NANO BINARY CONTRA CONTINUOUS FUNCTIONS IN NANO BINARY TOPOLOGICAL SPACES

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Abstract: The purpose of this paper, we introduce and study the nano binary contra continuous function in nano binary topological spaces. Also we introduce some nano binary contra continuous functions and their characterizations are also studied. We introduce and discuss the nano binary D-continuous function in nano binary topological spaces.

Keywords: \(N_B\) -contra continuous; \(N_B\) contra \(\alpha\) -continuous; \(N_B\) contra semi-continuous; \(N_B\) contra pre-continuous; \(N_B\) contra \(\beta\) -continuous; \(N_B\) perfectly continuous; \(N_B\) strongly continuous; \(N_B\) D- continuous.

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1. INTRODUCTION

M. Lellis Thivagar [1] introduced the concept of nano topological space with respect to a subset
X of a universe U. S. Nithyanantha Jothi and P. Thangavelu [2] introduced the concept of binary topological spaces. By combining these two concepts G. Hari Siva Annam and J. Jasmine Elizabeth [3] introduced nano binary topological spaces. J. Jasmine Elizabeth and G. Hari Siva Annam [4] introduced nano binary continuous function in nano binary topological spaces. In this paper we have introduced a new class of functions on nano binary topological spaces called nano binary contra continuous functions and derived their characterizations in terms of nano binary strongly continuous and nano binary perfectly continuous. Also the relationships between some nano binary contra continuous functions are studied. Also we have introduced the nano binary D-continuous function.

2. PRELIMINARIES
The concepts given here help us to recall our memories regarding the basic concepts of nano binary topological spaces.

Definition 2.1: [3] Let \((U_1, U_2)\) be a non-empty finite set of objects called the universe and \(R\) be an equivalence relation on \((U_1, U_2)\) named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair \((U_1, U_2, R)\) is said to be the approximation space. Let \((X_1, X_2) \subseteq (U_1, U_2)\)

1. The lower approximation of \((X_1, X_2)\) with respect to \(R\) is the set of all objects, which can be for certain classified as \((X_1, X_2)\) with respect to \(R\) and it is denoted by \(L_R(X_1, X_2)\).

   That is, \(L_R(X_1, X_2) = \bigcup_{(x_1, x_2) \in (U_1, U_2)} \{R(x_1, x_2) : R(x_1, x_2) \subseteq (X_1, X_2)\}\)

   Where \(R(x_1, x_2)\) denotes the equivalence class determined by \((x_1, x_2)\)

2. The upper approximation of \((X_1, X_2)\) with respect to \(R\) is the set of all objects, which can be possibly classified as \((X_1, X_2)\) with respect to \(R\) and it is denoted by \(U_R(X_1, X_2)\).

   That is, \(U_R(X_1, X_2) = \bigcup_{(x_1, x_2) \in (U_1, U_2)} \{R(x_1, x_2) : R(x_1, x_2) \cap (X_1, X_2) \neq \emptyset\}\)

3. The boundary region of \((X_1, X_2)\) with respect to \(R\) is the set of all objects, which can be classified neither as \((X_1, X_2)\) nor as not \(-(X_1, X_2)\) with respect to \(R\) and it is denoted by \(B_R(X_1, X_2)\).
That is, \( B_R(X_1, X_2) = U_R(X_1, X_2) - L_R(X_1, X_2) \)

**Proposition 2.2:** [3] If \((U_1, U_2, R)\) is an approximation space and \((X_1, X_2)\), \((Y_1, Y_2) \subseteq (U_1, U_2)\), then

1. \( L_R(X_1, X_2) \subseteq (X_1, X_2) \subseteq U_R(X_1, X_2) \)
2. \( L_R(\phi, \phi) = U_R(\phi, \phi) = (\phi, \phi) \) and \( L_R(U_1, U_2) = U_R(U_1, U_2) = (U_1, U_2) \)
3. \( U_R((X_1, X_2) \cup (Y_1, Y_2)) = U_R(X_1, X_2) \cup U_R(Y_1, Y_2) \)
4. \( U_R((X_1, X_2) \cap (Y_1, Y_2)) \subseteq U_R(X_1, X_2) \cap U_R(Y_1, Y_2) \)
5. \( L_R((X_1, X_2) \cup (Y_1, Y_2)) \supseteq L_R(X_1, X_2) \cup L_R(Y_1, Y_2) \)
6. \( L_R((X_1, X_2) \cap (Y_1, Y_2)) \subseteq L_R(X_1, X_2) \cap L_R(Y_1, Y_2) \)
7. \( L_R(X_1, X_2) \subseteq L_R(Y_1, Y_2) \) and \( U_R(X_1, X_2) \subseteq U_R(Y_1, Y_2) \) whenever \((X_1, X_2) \subseteq (Y_1, Y_2)\)
8. \( U_R(X_1, X_2)^c = [L_R(X_1, X_2)]^c \) and \( L_R(X_1, X_2)^c = [U_R(X_1, X_2)]^c \)
9. \( U_R U_R(X_1, X_2) = L_R U_R(X_1, X_2) = U_R(X_1, X_2) \)
10. \( L_R L_R(X_1, X_2) = U_R L_R(X_1, X_2) = L_R(X_1, X_2) \)

**Definition 2.3:** [3] Let \((U_1, U_2)\) be the universe, \( R \) be an equivalence on \((U_1, U_2)\) and \( \tau_R(X_1, X_2) = \{(U_1, U_2), (\phi, \phi), L_R(X_1, X_2), U_R(X_1, X_2), B_R(X_1, X_2)\} \) where \((X_1, X_2) \subseteq (U_1, U_2)\).

Then by the property \( R(X_1, X_2) \) satisfies the following axioms

1. \((U_1, U_2)\) and \((\phi, \phi) \in (X_1, X_2)\)
2. The union of the elements of any sub collection of \( \tau_R(X_1, X_2) \) is in \( \tau_R(X_1, X_2) \)
3. The intersection of the elements of any finite sub collection of \( \tau_R(X_1, X_2) \) is in \( \tau_R(X_1, X_2) \).

That is, \( \tau_R(X_1, X_2) \) is a topology on \((U_1, U_2)\) called the nano binary topology on \((U_1, U_2)\) with respect to \((X_1, X_2)\).

We call \((U_1, U_2, \tau_R(X_1, X_2))\) as the nano binary topological spaces. The elements of \( \tau_R(X_1, X_2) \) are called as nano binary open sets and it is denoted by \( N_B \) open sets. Their complement is called \( N_B \) closed sets.

**Definition 2.4:** [3] If \((U_1, U_2, \tau_R(X_1, X_2))\) is a nano binary topological spaces with respect to \((X_1, X_2)\) and if \((H_1, H_2) \subseteq (U_1, U_2)\), then the nano binary interior of \((H_1, H_2)\) is defined as the
union of all $N_B$ open subsets of $(A_1, A_2)$ and it is defined by $N^*_B(H_1, H_2)$
That is, $N^*_B(H_1, H_2)$ is the largest $N_B$ open subset of $(H_1, H_2)$. The nano binary closure of
$(H_1, H_2)$ is defined as the intersection of all $N_B$ closed sets containing $(H_1, H_2)$ and it is
denoted by $\overline{N_B}(H_1, H_2)$.

That is, $\overline{N_B}(H_1, H_2)$ is the smallest $N_B$ closed set containing $(H_1, H_2)$.

**Proposition 2.5:** [3] Let $(U_1, U_2, \tau_R(X_1, X_2))$ be a nano binary topological space and
$(A_1, A_2), (B_1, B_2) \in P(X_1) \times P(X_2)$ then

i) $N^*_B(\varphi, \varphi) = (\varphi, \varphi)$

$\overline{N_B}((\varphi, \varphi)) = ((\varphi, \varphi))$

ii) $N^*_B(U_1, U_2) = (U_1, U_2)$

$\overline{N_B}(U_1, U_2) = (U_1, U_2)$

iii) $N^*_B(A_1, A_2) \subseteq (A_1, A_2) \subseteq \overline{N_B}(A_1, A_2)$

iv) $(A_1, A_2) \subseteq (B_1, B_2)$ implies $N^*_B(A_1, A_2) \subseteq N^*_B(B_1, B_2)$ and

$\overline{N_B}(A_1, A_2) \subseteq \overline{N_B}(B_1, B_2)$

v) $N^*_B((A_1, A_2) \cap (B_1, B_2)) \subseteq N^*_B(A_1, A_2) \cap N^*_B(B_1, B_2)$

vi) $\overline{N_B}((A_1, A_2) \cap (B_1, B_2)) \subseteq \overline{N_B}(A_1, A_2) \cap \overline{N_B}(B_1, B_2)$

vii) $N^*_B((A_1, A_2) \cup (B_1, B_2)) \supseteq N^*_B(A_1, A_2) \cup N^*_B(B_1, B_2)$

viii) $\overline{N_B}((A_1, A_2) \cup (B_1, B_2)) \supseteq \overline{N_B}(A_1, A_2) \cup \overline{N_B}(B_1, B_2)$

ix) $N_B(|N_B(A_1, A_2)) \subseteq N^*_B(A_1, A_2)$

x) $\overline{N_B}(|N_B(A_1, A_2)) \supseteq \overline{N_B}(A_1, A_2)$

xi) $N^*_B(\overline{N_B}(A_1, A_2)) \supseteq N^*_B(A_1, A_2)$
\[ N_B \left( N_B^\circ (A_1, A_2) \right) \subseteq N_B (A_1, A_2) \]

**Definition 2.6:** [4] A subset \((H_1, H_2)\) of a nano binary topological spaces \((U_1, U_2, \tau_R(X_1, X_2))\) is called

1. \(N_B\ \alpha\)-open if \((H_1, H_2) \subseteq N_B^\circ (N_B^\circ (H_1, H_2))\).

2. \(N_B\) semi-open set if \((H_1, H_2) \subseteq N_B (N_B^\circ (H_1, H_2))\)

3. \(N_B\) pre-open set if \((H_1, H_2) \subseteq N_B^\circ (N_B^\circ (H_1, H_2))\)

4. \(N_B\) \(\beta\)-open if \((H_1, H_2) \subseteq N_B (N_B^\circ (N_B^\circ (H_1, H_2)))\).

The complements of the above mentioned sets are called their respective \(N_B\) closed sets.

**Results 2.7:** [4]

1. Every \(N_B\) open sets is \(N_B\ \alpha\)-open.
2. Every \(N_B\) \(\alpha\)-open is \(N_B\) semi-open.
3. Every \(N_B\) \(\alpha\)-open is \(N_B\) pre-open.
4. Every \(N_B\) pre-open is \(N_B\) \(\beta\)-open.
5. Every \(N_B\) semi-open is \(N_B\) \(\beta\)-open.
6. Every \(N_B\) open is \(N_B\) semi-open.
7. Every \(N_B\) open is \(N_B\) pre-open.
8. Every \(N_B\) \(\alpha\)-open is \(N_B\) \(\beta\)-open.
9. Every \(N_B\) open is \(N_B\) \(\beta\)-open.

**Note 2.8:** The above result is also true for every \(N_B\) closed sets.

**Note 2.9:** The converse of the above result is not true.

**Definition 2.10:** [4] Let \((U_1, U_2, \tau_R(X_1, X_2))\) and \((V_1, V_2, \tau_{R'}(Y_1, Y_2))\) be nano binary topological spaces. Then a mapping \(f: (U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) is nano binary continuous on \((U_1, U_2)\) if the inverse image of every \(N_B\) open in \((V_1, V_2)\) is \(N_B\) open in \((U_1, U_2)\) and it is denoted by \(N_B\)-continuous.

**Definition 2.11:** [4] Let \((U_1, U_2, \tau_R(X_1, X_2))\) and \((V_1, V_2, \tau_{R'}(Y_1, Y_2))\) be nano binary
topological spaces. A mapping f: \((U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) is said to be

1. \(N_B\) \(\alpha\)-continuous if \((f^{-1}(B_1, B_2))\) is \(N_B\) \(\alpha\)-open in \((U_1, U_2)\) for every \(N_B\) open \((B_1, B_2)\) in \((V_1, V_2)\).

2. \(N_B\) semi -continuous if \((f^{-1}(B_1, B_2))\) is \(N_B\) semi-open in \((U_1, U_2)\) for every \(N_B\) open \((B_1, B_2)\) in \((V_1, V_2)\).

3. \(N_B\) pre -continuous if \((f^{-1}(B_1, B_2))\) is \(N_B\) pre-open in \((U_1, U_2)\) for every \(N_B\) open \((B_1, B_2)\) in \((V_1, V_2)\).

4. \(N_B\) \(\beta\) -continuous if \((f^{-1}(B_1, B_2))\) is \(N_B\) \(\beta\)-open in \((U_1, U_2)\) for every \(N_B\) open \((B_1, B_2)\) in \((V_1, V_2)\).

**Results 2.12: [4]**

1. Every \(N_B\)- continuous is \(N_B\) \(\alpha\)-continuous.

2. Every \(N_B\) \(\alpha\)- continuous is \(N_B\) semi - continuous.

3. Every \(N_B\) \(\alpha\)- continuous is \(N_B\) pre - continuous.

4. Every \(N_B\) pre – continuous is \(N_B\) \(\beta\)- continuous.

5. Every \(N_B\) semi – continuous is \(N_B\) \(\beta\)- continuous.

**3. NANO BINARY CONTRA CONTINUITY**

**Definition 3.1:** Let \((U_1, U_2, \tau_R(X_1, X_2))\) and \((V_1, V_2, \tau_{R'}(Y_1, Y_2))\) be nano binary topological spaces. Then a mapping f: \((U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) is nano binary contra continuous function if the inverse image of every \(N_B\) open in \((V_1, V_2)\) is \(N_B\) closed in \((U_1, U_2)\) and it is denoted by \(N_B\)- contra continuous.

**Example 3.2:** Let \(U_1 = \{a, b, c\}, U_2 = \{1, 2\}\) with \(\frac{(U_1, U_2)}{R} = \{(a, b), \{2\}, \{c\}, \{1\}\}\) and \((X_1, X_2) = \{(a, c), \{1\}\}\). Then \(\tau_R(X_1, X_2) = \{\{\Phi, \Phi\}, \{U_1, U_2\}, \{\{c\}, \{1\}\}, \{a, b, \{2\}\}\}\). The \(N_B\) closed sets are \((U_1, U_2), \{\Phi, \Phi\}, \{\{a, b, \{2\}\}\}\). Let \(V_1 = \{x, y, z\}, V_2 = \{e, f\}\) with \(\frac{(V_1, V_2)}{R'} = \{(\{x, z\}, \{e\}), \{(y), \{f\}\}\}\) and \(Y_1, Y_2 = \{\{x, y, \{f\}\}\}. Then \(\tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), \{V_1, V_2\}, \{(x, z), \{e\}\}, \{(y), \{f\}\}\}\). Define f: \((U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) as f \((\{a\}, \{1\}) = \{(x), \{f\}\}, f(\{a\}, \{2\}) = \{(\Phi, \Phi), \{V_1, V_2\}, \{(x), \{f\}\}, \{(y), \{f\}\}\}.
Theorem 3.3: For a function \( f: (U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_R'(Y_1, Y_2)) \) the following conditions are equivalent:

(i) \( f \) is \( N_B \)- contra continuous.

(ii) The inverse image of each \( N_B \) closed set in \( (V_1, V_2) \) is \( N_B \) open in \( (U_1, U_2) \).

(iii) For each \( (x_1, x_2) \in (U_1, U_2) \) and each \( N_B \) closed set \( (B_1, B_2) \) in \( (V_1, V_2) \) with \( f(x_1, x_2) \in (B_1, B_2) \), there exists an \( N_B \) open set \( (A_1, A_2) \) in \( (U_1, U_2) \) such that \( f(A_1, A_2) \subseteq (B_1, B_2) \).

Proof: (i) \( \Rightarrow \) (ii) Let \( f \) be \( N_B \)- contra continuous. Let \( (B_1, B_2) \) be \( N_B \) closed in \( (V_1, V_2) \). That is, \( (V_1, V_2) - (B_1, B_2) \) is \( N_B \) open in \( (V_1, V_2) \). Since \( f \) is \( N_B \)- contra continuous, \( f^{-1}((V_1, V_2) - (B_1, B_2)) \) is \( N_B \) closed in \( (U_1, U_2) \). But \( f^{-1}((V_1, V_2) - (B_1, B_2)) = (U_1, U_2) - f^{-1}(B_1, B_2) \). Therefore, \( f^{-1}(B_1, B_2) \) is \( N_B \) open in \( (U_1, U_2) \). Thus the inverse image of each \( N_B \) closed set in \( (V_1, V_2) \) is \( N_B \) open in \( (U_1, U_2) \).

(ii) \( \Rightarrow \) (i) Let \( (B_1, B_2) \) be a \( N_B \) open in \( (V_1, V_2) \). Then \((V_1, V_2) - (B_1, B_2)\) is \( N_B \) closed in \( (V_1, V_2) \). By assumption, \( f^{-1}((V_1, V_2) - (B_1, B_2)) \) is \( N_B \) open in \( (U_1, U_2) \). Therefore, \( f^{-1}(B_1, B_2) \) is \( N_B \) closed in \( (U_1, U_2) \). Hence \( f \) is \( N_B \)- contra continuous.

(iii) \( \Rightarrow \) (ii) Let \( (B_1, B_2) \) be a \( N_B \) closed set such that \( f(x_1, x_2) \in (B_1, B_2) \). By assumption, \( (x_1, x_2) \in f^{-1}(B_1, B_2) \), which is \( N_B \) open. Let \( (A_1, A_2) = f^{-1}(B_1, B_2) \). Then \( f(A_1, A_2) \subseteq (B_1, B_2) \).

Remark 3.4: The concept of \( N_B \)- continuity and \( N_B \)- contra continuity are independent as
shown in the following example

**Example 3.5:** Let \( U_1 = \{a, b, c\}, U_2 = \{1, 2\} \) with \( (U_1, U_2)/R = (\{\{a, b\}, \{2\}\}, \{\{c\}, \{1\}\}) \) and \((X_1, X_2) = (\{b\}, \{2\})\). Then \( \tau_R(X_1, X_2) = (\{\Phi, \Phi\}, (U_1, U_2), (\{a, b\}, \{2\})) \). Let \( V_1 = \{x, y, z\}, V_2 = \{e, f\} \) with \( (V_1, V_2)/R' = (\{\{x, z\}, \{e\}\}, \{\{y\}, \{f\}\}) \) and \((Y_1, Y_2) = (\{z\}, \{e\})\). Then \( \tau_{R'}(Y_1, Y_2) = (\{\Phi, \Phi\}, (V_1, V_2), (\{y\}, \{f\})) \). Define \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) as \( f (\{a\}, \{1\}) = (\{x\}, \{f\}), f(\{a\}, \{2\}) = (\{x\}, \{e\}), f (\{b\}, \{1\}) = (\{z\}, \{f\}), f (\{b\}, \{2\}) = (\{z\}, \{e\}), f (\{c\}, \{1\}) = (\{y\}, \{f\}), f (\{c\}, \{2\}) = (\{y\}, \{e\}) \). Here \((B_1, B_2) = (\{x, z\}, \{e\})\). Then \( f^{-1}(\{x, z\}, \{e\}) = (\{a, b\}, \{2\}) \). Here \( f \) is \( N_B \)-continuous function, but not \( N_B \)- contra continuous. Because \((\{a, b\}, \{2\})\) is not \( N_B \) closed in \((U_1, U_2)\), where \((\{x, z\}, \{e\})\) is \( N_B \) open in \((V_1, V_2)\). Therefore, \( f \) is \( N_B \)-continuous but not \( N_B \)-contra continuous.

**Example 3.6:** Let \( U_1 = \{a, b, c\}, U_2 = \{1, 2\} \) with \( (U_1, U_2)/R = (\{\{a, b\}, \{2\}\}, \{\{c\}, \{1\}\}) \) and \((X_1, X_2) = (\{b\}, \{2\})\). Then \( \tau_R(X_1, X_2) = (\{\Phi, \Phi\}, (U_1, U_2), (\{a, b\}, \{2\})) \). The \( N_B \) closed sets are \((\Phi, \Phi), (U_1, U_2), (\{c\}, \{1\})\). Let \( V_1 = \{x, y, z\}, V_2 = \{e, f\} \) with \( (V_1, V_2)/R' = (\{\{x, z\}, \{e\}\}, \{\{y\}, \{f\}\}) \) and \((Y_1, Y_2) = (\{y\}, \{f\})\). Then \( \tau_{R'}(Y_1, Y_2) = (\{\Phi, \Phi\}, (V_1, V_2), (\{y\}, \{f\})) \). Define \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) as \( f (\{a\}, \{1\}) = (\{x\}, \{f\}), f(\{a\}, \{2\}) = (\{x\}, \{e\}), f (\{b\}, \{1\}) = (\{z\}, \{f\}), f (\{b\}, \{2\}) = (\{z\}, \{e\}), f (\{c\}, \{1\}) = (\{y\}, \{f\}), f (\{c\}, \{2\}) = (\{y\}, \{e\}) \). Therefore, \( f^{-1}(\{y\}, \{f\}) = (\{c\}, \{1\}) \), which is \( N_B \) closed in \((U_1, U_2)\) but not \( N_B \) open in \((U_1, U_2)\). Hence \( f \) is \( N_B \)-contra continuous but not \( N_B \)-continuous.

**Note 3.7:** Every \( N_B \)-continuous is \( N_B \)-contra continuous if every \( N_B \) open set is \( N_B \) closed.

**Definition 3.8:** A function \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) is said to be

(i) \( N_B \) perfectly continuous, if \( f^{-1}(A_1, A_2) \) is \( N_B \) clopen in \((U_1, U_2)\) for every \( N_B \) open set \((A_1, A_2)\) in \((V_1, V_2)\).

(ii) \( N_B \) strongly continuous, if \( f^{-1}(A_1, A_2) \) is \( N_B \) clopen in \((U_1, U_2)\) for every subset \((A_1, A_2)\) in \((V_1, V_2)\).

**Definition 3.9:** A function \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) is said to be
(i) $N_B$ contra $\alpha$-continuous, if $f^{-1}(B_1, B_2)$ is $N_B$ $\alpha$-closed in $(U_1, U_2)$ for every $N_B$ open $(B_1, B_2)$ in $(V_1, V_2)$.

(ii) $N_B$ contra pre-continuous, if $f^{-1}(B_1, B_2)$ is $N_B$ pre-closed in $(U_1, U_2)$ for every $N_B$ open $(B_1, B_2)$ in $(V_1, V_2)$.

(iii) $N_B$ contra semi-continuous, if $f^{-1}(B_1, B_2)$ is $N_B$ semi-closed in $(U_1, U_2)$ for every $N_B$ open $(B_1, B_2)$ in $(V_1, V_2)$.

(iv) $N_B$ contra $\beta$-continuous, if $f^{-1}(B_1, B_2)$ is $N_B$ $\beta$-closed in $(U_1, U_2)$ for every $N_B$ open $(B_1, B_2)$ in $(V_1, V_2)$.

Result 3.10:

i) Every $N_B$-contra continuous is $N_B$ contra $\alpha$-continuous.

ii) Every $N_B$ contra $\alpha$-continuous is $N_B$ contra pre-continuous.

iii) Every $N_B$ contra $\alpha$-continuous is $N_B$ contra semi-continuous.

iv) Every $N_B$-contra continuous is $N_B$ contra pre-continuous.

v) Every $N_B$-contra continuous is $N_B$ contra semi-continuous.

vi) Every $N_B$ contra pre-continuous is $N_B$ contra $\beta$-continuous.

vii) Every $N_B$ contra semi-continuous is $N_B$ contra $\beta$-continuous.

viii) Every $N_B$-contra continuous is $N_B$ contra $\beta$-continuous.

ix) Every $N_B$ contra $\alpha$-continuous is $N_B$ contra $\beta$-continuous.

Proof: (i) Given $f$ is $N_B$-contra continuous. Let $(B_1, B_2)$ be $N_B$ open in $(V_1, V_2)$. Since $f$ is $N_B$-contra continuous, $f^{-1}(B_1, B_2)$ is $N_B$ closed in $(U_1, U_2)$. Since every $N_B$ closed is $N_B$ $\alpha$-closed, $f^{-1}(B_1, B_2)$ is $N_B$ $\alpha$-closed in $(U_1, U_2)$. Therefore, $f$ is $N_B$ contra $\alpha$-continuous.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii), (viii) and (ix).

Remark 3.11: The concept of $N_B$ contra pre-continuous and $N_B$ contra semi-continuous are independent as shown in the following examples.

Example 3.12: Let $U_1 = \{a, b, c\}$, $U_2 = \{1, 2\}$ with $(U_1, U_2)/_R = \{\{(a, b), \{2\}\}, \{(c), \{1\}\}\}$ and $(X_1, X_2) = \{\{b\}, \{2\}\}$. Then $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{a, b\}, \{2\})\}$. Let $V_1 = \{x, y, z\}$, $V_2 =$
\{e,f\} with \((V_1,V_2)/_{R'} = \{(x,z),\{e\}\},\{(y),\{f\}\}\) and \((Y_1,Y_2) = (\{z\},\{e\}\) . Then \(\tau_{R'}(Y_1,Y_2) = \{(\Phi,\Phi),(Y_1,Y_2),(\{x,z\},\{e\}\}\) . Define \(f: (U_1,U_2,\tau_{R}(X_1,X_2)) \rightarrow (V_1,V_2,\tau_{R'}(Y_1,Y_2))\) as \(f([a],[1]) = ([x],[e]), f([a],[2]) = ([x],[f]), f([b],[1]) = ([y],[e]), f([b],[2]) = ([y],[f]), f([c],[1]) = ([z],[e]), f([c],[2]) = ([z],[f])\) . Here \((B_1,B_2) = ([x,z],[e])\) . Then \(f^{-1}([x,z],[e]) = ([a,c],[1])\) . Here \(f\) is \(N_B\) contra pre-continuous, but not \(N_B\) contra semi-continuous. Because \([a,c],[1]\) is \(N_B\) pre-closed and not \(N_B\) semi-closed in \((U_1,U_2)\), where \(([x,z],[e])\) is \(N_B\) open in \((V_1,V_2)\) . Therefore, \(f\) is \(N_B\) contra pre-continuous but not \(N_B\) contra semi-continuous.

**Example 3.13:** Let \(U_1 = \{a,b,c,d,e\}, U_2 = \{1,2,3,4\}\) with \((U_1,U_2)/_R = \{(a,b),\{2\}\},\{(c),\{4\}\},\{(d),\{3\}\},\{(e),\{1\}\}\) . Let \((X_1,X_2) = (\{a,c,d\},\{2,3,4\}\) . Then \(\tau_{R}(X_1,X_2) = \{(\Phi,\Phi),(U_1,U_2),\{(c,d),\{3,4\}\},\{(a,b,c,d),\{2,3,4\}\},\{(a,b),\{2\}\}\) .

Let \(V_1 = \{x,y,z\}, V_2 = \{e,f\}\) with \((V_1,V_2)/_{R'} = \{(x,z),\{e\}\},\{(y),\{f\}\}\) . Let \((Y_1,Y_2) = ([x],[e])\) . Then \(\tau_{R'}(Y_1,Y_2) = \{(\Phi,\Phi),(V_1,V_2),\{(x,z),\{e\}\}\) . Define \(f: (U_1,U_2,\tau_{R}(X_1,X_2)) \rightarrow (V_1,V_2,\tau_{R'}(Y_1,Y_2))\) as \(f([a],[1]) = ([x],[e]), f([a],[2]) = ([x],[f]), f([b],[1]) = ([z],[e]), f([b],[2]) = ([z],[f]), f([c],[1]) = ([y],[e]), f([c],[2]) = ([y],[f]), f([d],[1]) = ([y],[e]), f([d],[2]) = ([y],[f]), f([e],[1]) = ([y],[e]), f([e],[2]) = ([y],[f]), f([e],[3]) = ([y],[f]), f([e],[4]) = ([y],[f])\) .

\(f^{-1}([x,z],[e]) = ([a,b],[1,2])\) . Here \(f\) is \(N_B\) contra semi-continuous, but not \(N_B\) contra pre-continuous. Because \([a,b],[1,2]\) is \(N_B\) semi-closed and not \(N_B\) pre-closed in \((U_1,U_2)\) , where \(([x,z],[e])\) is \(N_B\) open in \((V_1,V_2)\) . Therefore, \(f\) is \(N_B\) contra semi-continuous but not \(N_B\) contra pre-continuous.

**Theorem 3.14:** Let \(f: (U_1,U_2,\tau_{R}(X_1,X_2)) \rightarrow (V_1,V_2,\tau_{R'}(Y_1,Y_2))\) be the function. If \(f\) is \(N_B\) strongly continuous then \(f\) is \(N_B\) perfectly continuous.

**Proof:** Given \(f\) is \(N_B\) strongly continuous. Let \((B_1,B_2)\) be \(N_B\) open in \((V_1,V_2)\) . Since \(f\) is \(N_B\) strongly continuous then \(f\) is \(N_B\) perfectly continuous.
strongly continuous, \( f^{-1}(B_1, B_2) \) is \( N_B \) clopen in \((U_1, U_2)\). Hence \( f \) is \( N_B \) perfectly continuous.

**Remark 3.15:** The converse of the above theorem need not be true by the following example.

**Example 3.16:** Let \( U_1 = \{a, b, c\}, U_2 = \{1, 2\} \) with \( (U_1, U_2)/R = \{([a, b], \{2\}), ([c], \{1\})\} \) and \((X_1, X_2) = ([a, c], \{1\})\). Then \( \tau_R(X_1, X_2) = ([\Phi, \Phi], (U_1, U_2), ([c], \{1\}), ([a, b], \{2\})\). The \( N_B \) closed sets are \((U_1, U_2), ([\Phi, \Phi], ([a, b], \{2\})([c], \{1\}))\). Let \( V_1 = \{x, y, z\}, V_2 = \{e, f\} \) with \((V_1, V_2)/R' = \{([x, z], \{e\}), ([y], \{f\})\}\). Let \((Y_1, Y_2) = ([z], \{e\})\). Then \( \tau_{R'}(Y_1, Y_2) = ([\Phi, \Phi], (V_1, V_2), ([x, z], \{e\}))\). Define \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) as \( f ([a], \{1\}) = ([x], \{e\}), f ([a], \{2\}) = ([x], \{e\}), f ([b], \{1\}) = ([z], \{e\}), f ([b], \{2\}) = ([z], \{e\}), f ([c], \{1\}) = ([y], \{e\}), f ([c], \{2\}) = ([y], \{e\})\). \( f^{-1}([y], \{e\}) = ([c], \{2\})\), which is not \( N_B \) clopen in \((U_1, U_2)\) and \( f^{-1}([x, z], \{e\}) = ([a, b], \{2\})\), which is \( N_B \) clopen in \((U_1, U_2)\). Therefore, \( f \) is \( N_B \) perfectly continuous but not \( N_B \) strongly continuous.

**Theorem 3.17:** Let \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) be the function. Then \( f \) is \( N_B \) perfectly continuous if and only if \( f \) is \( N_B \)-contra continuous and \( N_B \)-continuous.

**Proof:** Let \((B_1, B_2)\) be \( N_B \) open in \((V_1, V_2)\). Since \( f \) is \( N_B \) perfectly continuous, \( f^{-1}(B_1, B_2)\) is \( N_B \) clopen in \((U_1, U_2)\). Hence \( f^{-1}(B_1, B_2)\) is both \( N_B \) closed and \( N_B \) open in \((U_1, U_2)\). Hence \( f \) is both \( N_B \)-contra continuous and \( N_B \)-continuous. Conversely, let \( f \) be \( N_B \)-contra continuous and \( N_B \)-continuous. Let \((B_1, B_2)\) be \( N_B \) open in \((V_1, V_2)\). Since \( f \) is \( N_B \)-contra continuous, \( f^{-1}(B_1, B_2)\) is \( N_B \) closed in \((U_1, U_2)\). Since \( f \) is \( N_B \)-continuous, \( f^{-1}(B_1, B_2)\) is \( N_B \) open in \((U_1, U_2)\). Therefore, \( f^{-1}(B_1, B_2)\) is both \( N_B \) closed and \( N_B \) open in \((U_1, U_2)\). Hence \( f \) is \( N_B \) perfectly continuous.

**Remark 3.18:** The above theorem is true if \( f \) is both \( N_B \)-continuous and \( N_B \)-contra continuous otherwise it is not true by the following example.

**Example 3.19:** Let \( U_1 = \{a, b, c\}, U_2 = \{1, 2\} \) with \( (U_1, U_2)/R = \{([a, b], \{2\}), ([c], \{1\})\} \) and \((X_1, X_2) = ([b], \{2\})\). Then \( \tau_R(X_1, X_2) = ([\Phi, \Phi], (U_1, U_2), ([a, b], \{2\}))\). The \( N_B \) closed sets are \((\Phi, \Phi), (U_1, U_2), ([c], \{1\}))\). Let \( V_1 = \{x, y, z\}, V_2 = \{e, f\} \) with \((V_1, V_2)/R' = \{([x, z], \{e\}), ([y], \{f\})\}\). Let \((Y_1, Y_2) = ([z], \{e\})\). Then \( \tau_{R'}(Y_1, Y_2) = ([\Phi, \Phi], (V_1, V_2), ([x, z], \{e\}))\). Define \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) as \( f ([a], \{1\}) = ([x], \{e\}), f ([a], \{2\}) = ([x], \{e\}), f ([b], \{1\}) = ([z], \{e\}), f ([b], \{2\}) = ([z], \{e\}), f ([c], \{1\}) = ([y], \{e\}), f ([c], \{2\}) = ([y], \{e\})\). \( f^{-1}([y], \{e\}) = ([c], \{2\})\), which is not \( N_B \) clopen in \((U_1, U_2)\) and \( f^{-1}([x, z], \{e\}) = ([a, b], \{2\})\), which is \( N_B \) clopen in \((U_1, U_2)\). Therefore, \( f \) is \( N_B \) perfectly continuous but not \( N_B \) strongly continuous.
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\{([x,z], \{e\}), ([y], \{f\})\} and \((Y_1, Y_2) = ([y], \{f\})\). Then \(\tau_{R'}(Y_1, Y_2) = ([\Phi, \Phi], (V_1, V_2), ([y], \{f\}))\).

Define \(f: (U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) as \(f([a], \{1\}) = ([x], \{f\}), f([a], \{2\}) = ([x], \{e\}), f([b], \{1\}) = ([z], \{f\}), f([b], \{2\}) = ([z], \{e\}), f([c], \{1\}) = ([y], \{f\}), f([c], \{2\}) = ([y], \{e\})\). Therefore, \(f^{-1}([y], \{f\}) = ([c], \{1\})\), which is \(N_B\) closed in \((U_1, U_2)\) but not \(N_B\) open in \((U_1, U_2)\). Hence \(f\) is \(N_B\)–contra continuous but not \(N_B\)–continuous. Also \(f\) is not \(N_B\) perfectly continuous.

**Result 3.20:** Let \(f: (U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) be the function.

i) If \(f\) is \(N_B\) perfectly continuous then \(f\) is \(N_B\) contra \(\alpha\)-continuous and \(N_B\) \(\alpha\)–continuous.

ii) If \(f\) is \(N_B\) perfectly continuous then \(f\) is \(N_B\) contra pre-continuous and \(N_B\) pre–continuous.

iii) If \(f\) is \(N_B\) perfectly continuous then \(f\) is \(N_B\) contra semi-continuous and \(N_B\) semi-continuous.

iv) If \(f\) is \(N_B\) perfectly continuous then \(f\) is \(N_B\) contra \(\beta\)-continuous and \(N_B\) \(\beta\)–continuous.

By theorem 3.17 and definition 2.6, the above result is true but none of these implications is reversible as example 3.19.

**Theorem 3.21:** Every \(N_B\) strongly continuous function is both \(N_B\)-continuous and \(N_B\)-contra continuous.

**Proof:** Let \((B_1, B_2)\) be a subset of \((V_1, V_2)\). Since \(f\) is \(N_B\) strongly continuous, \(f^{-1}(B_1, B_2)\) is \(N_B\) clopen in \((U_1, U_2)\). That is, \(f^{-1}(B_1, B_2)\) is both \(N_B\) open and \(N_B\) closed in \((U_1, U_2)\). Since it holds for every subset of \((V_1, V_2)\), it is also true for all the \(N_B\) open sets in \((V_1, V_2)\).

Therefore, \(f\) is both \(N_B\)-continuous and \(N_B\)-contra continuous.

**Remark 3.22:** The converse of the above theorem need not be true by the following example.

**Example 3.23:** In example 3.19, the function \(f: (U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) is \(N_B\) – contra continuous. Consider \(([x,y], \{e\})\) is any subset of \((V_1, V_2)\). Then \(f^{-1}([x,y], \{e\}) = ([a,c], \{2\})\), which is not \(N_B\) open in \((U_1, U_2)\). Therefore, \(f\) is \(N_B\)–contra...
continuous. But \( f \) is neither \( N_B \) strongly continuous nor \( N_B \) - continuous.

**Result 3.24:** Let \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) be the function.

i) If \( f \) is \( N_B \) strongly continuous then \( f \) is \( N_B \) contra \( \alpha \)-continuous and \( N_B \) \( \alpha \) - continuous.

ii) If \( f \) is \( N_B \) strongly continuous then \( f \) is \( N_B \) contra pre-continuous and \( N_B \) pre – continuous.

iii) If \( f \) is \( N_B \) strongly continuous then \( f \) is \( N_B \) contra semi-continuous and \( N_B \) semi- continuous.

iv) If \( f \) is \( N_B \) strongly continuous then \( f \) is \( N_B \) contra \( \beta \)-continuous and \( N_B \) \( \beta \) – continuous.

By theorem 3.21 and definition 2.6, the above result is true but none of these implications is reversible as example 3.23.

**Remark 3.25:** Composition of two \( N_B \)- contra continuous functions need not be \( N_B \)- contra continuous as shown in the following example.

**Example 3.26:** Let \( U_1 = \{a, b, c\}, U_2 = \{1, 2\} \) with \( (U_1, U_2)/R = \{\{(a, b), \{2\}\}, \{(c), \{1\}\}\} \) and \( (X_1, X_2) = \{(b), \{2\}\}. \) Then \( \tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), \{(a, b), \{2\}\}\} \). The \( N_B \) closed sets are

\( (\Phi, \Phi), (U_1, U_2), \{(c), \{1\}\} \)

Let \( V_1 = \{a, b, c\}, V_2 = \{1, 2\} \) with \( (V_1, V_2)/R = \{(V_1, V_2)/R', = \{(a, c), \{1\}\}, \{(b), \{2\}\}\} \) and \( (Y_1, Y_2) = \{(b), \{2\}\}. \) Then \( \tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), (V_1, V_2), \{(b), \{2\}\}\} \). The \( N_B \) closed sets are

\( (\Phi, \Phi), (V_1, V_2), \{(a, c), \{1\}\} \)

Define \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) as \( f(\{a\}, \{1\}) = \{(a), \{2\}\}, f(\{a\}, \{2\}) = \{(a), \{1\}\}, f(\{b\}, \{1\}) = \{(c), \{2\}\}, f(\{b\}, \{2\}) = \{(c), \{1\}\}, f(\{c\}, \{1\}) = \{(b), \{2\}\}, f(\{c\}, \{2\}) = \{(b), \{1\}\}. \) Therefore, \( f^{-1}(\{b\}, \{2\}) = \{(c), \{1\}\}, \) which is \( N_B \) closed in \( (U_1, U_2). \) Hence \( f \) is \( N_B \)- contra continuous.

Let \( W_1 = \{a, b, c\}, W_2 = \{1, 2\} \) with \( (W_1, W_2)/R = \{(a), \{2\}\}, \{(b, c), \{1\}\} \) and \( (Z_1, Z_2) = \{(b), \{1\}\}. \) Then \( \tau_{R'}(Z_1, Z_2) = \{(\Phi, \Phi), (W_1, W_2), \{(b, c), \{1\}\}\}. \) The \( N_B \) closed sets are

\( (\Phi, \Phi), (W_1, W_2), \{(a), \{2\}\} \)

Define \( g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \to (W_1, W_2, \tau_{R'}(Z_1, Z_2), \) as \( g(\{a\}, \{1\}) = \{(b), \{2\}\}, g(\{a\}, \{2\}) = \{(c), \{1\}\}, g(\{b\}, \{1\}) = \{(a), \{2\}\}, g(\{b\}, \{2\}) = \{(c), \{1\}\}. \)
([a], [1]), g([c], [1]) = ([a], [2]), g([c], [2]) = ([a], [1]). Therefore, \( g^{-1}([b, c], [1]) = ([a], [2]) \). Hence \( f^{-1}(g^{-1}([b, c], [1])) = f^{-1}([a], [2]) = ([a], [1]) \), which is not a \( N_B \) closed set. Hence composition of two \( N_B \) – contra continuous functions is not a \( N_B \) – contra continuous function.

**Theorem 3.27:** Let \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) and \( g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \to (W_1, W_2, \tau_{R'}(Z_1, Z_2)) \) be the functions then \( g \circ f \) is \( N_B \) – contra continuous if \( g \) is \( N_B \) – continuous and \( f \) is \( N_B \) – contra continuous.

**Proof:** Given that \( g \) is \( N_B \) – continuous and \( f \) is \( N_B \) – contra continuous. Let \((B_1, B_2)\) be \( N_B \) open in \((W_1, W_2)\). Since \( g \) is \( N_B \) – continuous, \( g^{-1}(B_1, B_2) \) be \( N_B \) open in \((V_1, V_2)\). Since \( f \) is \( N_B \) – contra continuous, \( f^{-1}(g^{-1}(B_1, B_2)) \) is \( N_B \) closed in \((U_1, U_2)\). That is, \((g \circ f)^{-1}(B_1, B_2)\) is \( N_B \) closed in \((U_1, U_2)\). Hence \( g \circ f \) is \( N_B \) – contra continuous.

**Remark 3.28:** Let \( f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \) and \( g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \to (W_1, W_2, \tau_{R'}(Z_1, Z_2)) \) be the functions then

i) \( g \circ f \) is \( N_B \) perfectly continuous if \( g \) is \( N_B \) perfectly continuous and \( f \) is \( N_B \) – contra continuous.

ii) \( g \circ f \) is \( N_B \) strongly continuous if \( g \) is \( N_B \) strongly continuous and \( f \) is \( N_B \) – contra continuous.

iii) \( g \circ f \) is \( N_B \) perfectly continuous if \( g \) is \( N_B \) -continuous and \( f \) is \( N_B \) perfectly continuous.

iv) \( g \circ f \) is \( N_B \) perfectly continuous if \( g \) is \( N_B \) -continuous and \( f \) is \( N_B \) strongly continuous.

v) \( g \circ f \) is \( N_B \) -continuous if \( g \) is \( N_B \) – contra continuous and \( f \) is \( N_B \) -continuous.

vi) \( g \circ f \) is \( N_B \) -continuous if \( g \) is \( N_B \) -continuous and \( f \) is \( N_B \) -continuous.

vii) \( g \circ f \) is \( N_B \) perfectly continuous if \( g \) is \( N_B \) perfectly continuous and \( f \) is \( N_B \) strongly continuous.
vii) \(gof\) is \(N_B\) strongly continuous if \(g\) is \(N_B\) strongly continuous and \(f\) is \(N_B\) perfectly continuous.

**Note 3.29:** The above remark follows from theorems 3.14, 3.17, 3.21 and 3.27.

### 4. NANO BINARY D – CONTINUOUS

**Definition 4.1:** A nano binary subset \((A_1, A_2)\) of a nano binary topological space \((U_1, U_2, \tau_R(X_1, X_2))\) is called nano binary dense if \(\overline{N_B}(A_1, A_2) = (U_1, U_2)\) and it is denoted by \(N_B\)-dense.

**Definition 4.2:** Let \((U_1, U_2, \tau_R(X_1, X_2))\) and \((V_1, V_2, \tau_{R'}(Y_1, Y_2))\) be nano binary topological spaces. Then a mapping \(f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) is nano binary D-continuous function if the inverse image of every \(N_B\) open in \((V_1, V_2)\) is \(N_B\)-dense in \((U_1, U_2)\) and it is denoted by \(N_B\) D-continuous.

**Example 4.3:** Let \(U_1 = \{a, b, c\}, U_2 = \{1, 2\}\) with \(\frac{(U_1, U_2)}{R} = \{(a, b), \{2\}, \{c\}, \{1\}\}\) and \((X_1, X_2) = \{\{b\}, \{2\}\}\). Then \(\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), \{(a, b), \{2\}\}\}\). Let \(V_1 = \{x, y, z\}, V_2 = \{e, f\}\) with \(\frac{(V_1, V_2)}{R'} = \{\{x, z, \{e\}\}, \{y, \{f\}\}\}\) and \((Y_1, Y_2) = \{\{x, y, \{f\}\}\}\). Then \(\tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), (V_1, V_2), \{(x, z), \{e\}\}, \{y, \{f\}\}\}\). Define \(f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))\) as \(f(\{a\}, \{1\}) = \{\{y\}, \{f\}\}, f(\{a\}, \{2\}) = \{\{y\}, \{e\}, f(\{b\}, \{1\}) = \{\{x\}, \{f\}\}, f(\{b\}, \{2\}) = \{\{x\}, \{e\}\}, f(\{c\}, \{1\}) = \{\{z\}, \{f\}\}, f(\{c\}, \{2\}) = \{\{z\}, \{e\}\}\). Then \(f^{-1}(\{y\}, \{f\}) = \{(a), \{1\}\}\) and \(f^{-1}(\{x, z\}, \{e\}) = \{(b, c), \{2\}\}\), which is \(N_B\)-dense in \((U_1, U_2)\). That is, the inverse image of every \(N_B\) open in \((V_1, V_2)\) is \(N_B\)-dense in \((U_1, U_2)\). Therefore, \(f\) is \(N_B\) D-continuous.

**Definition 4.4:** A nano binary topological space \((U_1, U_2, \tau_R(X_1, X_2))\) is called a nano binary submaximal space if every \(N_B\)-dense subset of \((U_1, U_2)\) is \(N_B\) open in \((U_1, U_2)\) and it is denoted by \(N_B\) submaximal space.

**Definition 4.5:** A nano binary topological space \((U_1, U_2, \tau_R(X_1, X_2))\) is called a nano binary hyperconnected space if every \(N_B\) open subset of \((U_1, U_2)\) is \(N_B\)-dense in \((U_1, U_2)\) and it is denoted by \(N_B\) hyperconnected space.
**Remark 4.6:** Every $N_B$ D-continuous is not $N_B$-continuous as shown in the following example.

**Example 4.7:** In example 4.3, $f$ is $N_B$ D-continuous. But $f^{-1}(\{y\}, \{f\}) = (\{a\}, \{1\})$ and $f^{-1}(\{x,z\}, \{e\}) = (\{b, c\}, \{2\})$, which is not $N_B$ open in $(U_1, U_2)$. Therefore, $f$ is not $N_B$ – continuous. Hence $f$ is $N_B$ D-continuous but not $N_B$ –continuous.

**Note 4.8:** Similarly we can say the following:

i) Every $N_B$ D-continuous is not $N_B$ α – continuous.

ii) Every $N_B$ D-continuous is not $N_B$ semi-continuous.

iii) Every $N_B$ D-continuous is not $N_B$ pre –continuous

iv) Every $N_B$ D-continuous is not $N_B$ β – continuous.

**Remark 4.9:** In $N_B$ submaximal space, every $N_B$ D-continuous is $N_B$ –continuous.

**Note 4.10:** In $N_B$ submaximal space,

i) Every $N_B$ D-continuous is $N_B$ α –continuous.

ii) Every $N_B$ D-continuous is $N_B$ semi–continuous.

iii) Every $N_B$ D-continuous is $N_B$ pre–continuous.

iv) Every $N_B$ D-continuous is $N_B$ β –continuous.

**Remark 4.11:** Every $N_B$ –continuous is not $N_B$ D-continuous as shown in the following example.

**Example 4.12:** Let $U_1 = \{a, b, c\}, U_2 = \{1, 2\}$ with $(U_1, U_2)/R = \{((a, b), \{2\}), ((c), \{1\})\}$ and $(X_1, X_2) = ((a, c), \{1\})$. Then $τ_R(X_1, X_2) = ((\Phi, \Phi), (U_1, U_2), ((c), \{1\}), ((a, b), \{2\}))$. The $N_B$ closed sets are $(U_1, U_2), (\Phi, \Phi), ((a, b), \{2\}), ((c), \{1\})$. Let $V_1 = \{x, y, z\}, V_2 = \{e, f\}$ with $(V_1, V_2)/R' = \{([x, z], \{e\}), ([y], \{f\})\}$ and $(Y_1, Y_2) = ([x, y], \{f\})$. Then $τ_{R'}(Y_1, Y_2) = ((\Phi, \Phi), (V_1, V_2), ([x, z], \{e\}), ([y], \{f\}))$.

Define $f$: $(U_1, U_2, τ_R(X_1, X_2)) → (V_1, V_2, τ_{R'}(Y_1, Y_2))$ as $f(\{a\}, \{1\}) = ([x], \{f\}), f(\{a\}, \{2\}) = ([x], \{e\}), f(\{b\}, \{1\}) = ([z], \{f\}), f(\{b\}, \{2\}) = ([z], \{e\}), f(\{c\}, \{1\}) = ([y], \{f\}), f(\{c\}, \{2\}) = ([y], \{e\}).$ Then $f^{-1}(\{y\}, \{f\}) = (\{c\}, \{1\})$ and $f^{-1}(\{x, z\}, \{e\}) = (\{a, b\}, \{2\})$, which is $N_B$ open but not $N_B$ dense in $(U_1, U_2)$. Therefore, $f$ is $N_B$-continuous but not $N_B$ D-continuous.

**Remark 4.13:** In $N_B$ hyperconnected space, every $N_B$ –continuous is $N_B$ D-continuous.
Note 4.14: i) Every $N_B \alpha$ – continuous is not $N_B$ D-continuous.

ii) Every $N_B$ semi-continuous is not $N_B$ D-continuous.

iii) Every $N_B$ pre-continuous is not $N_B$ D–continuous.

iv) Every $N_B \beta$ D-continuous is not $N_B$ D– continuous.

5. CONCLUSION

In this paper, we have defined $N_B$- contra continuous function in nano binary topological spaces and their characterizations were studied. Also we have explored some nano binary contra continuous functions in nano binary topological spaces and their features were discussed. Also we have defined $N_B$ D– continuous and some properties are discussed.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES


NANO BINARY CONTRA CONTINUOUS FUNCTIONS


