges in $G_1$. It is clear that $b_1 < b$, since in each step balanced edges become unbalanced but no unbalanced edges become balanced. If $G_1$ is not totally segregated, let $u_1v_1$ be one of the balanced edges of $G_1$. Multiply the vertex $u_1$, $\Delta(G_1)$ times and let the resultant graph be $G_2$. Continue this process until no such balanced edges remain. Since $G$ is finite, the process will end in finite number of steps. Then the resulting graph is totally segregated graph.

Remark 2.2.

1. Let $G = (V, E)$ be a graph. If there exists a balanced edge $uv$ such that $\deg_G u = \deg_G v = \Delta(G)$, then $G$ is not perfect segregated-extendable.
2. If a graph $G$ which is not totally segregated has a universal vertex, then it is not perfect segregated-extendable.
3. Perfect segregating sequence of a graph may not be minimal segregating sequence and minimal segregating sequence may not be perfect segregating sequence.
Example 2.1. Take 3 copies of $P_4$. Let $G = (V, E)$ be the graph obtained by fusing 3 copies of end vertex of $P_4$ as in Figure 1.

The vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$. Then the sequence $t = (t_i)$, where

$$t_i = \begin{cases} 
2 & \text{for } i = 8, 9, 10 \\
1 & \text{otherwise}
\end{cases}$$

is the segregating sequence of $G$. Here note that $t$ is a perfect segregating sequence which is not minimal.

The sequence $t = (t_i)$ where

$$t_i = \begin{cases} 
3 & \text{if } i = 1 \\
1 & \text{otherwise}
\end{cases}$$

is a minimal segregating sequence which is not perfect.

![Figure 1](image)

**Figure 1.** $G'$: Perfect segregated-extension of the graph $G$, $G''$: Minimal segregated-extension of the graph $G$

Remark 2.3. Regular graphs and path $P_n, n \neq 3$ are not perfect segregated-extendable.

3. Segregated Extensions of Some Classes of Graphs

- Segregating Sequence of Paths

  Let $G = P_n$ be the path on $n$ vertices with vertex set $V = \{v_1, v_2, \ldots, v_n\}$, where $v_1$ and $v_n$ are end vertices. By Remark 2.2 (1), perfect segregating sequence does not exist for path $P_n, n \neq 3$.

Remark 3.1. To make paths $P_n$ segregated, at least one vertex among 4 consecutive vertices on path $P_n$, should be multiplied by a number $i$, where $i \geq 2$. 
By Remark 3.1, The segregating sequence \( t = (t_i) \) of \( G \) given below is minimal.

Case 1. \( n = 4k, k \geq 1 \).

\[
t_i = \begin{cases} 
2 & \text{if } i = 1, 5, \cdots, 4k-3 \\
1 & \text{otherwise}
\end{cases}
\]

Case 2. \( n = 4k + 1, k \geq 1 \).

\[
t_i = \begin{cases} 
2 & \text{if } i = 3, 7, \cdots, 4k-1 \\
1 & \text{otherwise}
\end{cases}
\]

Case 3. \( n = 4k + 2, k \geq 1 \).

\[
t_i = \begin{cases} 
2 & \text{if } i = 3, 7, \cdots, 4k-1 \\
1 & \text{otherwise}
\end{cases}
\]

Case 4. \( n = 4k + 3, k \geq 1 \).

\[
t_i = \begin{cases} 
2 & \text{if } i = 4, 8, \cdots, 4k \\
1 & \text{otherwise}
\end{cases}
\]

• Segregating Sequence of Fused Paths

(1) Paths fused at one end vertex

Let \( P_n \) be a path on \( n \) vertices and \( w \) is an end vertex. Take \( m(\geq 3) \) copies of \( P_n \). Let \( G = (V, E) \) be the graph obtained by fusing the \( m \) copies of \( P_n \) at \( w \) which is denoted by \( F_w(P_n)^m \). The vertex set \( V = \bigcup_{i=1}^{m} V_i \), where \( V_i = \{w, v_i, v_{i+1}, \cdots, v_{4i-3}\} \). It is nothing but subdivided star.

Remark 3.2. To make \( F_w(P_n)^m \) segregated, at least one vertex among 4 consecutive vertices on any branch of it at \( w \), should be multiplied by a number \( i \), where \( i \geq 2 \).
Also if $\deg w = 3$ and if $w$ is the only vertex with multiplicity at least 2 on the path $(w, v_{i2}, v_{i3}, v_{i4}), m(w) > 2$. Otherwise degree of any copy of $w$ is 3 and $\deg v_{i2} = 3$ which is a contradiction.

**Segregating sequence of $F_w(P_n)^m$**

$G = (V, E)$ where $V = \bigcup_{i=1}^{m} V_i$, $V_i = \{w, v_{i2}, v_{i3}, \ldots, v_{in}\}$ and $V$ is ordered as $V = \{w, v_{12}, \ldots, v_{1n}, v_{22}, \ldots, v_{2n}, \ldots, v_{mn}\}$.

Let $t = (t_w, t_{12}, \ldots, t_{1n}, t_{22}, \ldots, t_{2n}, \ldots, t_{mn})$ be defined as follows.

Case 1. $n = 4k, k \geq 1$.

Subcase 1.1 $m = 3$.

If $t_w = 3$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \ldots, 4k - 3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}$

Then $t$ is segregating sequence and it is minimal by Remark 3.2 but not perfect.

Subcase 1.2 $m \geq 4$.

If $t_w = 2$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \ldots, 4k - 3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}$

Then $t$ is a segregating sequence and it is perfect but not minimal.
The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

If \( t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \ldots, 4k \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is perfect but not minimal.

Case 2. \( n = 4k + 1, k \geq 1 \).

Subcase 2.1 \( m = 3 \).
If \( t_w = 3 \) and \( t_{ij} = \begin{cases} 2 & \text{if } j = 5, 9, \ldots, 4k + 1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 but not perfect.

In this case perfect segregating sequence does not exist.

Subcase 2.2 \( m \geq 4 \).
If \( t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{if } j = 3, 7, \ldots, 4k - 1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

Case 3. \( n = 4k + 2, 4k + 3, k \geq 1, m \geq 3 \).

If \( t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \ldots, 4k \text{ and for all } i \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

(2) **Paths fused at two end vertices**

Let \( P_n \) be a path on \( n \) vertices and \( u, w \) are end vertices. Take \( m(\geq 3) \) copies of \( P_n \). Let \( G = (V, E) \) be the graph obtained by fusing \( m(\geq 3) \) copies of end vertices \( u, w \) of \( P_n \) separately, which is denoted by \( F_{u,w}(P_n)^m \). The vertex set \( V = \bigcup_{i=1}^{m} V_i \cup \{u, w\} \) where \( V_i = \{u, v_{i2}, v_{i3}, \ldots, v_{i(n-1)}, w\} \).

**Remark 3.3.** To make the fused paths \( F_{u,w}(P_n)^m \) segregated, at least one vertex among 4 consecutive vertices on any \( u - w \) path \( F_{u,w}(P_n)^m \), should be multiplied by a number \( i \), where \( i \geq 2 \). Also if \( deg u = 3 \) and if \( u \) is the only vertex with multiplicity at least
2 on the path \((u, v_i, v_{i+1}, v_{i+2})\), \(m(u) > 2\). Otherwise degree of any copy of \(u\) is 3 and \(\deg v_i = 3\) which is a contradiction. The case is similar for \(w\).

**Segregating sequence of** \(F_{u,w}(P_n)^m\)

\(G = (V, E)\) where \(V = \bigcup_{i=1}^{m} V_i\), \(V_i = \{u, v_{i+1}, \cdots, v_{i(n-1)}, w\}\) and \(V\) is ordered as \(V = \{u, v_{12}, \cdots, v_{1(n-1)}, v_{2n}, \cdots, v_{2(n-1)}, \cdots, v_{m2}, \cdots, v_{m(n-1)}, w\}\).

Let \(t = (t_u, t_{12}, \cdots, t_{1(n-1)}, t_{22}, \cdots, t_{2(n-1)}, \cdots, t_{m2}, \cdots, t_{m(n-1)}, t_w)\) be defined as follows.

Case 1. \(n = 4k, k \geq 1\).

Subcase 1.1 \(m = 3\)

If \(t_u = 3, t_w = 1\) and \(t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k - 3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}\)

The segregating sequence \(t\) is minimal by Remark 3.3 but not perfect.

In this case perfect segregating sequence does not exist.

![Diagram](image-url)

**Figure 4.** \(G = F_{u,w}(P_8)^3\), \(G' = F_{u,w}(P_8)^3 \circ t\) which is minimal segregated extension of \(G\)

Subcase 1.2 \(m \geq 4\).

If \(t_u = 2, t_w = 1\) and \(t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k - 3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}\)
The segregating sequence \( t \) is minimal by Remark 3.3 and perfect.

Case 2. \( n = 4k + 1, \ k \geq 1 \).

Subcase 2.1 \( m = 3 \).

If \( t_u = 1, t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{if } j = 2, 6, \ldots, 4k - 2 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 but not perfect.

In this case perfect segregating sequence does not exist.

Subcase 2.2 \( m \geq 4 \).

If \( t_u = 1, t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{if } j = 3, 7, \ldots, 4k - 1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

Case 3. \( n = 4k + 2, \ k \geq 1 \).

Subcase 3.1 \( m = 3 \).

If \( t_u = 3, t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \ldots, 4k + 1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 but not perfect.

In this case perfect segregating sequence does not exist.

Subcase 3.2 \( m \geq 4 \).

If \( t_u = 1, t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{for } j = 3, 7, \ldots, 4k - 1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

Case 4. \( n = 4k + 3, \ k \geq 1, m \geq 3 \).

If \( t_u = 1, t_w = 1 \) and \( t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \ldots, 4k \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases} \)

The segregating sequence \( t \) is minimal by Remark 3.2 and perfect.

**Remark 3.4.** A segregating sequence of \( F_{u,w}(P_n)^3 \) is perfect only when \( n = 4k + 3, \ k \geq 1 \).
• **Segregating Sequence of Cycles**

Let $G = C_n$ be the cycle on $n$ vertices with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. By Remark 2.2 (1), perfect segregating sequence of cycle does not exist.

Let $t = (t_1, t_2, \cdots, t_n)$ be the segregating sequence $G$.

**Remark 3.5.** To make paths $C_n$ segregated, at least one vertex among 4 consecutive vertices on cycle $C_n$, should be multiplied by a number $i$, where $i \geq 2$.

By Remark 3.5, The segregating sequence $t = (t_i)$ of $G$ given below is minimal.

**Case 1.** $n = 4k, k \geq 1$.

$$
  t_i = \begin{cases} 
    2 & \text{if } i = 1, 5, \cdots, 4k - 3 \\
    1 & \text{otherwise}
  \end{cases}
$$

**Case 2.** $n = 4k + 1, k \geq 1$.

$$
  t_i = \begin{cases} 
    2 & \text{if } i = 1, 5, \cdots, 4k - 3 \\
    3 & \text{if } i = 4k - 1 \\
    1 & \text{otherwise}
  \end{cases}
$$

**Case 3.** $n = 4k + 2, k \geq 1$.

$$
  t_i = \begin{cases} 
    2 & \text{if } i = 1, 5, \cdots, 4k + 1 \\
    1 & \text{otherwise}
  \end{cases}
$$

**Case 4.** $n = 4k + 3, k \geq 1$.

$$
  t_i = \begin{cases} 
    2 & \text{if } i = 1, 5, \cdots, 4k - 3 \\
    3 & \text{if } i = 4k + 1 \\
    1 & \text{otherwise}
  \end{cases}
$$

• **Segregating Sequence of Complete Graphs**

Let $G = K_n$ be the complete graph on $n$ vertices with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. Then $t = (t_i)$ is the segregating sequence $G$ where $t_i = i$. Here the segregating sequence $t$ is the minimal but not perfect.
- **Segregating Sequence of Complete K-partite Graphs with same partite size**

Let $G = K_{r,r,\ldots,r}$ be the complete $k$-partite graph with partite size $r$. The Vertex set $V(G) = \bigcup_{i=1}^{k} V_i$, where $V_i = \{v_{i1}, v_{i2}, \ldots, v_{ir}\}$ denote the $i^{th}$ partite set. Then $t = (t_{ij})$ is the segregating sequence of $G$, where

\[
t_{ij} = \begin{cases} 
  i & \text{if } j = 1 \\
  1 & \text{otherwise}
\end{cases}
\]

Here the segregating sequence $t$ is minimal but not perfect.

- **Segregating Sequence of Petersen Graph**

Let $G = (V,E)$ be Petersen graph

![Petersen Graph](image)

and $V = \{v_i\}$, $i = 1, 2, \ldots, 10$. Then $t = (t_i)$ is the segregating sequence where

\[
t_i = \begin{cases} 
  2 & \text{if } i = 1 \\
  3 & \text{if } i = 7, 9 \\
  1 & \text{otherwise}
\end{cases}
\]

- **Segregating Sequence of Bistar**

Let $G = (V,E)$ be bistar graph where $V = \{u,v,u_1,u_2,\ldots,u_d,v_1,v_2,\ldots,v_d\}$ and $E = \{uv,uu_i,vv_i : u_i, v_i \in V\}$.

Perfect segregating sequence does not exist for $G$ by Remark 2.2 (1). Here $t = (t_u, t_v, \ldots, t_{u_1}, \ldots, t_{v_1}, \ldots)$ is the minimal segregating sequence of $G$ where $t_w = \begin{cases} 
  2 & \text{if } w = u_1 \\
  1 & \text{otherwise}
\end{cases}$

- **Segregating Sequence of Sun flower Graph**
Sunflower graph is as described in Figure 7. Let $G = W_1, n = (W, E), n \geq 3$ be the wheel graph where $W = \{w\} \cup V$. Let $V = \{v_1, v_2, \ldots, v_n\}$. Sunflower graph is the graph $\lambda(W_1, n)$ with the vertex set $\{w\} \cup V \cup V'$ where $V' = \{v'_i : v_i \in V\}$ which is disjoint from $V$ and with edge set $E \cup \{v_i v'_i, v_{i+1} v'_i : v_i \in V\}$ where $v_{n+1} v'_n$ is replaced by $v_1 v'_n$. Here the sequence $t = (t'_i)$ is the perfect segregating sequence of the sunflower graph

\begin{align*}
\lambda(W_1, n), \text{ for } n \geq 8.
\end{align*}

Case 1. $n = 4k, k \geq 2$.

\begin{align*}
\begin{cases}
t'_i = \\
2 & \text{for } i = 1, 2, 5, 6, \ldots, 4k - 7, 4k - 6, 4k - 3, 4k - 2. \\
1 & \text{otherwise}
\end{cases}
\end{align*}

Case 2. $n = 4k + 1, k \geq 2$.

\begin{align*}
\begin{cases}
t'_i = \\
2 & \text{for } i = 1, 2, 5, 6, \ldots, 4k - 7, 4k - 6, 4k - 3, 4k - 2. \\
3 & \text{for } i = 4k - 1 \\
1 & \text{otherwise}
\end{cases}
\end{align*}
Case 3. $n = 4k + 2, k \geq 2$.

\[
t'_i \begin{cases} 
2 & \text{for } i = 1, 2, 5, 6, \ldots, 4k - 7, 4k - 6, 4k - 2, 4k - 1. \\
3 & \text{for } i = 4k - 5, 4k \\
1 & \text{otherwise}
\end{cases}
\]

Case 4. $n = 4k + 3, k \geq 2$.

\[
t'_i \begin{cases} 
2 & \text{for } i = 1, 2, 5, 6, \ldots, 4k - 3, 4k - 2, 4k + 1. \\
3 & \text{for } i = 4k + 2 \\
1 & \text{otherwise}
\end{cases}
\]

Note that in this case the segregating sequence $t$ is perfect as well as minimal. But for $3 \leq n \leq 7$, perfect segregating sequence does not exist.

**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

**REFERENCES**