FCA-BASED APPROACH FOR MODELING THE DOMAIN KNOWLEDGE IN AN EDUCATION SYSTEM

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Abstract. In multidisciplinary courses, learners have diverse educational background and training on varying subjects. Subjects that such learners have studied may or may not relate to those of a course now being pursued. A requirement, therefore, arises to ascertain the level of understanding that the learners' have acquired respectively in their chosen subjects. This paper proposes a FCA-based teaching methodology for the learners on the basis of their background knowledge. In our approach, we first present a mathematical model to calculate the learners' prior knowledge on the subjects of pursuing course based on the learners' background knowledge. Using the calculated prior knowledge on the pursuing subjects, the learners are then clustered by generating proto-fuzzy to deliver appropriate learning material to each of the learners. Finally, we discuss the proposed model with an example.

Keywords: fuzzy sets; formal concept analysis; fuzzy concepts; proto-fuzzy concepts; domain knowledge; e-learning; university consortium.

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1. INTRODUCTION

Teaching means a purposeful and preplanned activity whose aim is to provide opportunities to facilitate and speed up the learning within learning and developing systems. Learning is a process to achieve knowledge, skill and ability to make decisions and performance. Learners of pursuing courses can have same educational background, significant differences are observed in them concerning their level of knowledge on the topics which they have already learnt. In particular, in case of interdisciplinary courses, learners may have diverse educational background and training on varying subjects. Subjects that such learners have studied may or may not relate to those of a course now being pursued.

A requirement, therefore, arises to ascertain the level of understanding that the learners' have acquired respectively in their chosen subjects. An e-learning environment could then segregate in groups so that the appropriate learning material can be delivered to each of the learners.

In other words, to support customized learning processes, the e-learning system [9] would use selection, organizing and presenting the learning materials to individual students based on the needs of individual students i.e., learning styles, pedagogical rules, and background knowledge. Online learning courses shall be used by learners who differ widely in terms of prior knowledge of the domain. Especially, in higher education and learning on interdisciplinary courses, some learners may have a background consisting of portions of knowledge of the course, while others could well be complete beginners.

However, regardless of the level and extent of prior knowledge, all the learners would need to have the same knowledge after completion of the course. On the one hand, users might lose active interest if they have to work on topics that they are already familiar with. On the other hand, they may not be in a position to estimate whether they do really know everything about a topic of a course without having gone through the chapters. Thus, letting users themselves decide whether they have enough knowledge or not, might result in an incomplete knowledge acquisition. Moreover, prior knowledge has an impact on the learning gain [6].

People learn by building on prior knowledge and abilities. This suggests it is important to design educational activities that are relevant to learners' prior knowledge so they can treat lessons meaningfully [7]. By the same token, learners need the appropriate prior knowledge

to start with. If learners do not have useful prior knowledge, then there is a risk and high probability that they will build new knowledge on a faulty foundation. They might develop misconceptions or brittle behavioral routines, too.

We have used FCA (Formal Concept Analysis)[3, 21] as the basis to build and expound on the methodology that is proposed in this paper. In present times, as an effective tool for data analysis, FCA (Formal Concept Analysis) has been extensively applied to fields such as decision making, information retrieval, data mining, knowledge discovery etc. Conceptual clustering is a machine learning paradigm for unsupervised classification developed mainly during the 1980s. Conceptual clustering is closely related to formal concept analysis (FCA). It is distinguished from ordinary data clustering by generating a concept description for each generated class. Most conceptual clustering methods are capable of generating hierarchical category structures. FCA is a formal technique for data analysis and knowledge presentation [5, 8]. It defines formal contexts to represent relationships between objects and attributes in a domain. From the formal contexts, FCA can then generate formal concepts and interpret the corresponding concept lattice so that information can be browsed or retrieved effectively.

In this paper, we will introduce a teaching methodology which gives the prior knowledge of the learners on the subjects of pursuing courses and the role of proto-fuzzy concepts [15] in this evaluation. The background and related works and basics of formal concept analysis are presented in the next two sections. Sections after that, analyzes the methodology (theoretical model) of determining the learners' prior knowledge, subsequent to the mathematical model for representing the domain knowledge and proto-fuzzy based clustering approach have been proposed. Finally, this model has been executed with a complete set of data.

2. BACKGROUND AND RELATED WORKS

The increased use of e-learning among educational institutions has led to a change in higher education. One of the main reasons for this is it gives students' greater access to education in comparison to traditional methods of teaching as students can undertake their study from anywhere and at any time as well as being given the option to study part-time or full-time [4]. E-learning has transformed the educational sector by enabling students to share information

and data in a relatively easy way. However, the problem of the "one-size-fits-all" may result in high dropout rates, and instill low levels of motivation and satisfaction. Human beings are different and as such they learn and process information in different ways. Past experience, prior knowledge, skills, learning style and interests are among the factors that may affect the individuals' requirements for effective learning. Instead of delivering the same content to all users, e-learning systems should be able to adapt to each individual's characteristics in order to increase the relevance and appropriateness of the learning material.

The prior knowledge assessment and the learning style questionnaire proved to be simple but useful tools to gather necessary information about the user in order to deliver personalized e-learning experience. Based on the learners' feedback concerning the respective learner's prior knowledge and the learning style, selection of the LOs (introductory, intermediate or advanced) is made, and then the organization of the LOs is framed [1, 14]. The process of personalized course construction starts with collecting information about the user's existing knowledge. This information is then used to match the level of LOs.

The interoperability and reusability of developed learning materials are other aims of e-learning. The instructors may face a large number of the learning objects that may make them recombine a new teaching material laboriously, and the instructors might have to take in more than one factor or criterion into account when they organize a well structured teaching material. Bearing this in mind, an approach - particle swarm optimization (PSO) is used to generate particular teaching materials automatically [24]. But instructors have to specify multiple criterion that are scaled according to the degrees of difficulty of selected learning objects, range of expected lecture time, and relationship between selected learning objects and specified topics.

Scheduling model for learning content organization has also developed based on pre-test, knowledge points learning (the granularity of learning object as a knowledge point), post-test and evaluation of learners [12].

The basic structure of SCORM's content aggregation model consists of content organization and LOs. This model presents a rule-driven approach that defines the intended sequencing and ordering of learning activities inside of a tree-structured content organization, but a process driven approach can be used to the SCORM content aggregation model [19]. The approach

intends to replace the tree-structured content organization scheme, which is clearly evident in SCORM, by the concept of a process-driven content organization scheme to realize the learning activities' sequencing and ordering functionality.

The process-driven e-Learning content organization model is based on a mathematical formalism and also aims to specify and analyze learning activity flows and processes for e-Learning instructions. So the learning process designer is able to easily build a process-driven content organization model through the graphical notation.

To enable a learner to pursue a course which is fitting and relevant, as well as being appropriate, on the basis of his / her background knowledge, we have discussed and introduced a teaching methodology for learners. This methodology aims at providing an insight into a subject of a course being pursued by each and every learner in advance based on prior subject knowledge.

The proposed method consists of the following steps: building a relation among "pursuing subjects", and union of the "pursuing subjects" and course related subjects; collect the information about the background knowledge on pursuing subjects and course related subjects; construction cross table of prior knowledge or, fuzzy context from previous two steps, and cluster or, proto-fuzzy concept generation [16, 17].

3. BASICS OF FORMAL CONCEPT ANALYSIS

Formal Concept Analysis (FCA) is a method often used for the analysis of data, i.e., for deriving implicit relationships between objects described through a set of attributes on the one hand and these attributes on the other. The data are structured into units which are formal abstractions of concepts of human thought, allowing meaningful comprehensible interpretation. The theory of formal concept analysis (FCA) was introduced by Wille [21] in 1982. But many of the information people facing are usually fuzzy and imprecise, so can not be described by a concept in the formal setting. By introducing Fuzzy sets and fuzzy logic [13, 10, 11. 18] into formal context, the theory of concept lattices has been generalized in [20, 22, 2]. In order to better interpret our approach, the following is a brief presentation of the FCA framework.

3.1. Formal context and formal Concept. Definition A formal context is a triplet $\langle X, Y, I \rangle$, where X and Y are sets and $I \subseteq X \times Y$ is a binary relation. The elements of X are called objects and the elements of Y are called attributes. $I \subseteq X \times Y$ is a binary relation between objects and attributes, i.e., the inclusion $(x, y) \in I$ means that object x has attribute y.

For $A \subseteq X$ and $B \subseteq Y$, if we define

$$A^{\uparrow} = \{y | \text{ for all } x \in A : (x, y) \in I\}$$
$$B^{\downarrow} = \{x | \text{ for all } y \in B : (x, y) \in I\}$$

With the above notation we define the concept.

Definition A formal concept in $\langle X, Y, I \rangle$ is a pair $\langle A, B \rangle$ of a set $A \subseteq X$ of objects and a set $B \subseteq Y$ of attributes such that $A^{\uparrow} = B$ and $B^{\downarrow} = A$. A is called extent and B is called intent of the concept $\langle A, B \rangle$.

If $B\langle X, Y, I \rangle$ denotes the set of all concepts, i.e., $B\langle X, Y, I \rangle = \{\langle A, B \rangle | A^{\uparrow} = B, B^{\downarrow} = A\}$ and $\leq$ is a partial order relation on $B\langle X, Y, I \rangle$ defined by $\langle A_1, B_1 \rangle \leq \langle A_2, B_2 \rangle$ iff $A_1 \subseteq A_2$(or, equivalently$B_1 \supseteq B_2$), then the $(B\langle X, Y, I \rangle, \leq)$ is an ordered set. It has some important properties:

$(B\langle X, Y, I \rangle, \leq)$ is a complete lattice, the concept lattice of $\langle X, Y, I \rangle$.

3.2. Fuzzy contexts and fuzzy concepts. We start with a set X of objects, a set Y of attributes, a complete residuated lattice **L** [23] and a fuzzy relation I between X and Y. The key idea of a fuzzy context (**L**-context) is as follows: it is a triplet $\langle X, Y, I \rangle$, where $I(x, y) \in L$ (the set of truth values of complete residuated lattice **L**) is interpreted as the truth value of the fact, "the object $x \in X$ has the attribute $y \in Y$ ". For fuzzy sets $A \in L^X$ and $B \in L^Y$, Belohlavek [20] and, independently, Pollandt [22] defined the fuzzy sets $A^{\uparrow} \in L^Y$ and $B^{\downarrow} \in L^X$ according to the formulae

$$A^{\uparrow}(y) = \Lambda_{x \in X}\{A(x) \rightarrow I(x,y)\}$$
$$B^{\downarrow}(x) = \Lambda_{y \in Y}\{B(y) \rightarrow I(x,y)\}$$

One can easily interpret the element $A^{\uparrow}(y) \in A^{\uparrow}$ as the truth degree of "y is shared by all objects from A"and $B^{\downarrow}(x) \in B^{\downarrow}$ as the truth degree of "x has all attributes from B".

A fuzzy concept $\langle A,B \rangle$ consists of a fuzzy set A of objects (the extent of the concept) and a fuzzy set B of attributes (the intent of the concept) such that $A^{\uparrow} = B$ and $B^{\downarrow} = A$.

3.3. Proto-fuzzy concepts[15].

Let $\langle X,Y,I \rangle$ be an L-context, where X and Y are set of objects (X) and set of properties (Y), respectively and I is a fuzzy relation between X and Y. Since the value $I(x,y)$ express the degree to which the object x carries the attribute y. If we set a threshold value $t \in L$ to eliminate the lower degree membership value from fuzzy relation then the resulting relation is called t-cut of L-context which is basically a binary relation between X and Y and is denoted by I_t. For every confidence threshold $t \in L$, consider two sets: $A' = \{y \in Y \mid \forall x \in A : I(x,y) \geq t\}$ for $A \subseteq X$, i.e., the set of all attributes from Y shared by all objects of A at least with the degree t and $B' = \{x \in X \mid \forall y \in B : I(x,y) \geq t\}$ for $B \subseteq Y$, i.e., the set of all objects from X sharing attributes from B at least in the degree t. The pair $\langle A,B \rangle \in 2^X \times 2^Y$ is called t-concept iff $A' = B$, $B' = A$. The set of all t-concept in the t-cut is denoted by C_t.

The Triples $\langle A,B,t \rangle \in 2^X \times 2^Y \times L$ such that $\langle A,B \rangle \in \bigcup_{k \in L} C_k$ and $l = sup\{k \in L : \langle A,B \rangle \in C_k\}$ are called proto-fuzzy concepts. i.e., the proto-fuzzy concept is triple of a subset of objects, a subsets of attributes and a value as a best common degree of membership of all pairs of objects and attributes from the above-mentioned sets to the $L-$context. The set of proto-fuzzy concepts denoted by C^P.

4. THEORETICAL MODEL FOR REPRESENTING THE DOMAIN KNOWLEDGE

In this section, we describe a learning methodology for the learners of pursuing courses. Though, the learners of pursuing courses can have same educational background (in case of interdisciplinary courses, learners have the diverse educational background and training on varying subjects), significant differences are observed in them concerning their level of knowledge

on the topics which they have already learnt. Also, their subjects may or may not relate to those of a course now being pursued. Therefore, considering the learners' previous knowledge and, further, considering a relation between the subjects which can be found in common to the one that a learner may have already studied, and those in the curriculum of the pursuing course, we propose the following approach:

- Relate all the "pursuing subjects" to the "union of the pursuing subjects and their related subjects (list of subjects which helps to learn the pursuing subjects)" by assigning a weight from a numerical scale graded from 0 to 10. The weight are given on the basis of the amount of subject matter/content related with the pursuing subject, where 0 indicates that constituent portion of a subject of the union in the pursuing subject is 0 percent, 1 indicates that constituent portion of a subject of the union in the pursuing subject is 10 percent, 2 indicates that constituent portion of a subject of the union in the pursuing subject is 20 percent, etc.

- Relate all learners with the pursuing subjects and their related subjects on the basis of their knowledge on those subjects. Again, a numerical scale graded from 0 to 10 is used to determine the level of the knowledge of a particular learner, on pursuing and related subjects.

- Using the above two relations, construct a data table, i.e., a fuzzy context which provides the truth degree of previous / prior knowledge of each learners on the pursuing subjects.

- Now from the table of learners' prior knowledge on the pursuing subjects, we generate all proto-fuzzy concepts of different truth degrees. Each proto-fuzzy concept divides all the learners into a group in which the level of the knowledge of each learner on different pursuing subjects (i.e., intent of proto-fuzzy concept) is represented by the truth degree of proto-fuzzy concept. Therefore according to the level of the knowledge on pursuing subjects, the learners may learn the subjects starting with basic of the subjects or at intermediate level of the subjects or may learn the subjects starting at the advanced level of the subjects.

5. MATHEMATICAL MODEL FOR REPRESENTING THE DOMAIN KNOWLEDGE

Let X be the set of all learners pursuing a particular course, Y be the set of all subjects of pursuing course, and Y' be the set of pursuing subjects together with their related subjects.

We relate Y and Y' by assigning a weight, $w(s,s')$, between $s \in Y$ and $s' \in Y'$, where $w(s,s')$ indicates the amount of constituent portions of $s \in Y$ are contained in $s' \in Y'$ and weight between same pursuing subject, i.e., $w(s,s)$ for $s \in Y$ indicates the percentage of originality of the subject, $s \in Y$. A numerical scale graded from 0 to 10 is used for $w(s,s')$ so that $\sum_{s' \in Y'} w(s.s') = 10$.

Similarly we relate X and Y' by assigning a weight, $w(x,s')$, between $x \in X$ and $s' \in Y'$, where $w(x,s')$ indicates the level of the knowledge of a particular learner $x \in X$ on the subject $s' \in Y'$. Again, a numerical scale graded from 0 to 10 is used to determine the knowledge of a particular learner with the assumption that if, a learner shares 100 percent knowledge in any pursuing subject then he should has 100 percent knowledge on all subjects related to pursuing subject. For the learner $x \in X$, we define the truth degree, $I(x,s_k)$ of the fact that"the student $x \in X$ has knowledge on the subject $x_k \in Y$" as

$$I(x,s_k) = \frac{\sum_i \{\frac{w(x,s_i)}{10}.w(s_k,s_i)\}}{\sum_i w(s_k,s_i)},$$ where $w(s_k,s_i)$ is the weight given on the subject $s_i \in Y'$ and $w(x,s_i)$ is the weight given on the subject $s_i \in Y'$. We denote this truth value by $I(x,s_k)$. This truth value $I(x,s_k)$ expresses the degree to which the learner carries x the knowledge on s_k.

Now we represent all calculated prior knowledge of the learners by means of a cross table with rows and columns corresponding to learner $x \in X$ and subject $s \in Y$, respectively, and table entries $I(x,s)$ is interpreted as the truth value of the fact, "the degree to which the learner $x \in X$ carries the knowledge on the pursuing subject $s \in Y$". This table is a concrete example of fuzzy formal context, $\langle X, Y, I \rangle$, where X and Y are the set of objects and attributes subjects, respectively, and the value $I(x,s)$ is interpreted as the truth value of the fact, the learner $x \in X$ has the attribute $s \in Y$. Finally, from the fuzzy context we find groups of the learners who have best common degree of knowledge on the pursuing subjects by generating proto-fuzzy concepts.

6. EXAMPLES

Let $Y = \{s_1, s_2 s_3, s_4 s_5\}$ be the set of pursuing subjects, $Y' = \{s_1, s_2 s_3, s_4 s_5 s_6$ $s_7 s_8\}$ be the set of the union of the pursuing subjects and their related subjects, and $X = \{x_1, x_2 x_3, x_4 x_5\}$ be the set of learners. We now consider the following two weighted relations between "Y and Y'", and "X and Y" given in Table 1 and Table 2, respectively.

TABLE 1. A weighted relation between the set of "pursuing subjects" and the union of "pursuing subjects and its related subjects"

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
s_1	5	0	0	0	0	2	2	1
s_2	3	5	0	0	0	0	2	0
s_3	2	0	8	0	0	0	0	0
s_4	0	1	0	7	1	1	0	0
s_5	0	0	3	0	5	2	0	0

TABLE 2. A weighted relation between the set of 'learners' and the union of 'pursuing subjects and its related subjects'

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
x_1	5	5	0	0	0	10	5	0
x_2	0	0	5	0	0	5	10	0
x_3	0	0	0	5	0	5	10	0
x_4	10	3	0	0	2	10	10	10
x_5	0	0	5	0	0	5	0	0

Now Using the transformation, $I(x, s_k) = \dfrac{\sum_i \{\frac{w(x,s_i)}{10} . w(s_k, s_i)\}}{\sum_i w(s_k, s_i)}$, where $w(s_k, s_i)$ is the weight given on the subject $s_i \in Y'$ and $w(x, s_i)$ is the weight given on the subject $s_i \in Y'$, described in the

section 4, we obtain the following table 3 as fuzzy context which which provides the knowledge information on the pursuing subjects for each learner.

TABLE 3. Data table of prior knowledge

	s_1	s_2	s_3	s_4	s_5
x_1	0.55	0.5	0.1	0.15	0.2
x_2	0.3	0.2	0.4	0.05	0.25
x_3	0.3	0.2	0	0.4	0.1
x_4	1	0.65	0.2	0.15	0.3
x_5	0.1	0	0.4	0.05	0.25

It may appear from the Table 2 that learners x_2 and x_3 have no knowledge on the pursuing subject s_1. But on the basis of their previous knowledge, we are able to infer from the Table 3 that both learners have good knowledge on the subject s_1. Especially, we can say that the learner x_2 could well be one of best learners for this course.

Now, based on their prior knowledge on the pursuing subjects, we divide all learners into different groups by generating proto-fuzzy concepts which are shown in table 4, so that learners belonging to the groups may learn the subjects starting with different level of the subjects, e.g, from the concepts $\langle \{x_4\}, \{s_1, s_2\}, 0.65 \rangle$ and $\langle \{x_4\}, \{s_1\}, 1 \rangle$ we can see that the learner x_4 has very good knowledge on the subjects s_1 and s_2. Therefore x_4 would able to learn the subjects s_1 and s_2 stating with the advance level. Similarly, from the concepts $\langle \{x_1, x_4\}, \{s_1, s_2\}, 0.5 \rangle$ and $\langle \{x_1, x_4\}, \{s_1\}, 0.55 \rangle$ we can see that the learner x_1 has good knowledge on the subjects s_1 and s_2 and he could learn the those subjects starting at the intermediate stages. Moreover, proto-fuzzy concepts may also help the learners to find their friends so that they can study together, e.g., it may well be possible that learner x_2 might be comfortable more with x_4 rather than x_5 when studying together. With this connection concept $\langle \{x_2, x_4, x_5\}, \{s_3, s_5\}, 0.20 \rangle$, it can be inferred that x_2, like x_4, can study four subjects $\{s_3, s_5\}$ together.

TABLE 4. Proto-fuzzy concepts of the fuzzy context given in Table 1

Truth value t	proto-fuzzy concepts of degree t
0	$\langle \{x_1, x_2, x_3, x_4\}, \{s_1, s_2, s_4, s_5\}, 0 \rangle$
	$\langle \{x_1, x_2, x_4, x_5\}, \{s_1, s_3, s_4, s_5\}, 0 \rangle$
0.05	$\langle \{x_1, x_2, x_3, x_4, x_5\}, \{s_1, s_4, s_5\}, 0.05 \rangle$
	$\langle \{x_1, x_2, x_4, x_5\}, \{s_1, s_3, s_4, s_5\}, 0.05 \rangle$
	$\langle \{x_1, x_2, x_4\}, \{s_1, s_2, s_3, s_4, s_5\}, 0.05 \rangle$
0.1	$\langle \{x_1, x_2, x_3, x_4, x_5\}, \{s_1, s_5\}, 0.1 \rangle$
	$\langle \{x_1, x_2, x_4, x_5\}, \{s_1, s_3, s_5\}, 0.1 \rangle$
	$\langle \{x_1, x_3, x_4\}, \{s_1, s_2, s_4, s_5\}, 0.1 \rangle$
	$\langle \{x_1, x_4\}, \{s_1, s_2, s_3, s_4, s_5\}, 0.1 \rangle$
0.15	$\langle \{x_1, x_3, x_4\}, \{s_1, s_2, s_4\}, 0.15 \rangle$
	$\langle \{x_1, x_4\}, \{s_1, s_2, s_4, s_5\}, 0.15 \rangle$
	$\langle \{x_4\}, \{s_1, s_2, s_3, s_4, s_5\}, 0.15 \rangle$
0.2	$\langle \{x_1, x_2, x_3, x_4\}, \{s_1, s_2\}, 0.2 \rangle$
	$\langle \{x_1, x_2, x_4, x_5\}, \{s_5\}, 0.2 \rangle$
	$\langle \{x_1, x_2, x_4\}, \{s_1, s_2, s_5\}, 0.2 \rangle$
	$\langle \{x_2, x_4, x_5\}, \{s_3, s_5\}, 0.2 \rangle$
	$\langle \{x_2, x_4\}, \{s_1, s_2, s_3, s_5\}, 0.2 \rangle$
	$\langle \{x_3\}, \{s_1, s_2, s_4\}, 0.2 \rangle$
0.3	$\langle \{x_1, x_2, x_3, x_4\}, \{s_1\}, 0.3 \rangle$
	$\langle \{x_2\}, \{s_1, s_3\}, 0.3 \rangle$
	$\langle \{x_3\}, \{s_1, s_4\}, 0.3 \rangle$
	$\langle \{x_4\}, \{s_1, s_2, s_5\}, 0.3 \rangle$
0.4	$\langle \{x_2, x_5\}, \{s_3\}, 0.4 \rangle$
	$\langle \{x_3\}, \{s_4\}, 0.4 \rangle$
0.5	$\langle \{x_1, x_4\}, \{s_1, s_2\}, 0.5 \rangle$
0.55	$\langle \{x_1, x_4\}, \{s_1\}, 0.55 \rangle$
0.65	$\langle \{x_4\}, \{s_1, s_2\}, 0.65 \rangle$
1	$\langle \{x_4\}, \{s_1\}, 1 \rangle$

7. Conclusion

In this paper, we have introduced a FCA-based teaching methodology for the learners on the basis of their background knowledge. This method aims at providing an insight into a subject of a pursuing course of each learner in advance, and also helps a learner to pursue a course which is fitting and appropriate in the context of his/her background knowledge.

The proposed method consists of the following steps: building a relation among "pursuing subjects", and union of the "pursuing subjects" and course related subjects; collect the information about the background knowledge of the learner on pursuing subjects and course related subjects; construction of fuzzy context from previous two steps and classification of learners by generating proto-fuzzy concepts.

More specifically, we have analyzed the data table of background knowledge using formal concept analysis. Students represent objects, subjects represent attributes and corresponding valuations represent values assigned to every object-attribute pair by fuzzy binary relation over the set [0,1]. Therefore using formal concept analysis, we have calculated clusters of students similar by their studying results of all subjects, or to find clusters of subjects similar by knowledge of all students. This clustering technique ultimately help in selecting and sequencing the appropriate, and consequently, the most effective learning materials for a given type of a learner.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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