MODIFIED RESIDUAL POWER SERIES METHOD FOR SOLVING SYSTEM OF DIFFERENTIAL ALGEBRAIC EQUATIONS

N. R. ANAKIRA[1,*], M. S. HIJAZI[2], A. F. JAMEEL[3], A. K. ALOMARI[4], R. M. BATYHA[4,5],

M. ALMAHAMEED[1]

[1]Department of Mathematics, Faculty of Science and Information Technology, Irbid National University, 2600

Irbid, Jordan

[2]Department of Mathematics, College of arts and Sciences Tabarjal, Jouf University, KSA

[3]School of Quantitative Sciences, Universiti Utara Malaysia (UUM), Kedah, Sintok, 06010 Malaysia

[4]Department of Mathematics, Faculty of Science, Yarmouk University, Irbid 211-63, Jordan

[5]Department of Computer Science, Faculty of Science and Information Technology, Irbid National University,

2600 Irbid, Jordan

Abstract. In this paper, a powerful modified technique based on the residual power series method (RPSM) has been

formulated for the solutions of system of linear and nonlinear algebraic differential equations. The performance

and effectiveness of this modification are verified throughout the results obtained of the tested numerical examples

and comparing it with that obtained by the standard RPSM and other method in literature.

Keywords: system of algebraic differential equation; RPSM; MRPSM; Laplace transform; Pade approximant.

2010 AMS Subject Classification: 34K28, 44A10.

1. INTRODUCTION

Linear and nonlinear system of differential equations appears in various fields of applied science and engineering. Obtaining exact solutions of these systems are not easy to find. Different

---

*Corresponding author

E-mail address: dr.nidal@inu.edu.jo

numerical or approximated methods have been applied such as homotopy analysis method, optimal homotopy analysis method, A domian decomposition method, Differential transformation method, cubic splin, and so on [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The existing of a powerful and perfect method with high performance results is much more difficulties in related to the size of computational work, especially when the system is strongly nonlinear. In this regard, the differential algebraic equation is kind of differential equations, but the unknown functions in these equations are satisfying additional algebraic equations, such tha the derivatives is not in general expressed explicitly and typically derivatives of some of the dependent variables may not arise in the equations at all. In the last decades, several studies about the differential algebraic equations have been appeared. In this point, it is commonly difficult to solve these types of equations analytically. Hence, there are many powerful numerical methods in literature that can be employed to find approximate solutions, for example, the backward differentiation formula is first numerical method employed to find the solutions of class of algebraic differential equations. The implicit Runge-Kutta method also has been employed to solve these type of equations numerically. Furthermore, the variational iteration method and the homotopy perturbation method have been applied successfully for solving different types of equations and their applications in engineering [14, 15, 16, 17, 18, 19].

The fundamental motivation of this article is to utilized the RPSM for developing a technique to obtain the exact solutions of strongly linear and nonlinear system of differential algebraic equations. This technique is simple, in addition it can be applied directly to the given problems and does't require big effort to achieve accurate approximate solutions. The RPSM is an effective, easy and powerful technique that was employed in extensive scale in the last years for different types of differential equations without any restrictions such as lineariazation, descretization and perturbation [20, 21, 22, 23, 24, 25, 26, 27]. However, the accuracy of the RPSM depend on the number of the approximation and the solution converge to the exact form by increasing the number of the approximation.

The fundamental motivation of this article is based on improving the efficiency of the RPSM solution by using an alternative technique throughout applying the laplace transformation to the truncated RPSM solution then convert the transformed series into a meromorphic function by

applying Pade approximants, and then applying the inverse laplace transformation to obtain the required solution of the given problem.

This article is organized as follows. Section 2 is considered to explain the basic concepts of the RPSM along with the method analysis besides to the preliminary of the Pade approximants. Numerical examples are given to demonstrate and prove the capability of the presented technique. Conclusions of this study are summarized in the last Section.

2. DESCRIPTION OF THE SOLUTIONS PROCEDURES

2.1. Residual Power Series Method RPSM. [28, 29]

In this research article, we explain the solution procedure for system of linear and nonlinear algebraic differential equations which has the form of a power series expansion about the initial point $t = t_0$ for the given problem

$$(1) \qquad u'(t) = f(t, u(t)), \quad u(0) = t_0, \quad t \in [0, a],$$

where $f : [0, a] \times R - > R$ are nonlinear continuous function, $u(t)$ are unknown functions of independent variable t to be determined, and $a > 0$. To reach our goal, we assume the solution in the following form

$$(2) \qquad u(t) = \sum_{m=0}^{\infty} u_m(t),$$

where $u_m(t)$ are terms of approximations, note that, when $m = 0$, we have $u_0(t) = u(t_0) = c_0$, which is the initial guess approximation, then we evaluate $u_m(t), \forall m = 1, 2, ...$ and approximate the solution $u(t)$ of the given problem by k'th truncated series

$$(3) \qquad u_k(t) = \sum_{m=0}^{k} c_m(t)^m$$

To apply the RPSM, we write the given problem (1) in the following form:

$$(4) \qquad u'(t) - f(t, u(t) = 0.$$

Now, the kth residual function will be obtained by substituting the $k'th$ truncated series (3) into Eq. (4), as given below

$$(5) \qquad Res_k(t) = \sum_{m=1}^{k} mc_m t^{m-1} - f(t, \sum_{m=0}^{k} u_m(t)),$$

and the following ∞'th residual function:

(6) $$Res^{\infty}(t) = \lim_{k \to \infty} Res^k(t)$$

Clearly, it is easy to see that $Res^{\infty}(t) = 0$ for each $t \in (t_0, T)$, are infinitely differentiable functions at $t = t_0$. Moreover, $\frac{d^m}{dt^m} Res^{\infty}(t_0) = \frac{d^m}{dt^m} Res^k(t_0) = 0, m = 1, 2, ..., k$, this relation is considered a basic rule in the RPSM and its applications.

Now, in order to obtain the first order-approximate solutions, we put $k = 1$, and substituting $t = 0$ into Eq. (5), and using the fact that $Res^{\infty}(0) = Res^1(0) = 0$, to evaluate $c_1 = f(0, c_0) = f(0, u(0))$. Thus, using first-truncated series the first approximation for the given problem can be written as

(7) $$u(t) = u(t_0) + f(t_0, u(t_0))t$$

Similarly, the second-order approximation will be will be obtained by substituting $k = 2$ into Eq. (2) to be $\sum_{m=0}^{k=2} u_m(t)$ and by differentiate both sides of Eq. (5) with respect to t which yields to $\frac{d}{dt} Res^2(0) = 2c_2 - \frac{\partial}{\partial t} f(0, c_0) - c_1 \frac{\partial}{\partial u^2} f(0, c_0)$. In fact, $\frac{d}{dt} Res^2(0) = Res^{\infty}(0) = 0$.

Thus, we can write $c_2 = \frac{1}{2}(\frac{\partial}{\partial t} f(0, u(0)) + c_1 \frac{\partial}{\partial u^2} f(0, u(0)))$. Therefore, by consider the values of c_1 and c_2 into Eq.(3) when $k = 2$, the second-order approximate solution for the given problem becomes:

(8) $$u(t) = u(0) + c_1 t + c_2 t^2$$

The same process will be repeated to compute more components of the solution-order to obtain higher accuracy. The next theorem shows convergence of the RPS method.

2.2. Padé approximation. [29, 30] The $[L/M]$ Padé approximants of a function $u(x)$ is given by

$$\left[\frac{L}{M}\right] = \frac{P_L(t)}{Q_M(t)}$$

where $P_L(t)$ and $Q_M(t)$ are polynomials of degrees at most L and M, respectively. We know the formal power series

$$u(t) = \sum_{i=1}^{\infty} a_i t^i.$$

The coefficients of the polynomials $P_L(t)$ and $Q_M(t)$ are obtained from the equation

$$(9) \qquad u(t) - \frac{P_L(t)}{Q_M(t)} = O\left(t^{L+M+1}\right)$$

When the fraction of the numerator and denominator $\frac{P_L(t)}{Q_M(t)}$ is multiplying by a nonzero constant the fractional values remain unchanged, then we can define the normalization condition as

$$(10) \qquad Q_M(0) = 1.$$

Hence, we note that $P_L(t)$ and $Q_M(t)$ have no public factors. If we express the coefficient of $P_L(t)$ and $Q_M(t)$ as

$$(11) \qquad \begin{cases} P_L(t) = p_0 + p_1 t + p_2 t^2 + \cdots + p_L t^L \\ Q_M(t) = q_0 + q_1 t + q_2 t^2 + \cdots + q_M t^M \end{cases}$$

then, by Eq.s (10) and (11), we may multiply (9) by $Q_M(x)$, which linearizes the coefficient equations. We can write out Eq. (9) in more detail as

$$(12) \qquad \begin{cases} a_{L+1} + a_L q_1 + \cdots + a_{L-M+1} q_M = 0 \\ a_{L+2} + a_{L+1} q_1 + \cdots + a_{L-M+2} q_M = 0 \\ \qquad\qquad . \\ \qquad\qquad . \\ a_{L+M} + a_{L+M-1} q_1 + \cdots + a_L q_M = 0 \end{cases}$$

$$(13) \qquad \begin{cases} a_0 = p_0 \\ a_0 + a_0 q_1 = p_1 \\ a_2 + a_1 q_1 + a_0 q_2 = p_2 \\ \qquad\qquad . \\ \qquad\qquad . \\ a_L + a_{L-1} q_1 + \cdots + a_0 q_L = p_L \end{cases}$$

To solve these equations, we start with Eq. (12), which is a set of linear equations for all the unknown $q's$. Once the $q's$ are known, then Eq. (13) gives an explicit formula for the unknown $p's$, which complete the solution.

If Eq. (12) and Eq. (13) are non-singular, then we can solve them directly and obtain Eq.(15),

where Eq. (15) holds, and if the lower index on a sum exceeds the upper, the sum is replaced by zero:

$$
(14) \quad \left[\frac{L}{M}\right] = \frac{det \begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_L & a_{L+1} & \cdots & a_{L+M} \\ \sum_{j=M}^{L} a_{j-M} x^j & \sum_{j=M-1}^{L} a_{j-M+1} x^j & \cdots & \sum_{j=0}^{L} a_j x^j \end{bmatrix}}{det \begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_L & a_{L+1} & \cdots & a_{L+M} \\ x^M & x^{M-1} & \cdots & 1 \end{bmatrix}}
$$

Now, we can obtain Padé approximants diagonal matrix of different order using software such as Mathematica, Matlab and son.

3. NUMERICAL RESULTS AND DISSECTIONS

This section is devoted to present some numerical examples to check the validity and performances of our procedure.

3.1. Numerical Results.

3.1.1. *Example 1.* Consider the following linear system of algebraic differential equation

$$u_1'(t) - tu_2'(t) + t^2 u_3'(t) + u_1(t) - (t+1)u_2(t) + (t^2 + 2t)u_3(t) = 0,$$

$$u_2'(t) - tu_3'(t) - u_2(t) + (t-1)u_3(t) = 0,$$

$$(15) \qquad\qquad\qquad\qquad\qquad\qquad\qquad u_3(t) = sin(t)$$

subject to the initial conditions

$$(16) \qquad\qquad u_1(0) = 1 \quad u_2(0) = 1, \quad u_3(0) = 0.$$

To apply RPSM, we start with the initial conditions $c_{1,0} = u_{1,0}(t) = 1, \quad c_{2,0} = u_{2,0}(t) = 1, \quad c_{3,0} = u_{3,0}(t) = 0$, as initial guess and then, we suppose the solution in the following form of kth-truncated series

$$u_{1,k}(t) = c_{1,0} + c_{1,1}t + c_{1,2}t^2 + \cdots + c_{1,k}t^k,$$

$$u_{2,k}(t) = c_{2,0} + c_{2,1}t + c_{2,2}t^2 + \cdots + c_{2,k}t^k,$$

(17) $$u_{3,k}(t) = c_{3,0} + c_{3,1}t + c_{3,2}t^2 + \cdots + c_{3,k}t^k.$$

To evaluate the values of unknown constants $c_{i,m}, i = 1, 2, 3 \quad m = 1, 2, ..., k$, we construct the following $k'th$ residual functions

$$\begin{aligned}
Res_1^k(t) &= \sum_{m=1}^{k} mc_{1,m}t^{m-1} - \sum_{m=1}^{k} c_{2,m}t^m + \sum_{m=1}^{k} mc_{3,m}t^{m+1} + \sum_{m=0}^{k} c_{1,m}(t)^m, \\
&\quad - \sum_{m=0}^{k} c_{1,m}t^m + (t^2 + 2t) \sum_{m=1}^{k} c_{3,m}t^m,, \\
Res_2^k(t) &= \sum_{m=1}^{k} mc_{2,m}t^{m-1} - \sum_{m=1}^{k} c_{3,m}t^m - \sum_{m=0}^{k} c_{2,m}t^m, \\
&\quad + (t+1) \sum_{m=1}^{k} c_{3,m}t^m,
\end{aligned}$$

(18) $$Res_3^k(t) = \frac{d}{dt}\left(\sum_{m=1}^{k} c_{3,m}t^m - sint\right).$$

Using $k = 1$ and $t = 0$ on the above residual function and by making it equal to zero, i.e $(Res_i^1(0) = 0, \quad i = 1, 2, 3)$, the values of $c_{i,1}, \quad i = 1, 2, 3$ will be $c_{1,1} = 0, c_{2,1} = 1$ and $c_{3,1} = 1$. The values of the coefficients $c_{1,2}, \quad c_{2,2}$ and $c3, 2$ can be evaluated by differentiate both sides of Eq.(18) with respect to t by using $k = 2$ and then substituting $t = 0$ to be $c_{1,2} = \frac{3}{2}, \quad c_{2,2} = \frac{3}{2}$ and $c_{3,2} = 0$. By follow the same proceeder, the following order of the RPSM series approximate solutions is given by

$$\begin{aligned}
u_{1,10}(t) &= 1 + \frac{3}{2}t^2 + \frac{1}{3}t^3 + \frac{5}{24}t^4 + \frac{1}{30}t^5 + \frac{7}{720}t^6 + \frac{8}{720}t^7 + \frac{1}{4480}t^8 \\
&\quad + \frac{1}{45360}t^9 + \frac{11}{3628800}t^{10} + \cdots
\end{aligned}$$

$$u_{2,10}(t) = 1 + t + \frac{3}{2}t^2 + \frac{1}{6}t^3 - \frac{1}{8}t^4 + \frac{1}{120}t^5 + \frac{7}{720}t^6 + \frac{1}{5040}t^7 - \frac{1}{5760}t^8$$
$$+ \frac{1}{362880}t^9 + \frac{11}{3628800}t^{10} + \cdots$$

(19) $$u_{3,10}(t) = t - \frac{1}{6}t^3 + \frac{1}{120}t^5 - \frac{1}{5040}t^7 + \frac{1}{362880}t^9 + \cdots$$

which is converge to the exact solution in the limit of infinity terms of the order of the approximate solutions. To get better or high accuracy of the RPSM, we will employ an effective and accurate technique depend on the truncated series solutions of the RPSM by applying the laplace transform to the first ten terms of Eq. (21) to be

$$L\{u_{1,10}(t)\} = \frac{11}{s^{11}} + \frac{8}{s^{10}} + \frac{9}{s^9} + \frac{6}{s^8} + \frac{7}{s^7} + \frac{4}{s^6} + \frac{5}{s^5} + \frac{2}{s^4} + \frac{3}{s^3} + \frac{1}{s},$$

$$L\{u_{2,10}(t)\} = \frac{11}{s^{11}} + \frac{1}{s^{10}} - \frac{7}{s^9} + \frac{1}{s^8} + \frac{7}{s^7} + \frac{1}{s^6} - \frac{3}{s^5} + \frac{1}{s^4} + \frac{3}{s^3} + \frac{1}{s^2} + \frac{1}{s},$$

(20) $$L\{u_{2,10}(t)\} = \frac{1}{s^{10}} - \frac{1}{s^8} + \frac{1}{s^6} - \frac{1}{s^4} + \frac{1}{s^2}.$$

Consider $s = \frac{1}{z}$, yields

$$L\{u_1(t)\} = 11z^{11} + 8z^{10} + 9z^9 + 6z^8 + 7z^7 + 4z^6 + 5z^5 + 2z^4 + 3z^3 + z,$$

$$L\{u_2(t)\} = 11z^{11} + z^{10} - 7z^9 + z^8 + 7z^7 + z^6 - 3z^5 + z^4 + 3z^3 + z^2 + z,$$

(21) $$L\{u_3(t)\} = z^{10} - z^8 + z^6 - z^4 + z^2.$$

Then, using the Pade approximants of $\left[\frac{5}{5}\right]$

$$L\{u_1\} = \frac{2z^3 - z^2 + z}{z^3 - z^2 - z + 1}$$

$$L\{u_2\} = \frac{z^5 - 2z^4 + 4z^3 + z}{-z^5 + z^4 - 2z^3 + 2z^2 - z + 1}$$

(22) $$L\{u_3\} = \frac{z^2}{z^2 + 1}.$$

Consider that $z = \frac{1}{s}$, then applying the inverse of Laplace transform, yields to

$$u_1(t) = e^t t + e^{-t}$$

$$u_2(t) = e^t + t\sin(t).$$

(23) $$u_3(t) = \sin(t).$$

which is the exact solution.

3.1.2. *Example 2.* The second examples considered is the following nonlinear system of algebraic differential equation

$$u_1'(t) - u_1(t) + u_1(t)u_3(t) + u_3(t) + u_3'(t) = 1,$$

$$u_3'(t) - u_2(t) + u_1^2(t) + u_3(t) = 0,$$

(24) $$2u_2(t) - 2u_1^2(t) \;=\; 0$$

subject to the initial conditions

(25) $$u_1(0) = \quad u_2(0) = \quad u_3(0) = 1.$$

To apply RPSM, we start with the initial conditions $c_{1,0} = u_{1,0}(t) = 1$, $c_{2,0} = u_{2,0}(t) = 1$, $c_{3,0} = u_{3,0}(t) = 1$, as initial guess and then, we suppose the solution in the following form of kth-truncated series

$$u_{1,k}(t) = \sum_{m=0}^{k} c_{1,m} t^m = c_{1,0} + c_{1,1}t + c_{1,2}t^2 + \cdots + c_{1,k}t^k,$$

$$u_{2,k}(t) = \sum_{m=0}^{k} c_{2,m} t^m = c_{2,0} + c_{2,1}t + c_{2,2}t^2 + \cdots + c_{2,k}t^k,$$

(26) $$u_{3,k}(t) = \sum_{m=0}^{k} c_{3,m} t^m = c_{3,0} + c_{3,1}t + c_{3,2}t^2 + \cdots + c_{3,k}t^k,$$

then we construct the *kth* residual function $Res_i^k(t)$, where $i = 1, 2, \ldots$ is

$$Res_1^k(t) \;=\; \sum_{m=1}^{k} mc_{1,m}t^{m-1} - \sum_{m=0}^{k} c_{1,m}t^m + \left(\sum_{m=0}^{k} c_{1,m}t^m + 1\right)\left(\sum_{m=0}^{k} c_{3,m}t^m\right)$$

$$+ \sum_{m=1}^{k} mc_{3,m}t^{m-1} - 1,$$

$$Res_2^k(t) \;=\; \sum_{m=1}^{k} mc_{3,m}t^{m-1} - \sum_{m=0}^{k} c_{2,m}t^m + \left(\sum_{m=0}^{k} c_{1,m}t^m\right)^2 + \sum_{m=0}^{k} c_{3,m}t^m,$$

(27) $$Res_3^k(t) \;=\; \frac{d}{dt}\left(\sum_{m=0}^{k} 2c_{2,m}t^m - 2\left(\sum_{m=0}^{k} c_{1,m}t^m\right)^2\right).$$

Using $k = 1$ and $t = 0$ on the above residual function and make it equal to zero, i.e $(Res_i^1(0) = 0, \quad i = 1, 2, 3)$, the values of $c_{i,1}$, $i = 1, 2, 3$ are evaluated to be $c_{1,1} = 1, c_{2,1} = 2$ and $c_{3,1} = -1$.

The values of the coefficients $c_{1,2}, \quad c_{2,2}$ and $c_{3,2}$ can be evaluated by differentiate both sides of Eq.(18) with respect to t using $k = 2$ and then substituting $t = 0$ to be $c_{1,2} = \frac{1}{2}, \quad c_{2,2} = 2$ and $c_{3,1} = \frac{1}{2}$. By applying the same proceeder, the following order of the RPSM series approximate solutions will be

$$u_{1,k}(t) = 1 + t + \frac{1}{2}t^2 + \frac{1}{3!}t^3 + \frac{1}{4!}t^4 + \frac{1}{5!}t^5 + \frac{1}{6!}t^6 + \frac{1}{7!}t^7 + \frac{1}{8!}t^8 + \cdots,$$

$$u_{2,k}(t) = 1 + 2t + 2t^2 + \frac{4}{3}t^3 + \frac{2}{3}t^4 + \frac{4}{15}t^5 + \frac{4}{45}t^6 + \frac{8}{315}t^7 + \frac{2}{315}t^8 + \cdots,$$

(28) $$u_{3,k}(t) = 1 - t + \frac{1}{2}t^2 - \frac{1}{3!}t^3 + \frac{1}{4!}t^4 - \frac{1}{5!}t^5 + \frac{1}{6!}t^6 - \frac{1}{7!}t^7 + \frac{1}{8!}t^8 + \cdots,$$

which is converge to the exact solution in the limit of infinity terms of the order of the approximate solutions, i.e $(\lim_{x \to \infty} u_i(t)) = t\sin(t), \quad \tan(t)$ and $t\cos(t), \forall i = 1,2,3$, respectively. To obtain high accuracy, we use powerful and effective technique based on RPSM truncated series solutions by applying the laplace transform to the first ten terms of Eq. (28) as follows

$$L\{u_{1,k}(t)\} = \frac{1}{s^9} + \frac{1}{s^8} + \frac{1}{s^7} + \frac{1}{s^6} + \frac{1}{s^5} + \frac{1}{s^4} + \frac{1}{s^3} + \frac{1}{s^2} + \frac{1}{s},$$

$$L\{u_{2,k}(t)\} = \frac{256}{s^9} + \frac{128}{s^8} + \frac{64}{s^7} + \frac{32}{s^6} + \frac{16}{s^5} + \frac{8}{s^4} + \frac{4}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

(29) $$L\{u_{3,k}(t)\} = \frac{1}{s^9} - \frac{1}{s^8} + \frac{1}{s^7} - \frac{1}{s^6} + \frac{1}{s^5} - \frac{1}{s^4} + \frac{1}{s^3} - \frac{1}{s^2} + \frac{1}{s}.$$

For the purpose of simplicity, let $s = \frac{1}{z}$, yields to

$$L\{u_{1,k}(t)\} = z^9 + z^8 + z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z,$$

$$L\{u_{2,k}(t)\} = 256z^9 + 128z^8 + 64z^7 + 32z^6 + 16z^5 + 8z^4 + 4z^3 + 2z^2 + z,$$

(30) $$L\{u_{3,k}(t)\} = z^9 - z^8 + z^7 - z^6 + z^5 - z^4 + z^3 - z^2 + z.$$

Now, we use the Pade approximants of $\left[\frac{5}{5}\right]$

$$L\{u_1\} = \frac{z}{1-z}$$

$$L\{u_2\} = \frac{z}{1-2z}.$$

$$L\{u_3\} = \frac{z}{z+1}.$$

(31)

(A)   (B)   (C)

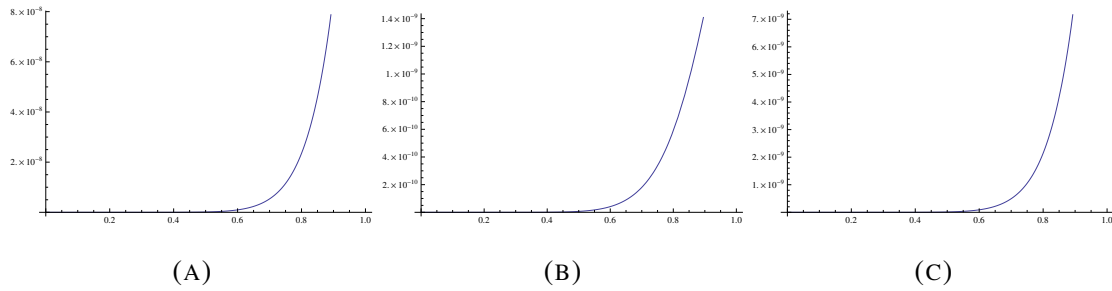FIGURE 1. Absolute errors related to $u_1(t)$, $u_2(t)$ and $u_3(t)$ of example 1

(A)   (B)   (C)
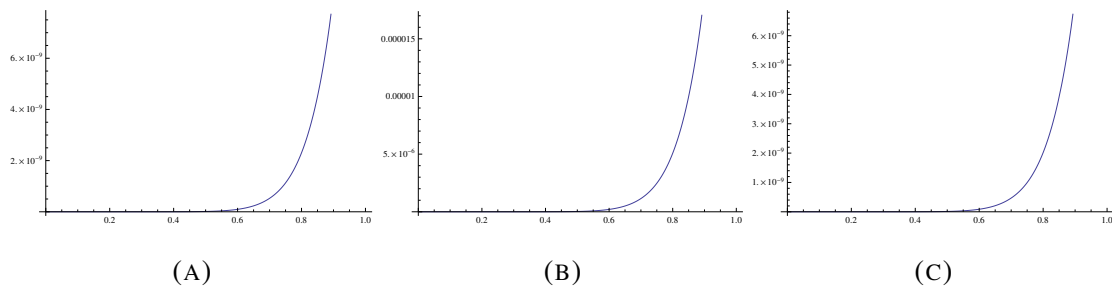
FIGURE 2. Absolute errors related to $u_1(t)$, $u_2(t)$ and $u_3(t)$ of example 2

Consider $z = \frac{1}{s}$, and applying the inverse of the Laplace transform, yields to

(32)
$$
\begin{aligned}
u_1(t) &= e^t \\
u_2(t) &= e^{2t}. \\
u_3(t) &= e^{-t}.
\end{aligned}
$$

which is the exact solution.

The obtained results, show that the proposed technique give us high accurate solutions iden-
tical to the exact solution. This obtained throughout using a few number's of truncated series
solution of the standard RPSM, this advantage overcomes the difficulties and efforts of evalu-
ated more terms of the solutions order. Figs. 1 and 2, are displayed to represent the absolute
errors of the standard RPSM solutions, from these figures we observed that high accuracy will
be obtained by evaluating more and more terms of the approximation.

4. Conclusions

In this research paper, an accurate and efficient modified procedure is presented and employed to handle linear and nonlinear system of algebraic differential equations based on the RPSM. The obtained results confirmed that the presented procedure is precise, convenient and straightforward for the solution of such systems of algebraic equations.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] A. F. Jameel, N. R. Anakira, A. K. Alomari, N. H. Man, Solution and Analysis of the Fuzzy Volterra Integral Equations via Homotopy Analysis Method. Computer Model. Eng. Sci. 127(3) (2021), 875-899.

[2] A. Hamoud, K. Ghadle, Homotopy analysis method for the first order fuzzy Volterra-Fredholm integro-differential equations. Indonesian J. Electric. Eng. Computer Sci. 11(3) (2018), 857-867.

[3] N. R. Anakira, Optimal homotopy asymptotic method for solving multi-pantograph type delay differential equations. Adv. Differ. Equ. Control Process. 19(3) (2018), 191-204.

[4] N. R. Anakira, A. K. Alomari, I. Hashim, Optimal homotopy asymptotic method for solving delay differential equations. Mathematical Problems in Engineering, 2013 (2013), 498902.

[5] A. F. Jameel, A. H. Shather, N. R. Anakira, A. K. Alomari, A. Saaban. Comparison for the Approximate Solution of the Second-Order Fuzzy Nonlinear Differential Equation with Fuzzy Initial Conditions. Math. Stat. 8(5) (2020), 527-534.

[6] J. Biazar, E. Babolian, R. Islam, Solution of the system of ordinary differential equations by Adomian decomposition method. Appl. Math. Comput. 147(3) (2004), 713-719.

[7] N. R. Anakira, A. Jameel, A. K. Alomari, A. Saaban, M. Almahameed, I. Hashim, Approximate solutions of multi-pantograph type delay differential equations using multistage optimal homotopy asymptotic method. J. Math. Fundam. Sci. 50(3) (2018), 221-232.

[8] I. A. H. Hassan, Application to differential transformation method for solving systems of differential equations. Appl. Math. Model. 32(12) (2008), 2552-2559.

[9] A. K. Alomari, N. R. Anakira, A. S. Bataineh, I. Hashim, Approximate solution of nonlinear system of BVP arising in fluid flow problem. Math. Probl. Eng. 2013 (2013), 136043.

[10] A. S. Heilat, R. S. Hailat, Extended cubic B-spline method for solving a system of non-linear second-order boundary value problems. J. Math. Computer Sci. 21 (2020), 231?242

[11] A. S. Heilat, N. N. Abd Hamid, A. I. M. Ismail, Extended cubic B-spline method for solving a linear system of second-order boundary value problems. SpringerPlus, 5(1) (2016), 1314.

[12] A. K. Alomari, M. I. Syam, N. R. Anakira, A. F. Jameel, Homotopy Sumudu transform method for solving applications in physics. Results Phys. 18 (2020), 103265.

[13] N. R. Anakira, M. M. Al-Shorman, A. F. Jameel, An Accurate Approximate Solutions of Multipoint Boundary Value Problems. General Lett. Math. 7 (2019), 31-38

[14] C. W. Gear, The simultaneous numerical solution of differential-algebraic equations, IEEE Transactions on Circuit Theory, 18(1) (1971), 89-95.

[15] E. Celik, E. Karaduman, M. Bayram, Numerical method to solve chemical differential-algebraic equations. Int. J. Quantum Chem. 89(5) (2002), 447-451.

[16] E. Celik, T. Yeloglu, Chebyshev series approximation for solving differential-algebraic equations (DAEs). Int. J. Computer Math. 83(8-9) (2006), 651-662.

[17] M. Ghovatmand, M. M. Hosseini, M. Nilli, Application of Chebyshev Approximation in the Process of Variational Iteration Method for Solving Differentialalgebraic Equations. Math. Comput. Appl. 16(4) (2011), 969-978.

[18] F. Soltanian, M. Dehghan, S. M. Karbassi, Solution of the differential algebraic equations via homotopy perturbation method and their engineering applications. Int. J. Computer Math. 87(9) (2010), 1950-1974.

[19] F. Salehi, M. A. Asadi, M. M. Hosseini, Solving system of DAEs by modified homotopy perturbation method. J. Computer Sci. Comput. Math. 2(6) (2012), 1-5.

[20] S. Thota, On Solving System of Linear Differential-Algebraic Equations Using Reduction Algorithm. Int. J. Math. Math. Sci. 2020 (2020), 6671926.

[21] M. Alshammari, M. Al-Smadi, O. A. Arqub, I. Hashim, M. A. Alias, Residual Series representation algorithm for solving fuzzy duffing oscillator equations. Symmetry, 12(4) (2020), 572.

[22] A. Kumar, S. Kumar, S. P. Yan, Residual power series method for fractional diffusion equations. Fundam. Inform. 151(1-4) (2017), 213-230.

[23] M. Modanli, S. T. Abdulazeez, A. M. Husien, A residual power series method for solving pseudo hyperbolic partial differential equations with nonlocal conditions. Numer. Meth. Part. Differ. Equ. 37(3) (2021), 2235-2243.

[24] M. Al-Smadi, O. A. Arqub, S. Hadid, Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method. Physica Scripta, 95(10) (2020), 105205.

[25] B. A. Mahmood, M. A. Yousif, A novel analytical solution for the modified Kawahara equation using the residual power series method. Nonlinear Dyn. 89(2) (2017), 1233-1238.

[26] M. Alquran, F. Yousef, F. Alquran, T. A. Sulaiman, A. Yusuf, Dual-wave solutions for the quadratic-cubic conformable-Caputo time-fractional Klein-Fock-Gordon equation. Math. Computers Simul. 185 (2021), 62-76.

[27] M. Al-Smadi, Solving initial value problems by residual power series method. Theor. Math. Appl. 3(1) (2013), 199-210.

[28] N. R. Anakira, A new accurate procedure for solving nonlinear delay differential equations. J. Math. Comput. Sci. 11(4) (2021), 4673-4685.

[29] S. Al-Ahmad, N. R. Anakira, M. Mamat, I. M. Suliman, R. AlAhmad, Modified Differential Transformation Scheme for Solving Classes of Non-Linear Differential Equations. TWMS J. Appl. Eng. Math. (2021), in press.

[30] S. Momani, V. S. Erturk, Solutions of non-linear oscillators by the modified differential transform method. Computers Math. Appl. 55(4) (2008), 833-842.