A NOTE ON \((\varepsilon, \varepsilon \vee q)\)-FUZZY PRIME IDEALS OF NEAR-RINGS

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Abstract. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, we discuss a new kind of \((\varepsilon, \varepsilon \vee q)\)-fuzzy prime (semiprime) ideals of near-rings. The concept of \((\varepsilon, \varepsilon \vee q_0^\delta)\)-fuzzy prime (semiprime) ideals of near-rings are introduced and some of its related properties are investigated.

Keywords: near-rings; subnear-rings (ideals); \((\varepsilon, \varepsilon \vee q_0^\delta)\)-fuzzy ideals; \((\varepsilon, \varepsilon \vee q_0^\delta)\)-fuzzy prime (semiprime) ideals.

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1. INTRODUCTION

In 1965, Zadeh [16] introduced the concept of a fuzzy set. Using this concept, Rosenfeld [15] defined fuzzy subgroups in 1971. Since then, the study of fuzzy algebraic structure has been pursued in many directions such as groups, rings, near-rings, modules and so on. Abou-zaid [1] introduced the concept of fuzzy subnear-rings (ideals) and defined fuzzy prime ideals of near-rings. The concept of quasi-coincidence of a fuzzy point with a fuzzy set was introduced by Ming and Ming [14]. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, Bhakat and Das [2] defined different types of fuzzy subgroups called \((\alpha, \beta)\)-fuzzy subgroups. In particular, they introduced \((\varepsilon, \varepsilon \vee q)\)-fuzzy subgroup as the most viable generalization of

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a fuzzy subgroup defined by Rosenfeld. Narayanan and Manikantan [13] defined \((e,e \lor q^\delta)\)-fuzzy subnear-rings (ideals) of near-rings. The notions of \((e,e \lor q^\delta)\)-fuzzy subgroups, \(q^\delta_0\)-level sets and \((e \lor q^\delta)\)-level sets were introduced by Jun et al. [9]. The notion of \((e,e \lor q^\delta)\)-fuzzy subnear-rings (ideals) of near-rings was introduced by Gangmei and Devi [8]. Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, Bhakat and Das [4] defined \((e,e \lor q^\delta)\)-fuzzy semiprime (prime) of rings. Jian-ming and Davvaz [10] defined \((e,e \lor q^\delta)\)-fuzzy prime ideals of near-rings in the context of a fuzzy point with a fuzzy set. In this paper, \((e,e \lor q^\delta)\)-fuzzy semiprime (prime) ideals of near-rings in context of a fuzzy point with a fuzzy set are introduced and some of its related properties are investigated.

2. Preliminaries

We first recall the definition of a near-ring. A non-empty subset \(N\) with two binary operations “+” (addition) and “.” (multiplication) is called a near-ring if it satisfies the following axioms:

i) \((N,+)\) is a group,

ii) \((N,\cdot)\) is a semigroup,

iii) \((x+y)\cdot z = x\cdot z + y\cdot z\) \(\forall x,y,z \in N\). It is a right near-ring because it satisfies the right distributive law. If it satisfies left distributive law it is called left near-ring.

Unless otherwise stated, we shall consider only right near-rings throughout this paper.

Definition 2.1. [1] Let \(N\) be a near-ring. A normal subgroup \(I\) of \((N,+)\) is called

i) a right ideal if \(IN \subseteq I\),

ii) a left ideal if \(n(m+i) - nm \in I \forall n,m \in N\) and \(i \in I\),

iii) an ideal if it is both right and left ideal.

An ideal \(I\) of a near-ring \(N\) is called prime if \(xy \in I \Rightarrow x \in I\) or \(y \in I \forall x,y \in N\).

Definition 2.2. [16] A fuzzy set in a set \(X\) is a function \(\mu : G \to [0,1]\).

Definition 2.3. [14] A fuzzy set \(\mu\) in a set \(X\) of the form

\[
\mu(y) = \begin{cases} 
    t & \text{if } y = x, \\
    0 & \text{if } y \neq x, 
\end{cases}
\]

is said to be a fuzzy point with support \(x\) and value \(t\) and is denoted by \(x_t\).
Definition 2.4. [14] For a fuzzy point \( x_t \) and a fuzzy set \( \mu \) in a set \( X \), we say that
\[
i) \ x_t \in \mu \text{ (resp. } x_t q\mu) \text{ if } \mu (x) \geq t \text{ (resp. } \mu (x) + t > 1),
\]
\[
ii) \ x_t \in \vee q\mu \text{ if } x_t \in \mu \text{ or } x_t q\mu.
\]

Definition 2.5. [2],[3] A fuzzy set \( \mu \) of a group \( G \) is said to be an \((\epsilon, \in \vee q)\)-fuzzy subgroup of \( G \) if \( \forall x, y \in G \text{ and } t, r \in (0, 1], \)
\[
i) \ x_t, y_r \in \mu \Rightarrow (xy)_{\min \{t, r\}} \in \vee q\mu \text{ and }
\]
\[
ii) \ x_t \in \mu \Rightarrow (-x)_t \in \vee q\mu.
\]

Definition 2.6. [13] A fuzzy set \( \mu \) of a near-ring \( N \) is said to be an \((\epsilon, \in \vee q)\)-fuzzy subnear-ring of \( N \) if \( \forall x, y \in N \text{ and } t, r \in (0, 1], \)
\[
i) \ x_t, y_r \in \mu \Rightarrow (x+y)_{\min \{t, r\}} \in \vee q\mu.
\]
\[
ii) \ x_t \in \mu \Rightarrow (-x)_t \in \vee q\mu.
\]
\[
iii) \ x_t, y_r \in \mu \Rightarrow (xy)_{\min \{t, r\}} \in \vee q\mu.
\]

Definition 2.7. [13] A fuzzy set \( \mu \) of a near-ring \( N \) is said to be an \((\epsilon, \in \vee q)\)-fuzzy ideal of \( N \) if
\[
i) \ \mu \text{ is an } (\epsilon, \in \vee q)\text{-fuzzy subnear-ring of } N,
\]
\[
ii) \ x_t \in \mu \text{ and } y \in N \Rightarrow (y+x-y)_t \in \vee q\mu,
\]
\[
iii) \ x_t \in \mu \text{ and } y \in N \Rightarrow (xy)_t \in \vee q\mu,
\]
\[
iv) \ a_t \in \mu \text{ and } x, y \in N \Rightarrow (y(x+a)-yx)_t \in \vee q\mu \forall x, y, a \in N.
\]

Definition 2.8. [10] An \((\epsilon, \in \vee q)\)-fuzzy ideal \( \mu \) of a near-ring \( N \) is prime if \( \forall x, y \in N \text{ and } t \in (0, 1], \) we have \((xy)_t \in \mu \Rightarrow x_t \in \vee q\mu \text{ or } y_t \in \vee q\mu. \)

An \((\epsilon, \in \vee q)\)-fuzzy ideal \( \mu \) of a near-ring \( N \) is semiprime if \( \forall x, y \in N \text{ and } t \in (0, 1], \) we have \((x^2)_t \in \mu \Rightarrow x_t \in \vee q\mu. \)

Jun et al. [9] introduced the concept of \( \delta \)-quasi-coincidence of a fuzzy point with a fuzzy set as a generalization of quasi-coincidence of a fuzzy point with a fuzzy set in view of Bhakat and Das [2]. Let \( \delta \in (0, 1]. \) For a fuzzy point \( x_t \) and a fuzzy set \( \mu \) in a set \( X, \) we say that
\[
\bullet \ x_t \text{ is a } \delta \text{-quasi-coincident with } \mu, \text{ written } x_t q^\delta_\mu, \text{ if } \mu(x) + t > \delta.
\]
\[
\bullet \ x_t \in \vee q^\delta_\mu, \text{ if } x_t \in \mu \text{ or } x_t q^\delta_\mu.
\]
If \( \delta = 1, \) then the \( \delta \)-quasi-coincident with \( \mu \) is the quasi-coincident with \( \mu, \) i.e. \( x_t q^1_\mu = x_t q\mu. \)
**Definition 2.9.** [8] A fuzzy set $\mu$ of a near-ring $N$ is called an $(\in, \in \lor \mathcal{F})$-fuzzy subnear-ring of $N$ if $\forall x, y \in N$ and $t, r \in (0, \delta]$,  

i) $x_t \in \mu, y_r \in \mu \Rightarrow (x - y)_{\min(t, r)} \in \mathcal{F}_0 \mu$ and  

ii) $x_t \in \mu, y_r \in \mu \Rightarrow (xy)_{\min(t, r)} \in \mathcal{F}_0 \mu$.

**Definition 2.10.** [8] A fuzzy set $\mu$ of a near-ring $N$ is called an $(\in, \in \lor \mathcal{F})$-fuzzy ideal in $N$ if $\forall x, a \in N$ and $t \in (0, \delta]$,  

i) it is an $(\in, \in \lor \mathcal{F})$-fuzzy subnear-ring of $N$,  

ii) $x_t \in \mu, y \in N \Rightarrow (y + x - y)_t \in \mathcal{F}_0 \mu$,  

iii) $x_t \in \mu, y \in N \Rightarrow (xy)_t \in \mathcal{F}_0 \mu$ and  

iv) $a_t \in \mu, x, y \in \mu \Rightarrow (y(x + a) - yx)_t \in \mathcal{F}_0 \mu$.

A fuzzy set with condition i), ii), iii) is called an $(\in, \in \lor \mathcal{F})$-fuzzy right ideal of $N$ and if it satisfies i), ii), iv), then it is called an $(\in, \in \lor \mathcal{F})$-fuzzy left ideal of $N$.

### 3. Main Results

Throughout this section, $\delta \in (0, 1]$.

**Definition 3.1.** An $(\in, \in \lor \mathcal{F})$-fuzzy ideal $\mu$ of a near-ring $N$ is said to be  

i) an $(\in, \in \lor \mathcal{F})$-fuzzy prime, if $\forall x, y \in N$ and $t \in (0, \delta]$,  

$(xy)_t \in \mu \Rightarrow x_t \in \mathcal{F}_0 \mu$ or $y_t \in \mathcal{F}_0 \mu$.  

ii) an $(\in, \in \lor \mathcal{F})$-fuzzy semiprime, if $\forall x \in N$ and $t \in (0, \delta]$,  

$(x^2)_t \in \mu \Rightarrow x_t \in \mathcal{F}_0 \mu$.

**Example 3.2.** Let $N = \{0, a, b, c\}$ with $(N, +)$ as the Klein 4-group and $(N, \cdot)$ as given in table below,

<table>
<thead>
<tr>
<th>·</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
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<td>0</td>
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<td>a</td>
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<td>b</td>
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<td>c</td>
<td>0</td>
<td>c</td>
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<td>c</td>
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</table>
Then \((N,+,\cdot)\) is a right near-ring.

Define a fuzzy set \(\mu\) in \(N\) as \(\mu(0) = 0.7, \mu(a) = 0.8, \mu(b) = 0.6, \mu(c) = 0.5\). Then \(\mu\) is an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime of \(N\) with \(\delta \in (0, 1]\).

**Theorem 3.3.** An \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy ideal \(\mu\) of a near-ring \(N\) is an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime if and only if \(\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\} \forall x, y \in N\).

**Proof:** Let \(\mu\) be an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime.

If possible, let there exist \(x, y \in N\) such that \(\max\{\mu(x), \mu(y)\} < \min\{\mu(xy), \frac{\delta}{2}\}\). Choose \(t\) such that \(\max\{\mu(x), \mu(y)\} < t < \min\{\mu(xy), \frac{\delta}{2}\}\). Then \((xy)_t \in \mu\) but \(x_t, y_t \in \vee q^\delta_0\mu\) which is a contradiction.

Therefore, \(\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\}\).

Conversely, suppose \(\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\}\).

Let \((xy)_t \in \mu\), now \(\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\} \geq \min\{t, \frac{\delta}{2}\}\).

\(\Rightarrow \max\{\mu(x), \mu(y)\} \geq t\) if \(t \leq \frac{\delta}{2}\) or \(\max\{\mu(x), \mu(y)\} + t > \delta\) if \(t > \frac{\delta}{2}\).

So, either \(x_t \in \vee q^\delta_0\mu\) or \(y_t \in \vee q^\delta_0\mu\). Hence, \(\mu\) is an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime.

**Proposition 3.4.** Let \(\mu\) be an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime of a near-ring \(N\), then

i) \(\mu(x) \geq \min\{\mu(x^n), \frac{\delta}{2}\}\) and

ii) if \(\mu(x) < \frac{\delta}{2}\), then \(\mu(x) \geq \mu(x^n) \forall n\) belong to natural number.

**Proof:**

i) Since \(\mu\) is an \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy prime ideal of \(N\),

\(\max\{\mu(x), \mu(x)\} \geq \min\{\mu(xx), \frac{\delta}{2}\}\). This implies that \(\mu(x) \geq \min\{\mu(x^2), \frac{\delta}{2}\}\).

Thus, for \(n = 2\) is true. Suppose for \(n = k\) is true, that is, \(\mu(x) \geq \min\{\mu(x^k), \frac{\delta}{2}\}\).

Now, \(\max\{\mu(x), \mu(x^k)\} \geq \min\{\mu(x^{k+1}), \frac{\delta}{2}\}\). If \(\min\{\mu(x^k), \frac{\delta}{2}\} = \mu(x^k)\), then \(\mu(x) \geq \mu(x^k)\).

\(\Rightarrow \max\{\mu(x), \mu(x^k)\} = \mu(x)\). This gives \(\mu(x) \geq \min\{\mu(x^{k+1}), \frac{\delta}{2}\}\).

If \(\min\{\mu(x^k), \frac{\delta}{2}\} = \frac{\delta}{2}\), then \(\mu(x) \geq \frac{\delta}{2} \geq \min\{\mu(x^{k+1}), \frac{\delta}{2}\}\).

\(\Rightarrow \mu(x) \geq \min\{\mu(x^{k+1}), \frac{\delta}{2}\}\). Hence, by principle of mathematical induction, \(\mu(x) \geq \min\{\mu(x^n), \frac{\delta}{2}\} \forall n\) belong to natural number.

ii) If \(\mu(x) < \frac{\delta}{2}\), then \(\mu(x) \geq \min\{\mu(x^n), \frac{\delta}{2}\}\) [by (i)].

\(\Rightarrow \mu(x) \geq \mu(x^n) \forall n\) belong to natural number.

**Theorem 3.5.** An \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy ideal \(\mu\) of a near-ring \(N\) is \((\varepsilon, \in \vee q^\delta_0)\)-fuzzy semiprime if and only if \(\mu(x) \geq \min\{\mu(x^2), \frac{\delta}{2}\} \forall x \in N\).
Proof: Let \( \mu \) be an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy semiprime. Let \( x \in N \) such that \( \mu(x) < t < min\{\mu(x^2), \frac{\delta}{2}\} \).

Then \( \mu(x^2) \geq t \). This implies that \( (x^2)_t \in \mu \). That is, \( x_t \in \lor q\delta \mu \) which is a contradiction.

Therefore, \( \mu(x) \geq min\{\mu(x^2), \frac{\delta}{2}\} \).

Conversely, we assume that \( \mu(x) \geq min\{\mu(x^2), \frac{\delta}{2}\} \). Let \( x \in N \) and \( (x^2)_t \in \mu \).

Then \( \mu(x^2) \geq t \). Now, \( \mu(x) \geq min\{\mu(x^2), \frac{\delta}{2}\} \geq min\{t, \frac{\delta}{2}\} \).

\( \Rightarrow \mu(x) \geq t \) if \( t \leq \frac{\delta}{2} \) or \( \mu(x) \geq \frac{\delta}{2} \) if \( t > \frac{\delta}{2} \).

\( \Rightarrow \mu(x) \geq t \) if \( t \leq \frac{\delta}{2} \) or \( \mu(x) + t > \delta \) if \( t > \frac{\delta}{2} \).

Therefore, \( (x)_t \in \lor q\delta \mu \). Hence, \( \mu \) is an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy semiprime.

Definition 3.6. [6] Let \( \mu \) be a fuzzy set of a group \( G \). Then \( \forall t \in (0, 1] \), the set \( \mu_t = \{x \in G | \mu(x) \geq t\} \) is called level set of \( \mu \).

Definition 3.7. [5] The subset \( \mu_t = \{x \in X | \mu(x) \geq t \) or \( \mu(x) + t > 1\} \) is called \((\epsilon \lor q)\)-level set of \( X \) determined by \( \mu \) and \( t \).

Theorem 3.8. [8] A fuzzy set \( \mu \) of a near-ring \( N \) is an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy subnear-ring (ideal) of \( N \) if and only if the level set \( \mu_t \) is a subnear-ring (ideal) of \( N \) \( \forall t \in (0, \frac{\delta}{2}] \) and \( \delta \in (0, 1] \).

Collorary 3.9. [13] A fuzzy set \( \mu \) of a near-ring \( N \) is an \((\epsilon, \epsilon \lor q)\)-fuzzy subnear-ring (ideal) of \( N \) if and only if the level set \( \mu_t \) is a subnear-ring (ideal) of \( N \) \( \forall t \in (0, 0.5] \).

Theorem 3.10. An \((\epsilon, \epsilon \lor q\delta)\)-fuzzy ideal \( \mu \) of a near-ring \( N \) is \((\epsilon, \epsilon \lor q\delta)\)-fuzzy prime if and only if \( \mu_t \) is a prime ideal \( \forall t \in (0, \frac{\delta}{2}] \) and \( \delta \in (0, 1] \).

Proof: Let \( \mu \) be an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy prime.

Then by Theorem 3.8., \( \mu_t \) is an ideal of \( N \) \( \forall t \leq \frac{\delta}{2} \).

Let \( xy \in \mu_t \). Since \( \mu \) is an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy prime, by Theorem 3.3.,

\( max\{\mu(x), \mu(y)\} \geq min\{\mu(xy), \frac{\delta}{2}\} \geq min\{t, \frac{\delta}{2}\} = t \).

Then \( \mu(x) \geq t \) or \( \mu(y) \geq t \). This implies that \( x \in \mu_t \) or \( y \in \mu_t \).

Hence, \( \mu_t \) is a prime ideal.

Conversely, let \( \mu_t \) be a prime ideal. Then by Theorem 3.8.,

\( \mu \) is an \((\epsilon, \epsilon \lor q\delta)\)-fuzzy ideal of \( N \). Let \( (xy)_t \in \mu \), then \( xy \in \mu_t \).

\( \Rightarrow x \in \mu_t \) or \( y \in \mu_t \). [since \( \mu_t \) is a prime].

\( \Rightarrow \mu(x) \geq t \) or \( \mu(y) \geq t \).

\( \Rightarrow x_t \in \mu \) or \( y_t \in \mu \).
\[ x_t \in \bigvee_{0}^{t} \mu \text{ or } y_t \in \bigvee_{0}^{t} \mu. \]

Therefore, \( \mu \) is an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy prime.

If \( t > \delta \), then \( \mu_t \) may not be a prime ideal.

**Example 3.11.** Let \( N = \{0, a, b, c\} \) be a near-ring with \((N, +)\) as the Klein 4-group and \((N, \cdot)\) as given in table below,

\[
\begin{array}{c|cccc}
\cdot & 0 & a & b & c \\
\hline
0 & 0 & 0 & 0 & 0 \\
a & a & a & a & a \\
b & 0 & 0 & 0 & 0 \\
c & a & a & a & a \\
\end{array}
\]

Define a fuzzy set \( \mu \) in \( N \) as \( \mu(0) = 0.7, \mu(a) = 0.8, \mu(b) = 0.6, \mu(c) = 0.5. \)

Then \( \mu \) is an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy prime ideal of \( N \) with \( \delta \in (0, 1] \).

When \( t = 0.65 > \frac{\delta}{2}, \mu_t = \{0, a\} \). Then \( \mu_t \) is an ideal of \( N \),

but \( \mu_t \) is not a prime ideal since \( c \cdot b = a \in \mu_t \) but \( c \notin \mu_t \) and \( b \notin \mu_t \).

**Collorary 3.12.** [13] An \((\in, \in \bigvee q)\)-fuzzy ideal \( \mu \) of a near-ring \( N \) is prime if and only if the level set \( \mu_t \) is a prime ideal of \( N \) \( \forall \ t \in (0, 0.5] \).

**Theorem 3.13.** An \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy ideal \( \mu \) of a near-ring \( N \) is an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy semiprime if and only if \( \mu_t \) is a semiprime ideal \( \forall \ t \in (0, \frac{\delta}{2}] \).

**Proof** is similar to the proof of Theorem 3.10.

**Definition 3.14.** [9] Let \( \mu \) be a fuzzy set of a group \( G \). Then the subset \( \tilde{\mu}^\delta_t = \{x \in G; \mu(x) \geq t \} \) or \( \mu(x) + t > \delta \} \) is called \((\in \bigvee_{0}^{\delta})\)-level set of \( G \).

**Theorem 3.15.** [8] A fuzzy set \( \mu \) of a near-ring \( N \) is an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy ideal of \( N \) if and only if \( \tilde{\mu}^\delta_t (\neq \phi) \) is an ideal of \( N \) \( \forall \ t \in (0, \delta] \) and \( \delta \in (0, 1] \).

**Theorem 3.16.** [10] A fuzzy set \( \mu \) of a near-ring \( N \) is an \((\in, \in \bigvee q)\)-fuzzy ideal of \( N \) if and only if \( \tilde{\mu}^\delta_t (\neq \phi) \) is an ideal of \( N \) \( \forall \ t \in (0, 1] \).

**Theorem 3.17.** A fuzzy set \( \mu \) of a near-ring \( N \) is an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy prime of \( N \) if and only if \( \tilde{\mu}^\delta_t (\neq \phi) \) is a prime ideal of \( N \) \( \forall \ t \in (0, \delta] \).

**Proof:** Let \( \mu \) be an \((\in, \in \bigvee_{0}^{\delta})\)-fuzzy prime of \( N \).
Then by Theorem 3.15., $\mu_t^\delta$ is an ideal of $N \forall \ t \in (0, \delta]$.

Let $xy \in \mu_t^\delta$. Then $\mu(xy) \geq t$ or $\mu(xy) + t > \delta$. This implies that $(xy)_t \in \mu$ or $(xy)_q^0 \mu$.

Case 1. Let $(xy)_t \in \mu$.

a) if $t > \frac{\delta}{2}$, then $\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\} \geq \min\{t, \frac{\delta}{2}\} = \frac{\delta}{2}$

$\Rightarrow \max\{\mu(x), \mu(y)\} + t > \delta$.

$\Rightarrow \mu(x) + t > \delta$ or $\mu(y) + t > \delta$.

$\Rightarrow x \in \mu_t^\delta$ or $y \in \mu_t^\delta$.

b) if $t \leq \frac{\delta}{2}$, then $\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\} \geq \min\{t, \frac{\delta}{2}\} = t$

$\Rightarrow \mu(x) \geq t$ or $\mu(y) \geq t$

$\Rightarrow x \in \mu_t^\delta$ or $y \in \mu_t^\delta$.

Case 2. Let $\mu(xy) < t$ and $\mu(xy) + t > \delta$.

a) if $\mu(xy) \leq \frac{\delta}{2}$, then $\max\{\mu(x), \mu(y)\} + t \geq \min\{\mu(xy), \frac{\delta}{2}\} + t = \mu(xy) + t > \delta$, implies that $\mu(x) + t > \delta$ or $\mu(y) + t > \delta$. That is, $x \in \mu_t^\delta$ or $y \in \mu_t^\delta$.

b) if $\mu(xy) > \frac{\delta}{2}$, then $\max\{\mu(x), \mu(y)\} \geq \min\{\mu(xy), \frac{\delta}{2}\} = \frac{\delta}{2}$.

$\Rightarrow \max\{\mu(x), \mu(y)\} + t > \delta$ [since $t > \mu(xy) > \frac{\delta}{2}$]

$\Rightarrow \mu(x) + t > \delta$ or $\mu(y) + t > \delta$. That is, $x \in \mu_t^\delta$ or $y \in \mu_t^\delta$.

Thus, in both cases, $x \in \mu_t^\delta$ or $y \in \mu_t^\delta$. Hence, $\mu_t^\delta$ is a prime ideal of $N$.

Conversely, suppose $\mu_t^\delta$ is a prime ideal of $N$.

Then by Theorem 3.15., $\mu$ is an $(\varepsilon, \in \vee \delta^0)$-fuzzy ideal of $N$.

Let $(xy)_t \in \mu$. Then $\mu(xy) \geq t$, this implies that $xy \in \mu_t^\delta$.

$\Rightarrow x \in \mu_t^\delta$ or $y \in \mu_t^\delta$ [since $\mu_t^\delta$ is a prime ideal].

$\Rightarrow x_t \in \vee q^0 \mu$ or $y_t \in \vee q^0 \mu$. Thus, $\mu$ is an $(\varepsilon, \in \vee q^0 \delta)$-fuzzy prime ideal of $N$.

**Collorary 3.18.** [10] A fuzzy set $\mu$ of a group $G$ is an $(\varepsilon, \in \vee q^0 \delta)$-fuzzy prime of $G$ if and only if $\mu_t(\neq \phi)$ is a prime ideal of $N \forall \ t \in (0, 1]$.

**Definition 3.19.** [8] For a subset $A$ of a near-ring $N$, a fuzzy set $\chi_A^\delta$ in $N$ defined by $\chi_A^\delta : N \rightarrow [0, \delta]$ as

$$\chi_A^\delta(x) = \begin{cases} 
\delta & \text{if } x \in A, \\
0 & \text{otherwise},
\end{cases}$$

is called a $\delta$-characteristic fuzzy set of $A$ in $N$. 
**Theorem 3.20.** [8] A non-empty subset $A$ of a near-ring $N$ is a subnear-ring(ideal) of $N$ if and only if $\mathcal{X}^\delta_A$ is an $(\in, \lor q^\delta_0)$-fuzzy subnear-ring(ideal) of $N$.

**Theorem 3.21.** A non-empty subset $A$ of a near-ring $N$ is a prime (semiprime) ideal if and only if $\mathcal{X}^\delta_A$ is an $(\in, \lor q^\delta_0)$-fuzzy prime (semiprime) ideal of $N$.

The proof is straightforward

**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

**REFERENCES**