Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 6, 6657-6665
https://doi.org/10.28919/jmcs/6393
ISSN: 1927-5307

ON INTERVAL VALUED FUZZY ALMOST (m,n)-BI-IDEAL IN SEMIGROUPS

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Abstract. In this paper, we study the concept of an interval valued fuzzy almost (m,n)- bi-ideals. We investigate properties of an interval valued fuzzy almost (m,n)-bi- ideal in semigroups.

Keywords: almost (m,n)-bi-ideal; bi-ideal; interval valued fuzzy almost (m,n)-bi-ideal.

2010 AMS Subject Classification: 03E72, 18B40.

1. Introduction

The theory of fuzzy set was presented in 1965 by Zadeh [12]. The theory of fuzzy semigroups contained by Kuroki in 1979 [8]. Later the theory of interval valued fuzzy sets was introduced in 1975 by Zadeh [13], as a generalization of the notion of fuzzy sets. Interval valued fuzzy sets have various applications in several areas like medical science [3], image processing [2], decision making [14], etc. In 2006, Narayanan and Manikantan [7] developed the theory of interval valued fuzzy subsemigroup and studied types interval valued fuzzy ideals in semigroups. In 1961, Lajos [5] studied the concepts of (m,n)-ideals in semigroups which generalized of ideals of semigroups. The reseach of (m,n)-ideals of semigroups has interested many such as Akram et al. [1], N. Yaqoob and M. Aslam [10] and many others. In 2020 Ahsan et al. [6] extened

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Received July 15, 2021

the idelas of (m,n)-ideals in semigroups to fuzzy sets in semigroup and they characterize the regular semigroup by using fuzzy (m,n)-ideals.

In this paper, we give the concept of an interval valued fuzzy almost (m,n)-bi-ideals. We prove properties of an interval valued fuzzy almost (m,n)-bi-ideal in semigroups.

2. PRELIMINARIES

In this section, we give some definition and theory helpful in later sections.

A non-empty subset L of a semigroup G is called

(1) a *subsemigroup* of G if $L^2 \subseteq L$,

(2) a *left* (right) ideal of G if $GL \subseteq L$ ($LG \subseteq G$),

(3) an *ideal* of a semigroup G we mean a left ideal and a right ideal of G,

(4) a *almost bi-ideal* of G if L is a subsemigroup and $LgL \cap L \neq \emptyset$.

A non-empty subset L of a semigroup G. We denote the

$$[L](m,n) = \bigcup_{r=1}^{m+n} L^r \cap L^m G L^n \text{ is principal } (m,n)\text{-ideal,}$$

$$[L](m,0) = \bigcup_{r=1}^{m} L^r \cap L^m G \text{ is principal } (m,0)\text{-ideal,}$$

$$[L](0,n) = \bigcup_{r=1}^{n} L^r \cap G L^n \text{ is the principal } (0,n)\text{-ideal,}$$

i.e., the smallest (m,n)-ideal, the smallest $(m,0)$-ideal and the smallest $(0,n)$-ideal of G containing L, respectively.

Lemma 2.1. [4] *Let G be a semigroup and m,n positive integers, $[\pi]_{(m,n)}$ the principal (m,n)-ideal generated by the element π. Then*

(1) $([\pi]_{(m,0)})^m G = \pi^m G$.

(2) $G([\pi]_{(0,n)})^n = G\pi^n$.

(3) $([\pi]_{(m,0)})^m G([\pi]_{(0,n)})^n = \pi^m G \pi^n$.

For any $p_i \in [0,1]$, where $i \in \mathscr{A}$, define

$$\bigvee_{i\in\mathscr{A}} p_i := \sup_{i\in\mathscr{A}}\{p_i\} \quad \text{and} \quad \bigwedge_{i\in\mathscr{A}} p_i := \inf_{i\in\mathscr{A}}\{p_i\}.$$

We see that for any $p,q \in [0,1]$, we have

$$p \vee q = \max\{p,q\} \quad \text{and} \quad p \wedge q = \min\{p,q\}.$$

A *fuzzy set* of a non-empty set T is a function $\omega : L \to [0,1]$.

Definition 2.2. [6] A fuzzy set ω of a semigroup G is said to be

(1) a *fuzzy subsemigroup* of G if $\omega(e_1e_2) \geq \omega(e_1) \wedge \omega(e_2)$ for all $e_1, e_2 \in G$,

(2) a *fuzzy left (right) ideal* of G if $\omega(e_1e_2) \geq \omega(e_2)$ $(\omega(e_1e_2) \geq \omega(e_1))$ for all $e_1, e_2 \in G$,

(3) a *fuzzy ideal* of G if it is both a fuzzy left ideal and a fuzzy right ideal of G,

(4) a *fuzzy (m,n)-ideal* of G if $\omega(e_1e_2\ldots e_mbd_1d_2\ldots d_n) \geq \omega(e_1) \wedge \omega(e_2) \wedge \cdots \wedge \omega(e_n) \wedge \omega(d_1) \wedge \omega(d_2) \wedge \cdots \wedge \omega(d_n)$ for all $e_1, e_2, ..., e_m, d_1, d_2, ..., d_n, b \in G$ and m, n are positive integers.

Definition 2.3. [11] A fuzzy set ω of a semigroup G such that $\omega \neq 0$ is called *fuzzy almost bi-ideal* of G if $(\omega \circ \chi_G \circ \omega) \cap \omega \neq 0$.

Let $\Omega[0,1]$ be the set of all closed subintervals of $[0,1]$, i.e.,

$$\Omega[0,1] = \{\overline{p} = [p^-, p^+] \mid 0 \leq p^- \leq p^+ \leq 1\}.$$

We note that $[p,p] = \{p\}$ for all $p \in [0,1]$. For $p = 0$ or 1 we shall denote $[0,0]$ by $\overline{0}$ and $[1,1]$ by $\overline{1}$.

Let $\overline{p} = [p^-, p^+]$ and $\overline{q} = [q^-, q^+] \in \Omega[0,1]$. Define the operations $\preceq$, $=$, $\curlywedge$ and $\curlyvee$ as follows:

(1) $\overline{p} \preceq \overline{q}$ if and only if $p^- \leq q^-$ and $p^+ \leq q^+$

(2) $\overline{p} = \overline{q}$ if and only if $p^- = q^-$ and $p^+ = q^+$

(3) $\overline{p} \curlywedge \overline{q} = [(p^- \wedge q^-), (p^+ \wedge q^+)]$

(4) $\overline{p} \curlyvee \overline{q} = [(p^- \vee q^-), (p^+ \vee q^+)]$.

If $\overline{p} \succeq \overline{q}$, we mean $\overline{q} \preceq \overline{p}$.

For each interval $\overline{p}_i = [p_i^-, p_i^+] \in \Omega[0,1]$, $i \in \mathscr{A}$ where $\mathscr{A}$ is an index set, we define

$$\curlywedge_{i \in \mathscr{A}} \overline{p}_i = [\wedge_{i \in \mathscr{A}} p_i^-, \wedge_{i \in \mathscr{A}} p_i^+] \quad \text{and} \quad \curlyvee_{i \in \mathscr{A}} \overline{p}_i = [\vee_{i \in \mathscr{A}} p_i^-, \vee_{i \in \mathscr{A}} p_i^+].$$

Definition 2.4. [9] Let L be a non-empty set. Then the function $\overline{f} : L \to \Omega[0,1]$ is called *interval valued fuzzy set* (shortly, IVF set) of T.

Definition 2.5. [9] Let L be a subset of a non-empty set G. An *interval valued characteristic function* of L is defined to be a function $\overline{\chi}_L : T \to \Omega[0,1]$ by

$$\overline{\chi}_L(e) = \begin{cases} \overline{1} & \text{if} \quad e \in L, \\ \overline{0} & \text{if} \quad e \notin L \end{cases}$$

for all $e \in G$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ of a non-empty set G, define

(1) $\overline{\omega} \sqsubseteq \overline{\varpi} \Leftrightarrow \overline{\omega}(e) \preceq \overline{\varpi}(e)$ for all $e \in G$,

(2) $\overline{\omega} = \overline{\varpi} \Leftrightarrow \overline{\omega} \sqsubseteq \overline{\varpi}$ and $\overline{\varpi} \sqsubseteq \overline{\omega}$,

(3) $(\overline{\omega} \sqcap \overline{\varpi})(e) = \overline{\omega}(e) \curlywedge \overline{\varpi}(e)$ for all $e \in G$,

(4) $(\overline{\omega} \sqcup \overline{\varpi})(e) = \overline{\omega}(e) \curlyvee \overline{\varpi}(e)$ for all $e \in G$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ in a semigroup G, define the product $\overline{\omega} \circ \overline{\varpi}$ as follows : for all $e \in G$,

$$(\overline{\omega} \circ \overline{\varpi})(e) = \begin{cases} \underset{(t,h) \in F_e}{\curlyvee} \{\overline{f}(t) \curlywedge \overline{\varpi}(h)\} & \text{if } F_e \neq \emptyset, \\ \overline{0} & \text{if } F_u = \emptyset, \end{cases}$$

where $F_e := \{(t,h) \in G \times G \mid e = th\}$.

Next, we shall give definitions of various types of IVF subsemigroups.

Definition 2.6. [7] An IVF set $\overline{\omega}$ of a semigroup G is said to be an *IVF subsemigroup* of G if $\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_1) \curlywedge \overline{\omega}(e_2)$ for all $e_1, e_2 \in G$.

Definition 2.7. [7] An IVF set $\overline{\omega}$ of a semigroup G is said to be an *IVF left (right) ideal* of G if $\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_2)$ $(\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_1))$ for all $e_1, e_2 \in G$. An IVF subset $\overline{\omega}$ of G is called an *IVF ideal* of G if it is both an IVF left ideal and an IVF right ideal of G.

Theorem 2.8. [7] *Let L be a non-empty subset of a semigroup G. Then $\overline{\chi}_L$ is an IVF subsemigroup of G if and only if L is a subsemigroup of G.*

Theorem 2.9. *Let $\overline{\omega}$ be an IVF set of a semigroup G. Then $\overline{\omega}$ is a subsemigroup of G if and only if sup$(\overline{\omega})$ is an IVF subsemigroup of G.*

Let $\overline{\omega}$ be an IVF set of a semigroup G and $m \in \mathbb{Z}$. Then

(1) $\omega^0 := \overline{\chi}_g$ and $\overline{\varpi}^0 \circ \overline{\chi}_G \circ \overline{\varpi}^0 := \overline{\chi}_G$,

(2) $\overline{\varpi}^m := \underbrace{\overline{\varpi} \circ \overline{\varpi} \circ \cdots \circ \overline{\varpi}}_{m-times}$,

(3) $\overline{\varpi}^m \circ \overline{\chi}_G \circ \overline{\varpi}^0 := \overline{\varpi}^m \circ \overline{\chi}_G$,

(4) $\overline{\varpi}^0 \circ \overline{\chi}_G \circ \overline{\varpi}^m \circ \overline{\varpi}^0 := \overline{\chi}_G \circ \overline{\varpi}^m$.

The following theorem we can easy to prove.

Theorem 2.10. *Lef $\overline{\varpi}, \overline{\boldsymbol{\varpi}}$ and $\overline{\kappa}$ be IVF set of a semigroup G. Then the following statements hold:*

(1) *If $\overline{\varpi} \sqsubseteq \overline{\boldsymbol{\varpi}}$ then $\overline{\varpi}^m \sqsubseteq \overline{\boldsymbol{\varpi}}^m$ for all $m \in \mathbb{Z}$.*

(2) *If $\overline{\varpi} \sqsubseteq \overline{\boldsymbol{\varpi}}$ then $\overline{\varpi} \circ \overline{\kappa} \sqsubseteq \overline{\boldsymbol{\varpi}} \circ \overline{\kappa}$.*

(3) *If $\overline{\varpi} \sqsubseteq \overline{\boldsymbol{\varpi}}$ then $\overline{\varpi} \circ \overline{\kappa} \sqcap \overline{\boldsymbol{\varpi}} \circ \overline{\kappa}$.*

Definition 2.11. An IVF set $\overline{\varpi}$ of a semigroup G is called an *IVF almost (m,n)-ideal* of G if $(\overline{\varpi}^m \circ \overline{G} \circ \overline{\varpi}^n) \sqcap \overline{\varpi} \neq \overline{0}$ for all $m,n \in \mathbb{Z}$.

3. On Interval Valued Fuzzy Almost (m,n)-Bi-Ideal in Semigroups

In this section, we give the concept of an interval valued fuzzy almost (m,n)-bi-ideals and investigate properties of an interval valued fuzzy almost (m,n)-bi-ideal in semigroups.

Definition 3.1. An IVF subsemigroup $\overline{\varpi}$ of a semigroup G is called an *IVF almost (m,n)-bi-ideal* of G if $(\overline{\varpi}^m \circ \overline{G} \circ \overline{\varpi}^n) \sqcap \overline{\varpi} \neq \overline{0}$ for all $m,n \in \mathbb{Z}$.

Theorem 3.2. *Suppose that $\overline{\varpi}$ is an IVF almost (m,n)-bi-ideal and $\overline{\boldsymbol{\varpi}}$ is an IVF subsemigroup of a semigroup G and $m,n \in \mathbb{Z}$. Then the following statements hold:*

(1) *If $\overline{\varpi} \sqsubseteq \overline{\boldsymbol{\varpi}}$, then $\overline{\boldsymbol{\varpi}}$ is an IVF almost (m,n)-bi-ideal of G.*

(2) *$\overline{\varpi} \sqcup \overline{\boldsymbol{\varpi}}$ is an IVF almost (m,n)-bi-ideal of G.*

Proof. (1) Suppose that $\overline{\varpi} \sqsubseteq \overline{\boldsymbol{\varpi}}$. Then $\overline{0} \neq (\overline{\varpi}^m \circ \overline{G} \circ \overline{\varpi}^n) \sqcap \overline{\varpi} \sqsubseteq (\overline{\boldsymbol{\varpi}}^m \circ \overline{G} \circ \overline{\boldsymbol{\varpi}}^n) \sqcap \overline{\boldsymbol{\varpi}}$. Thus $\overline{\boldsymbol{\varpi}}$ is an IVF almost (m,n)-bi-ideal of G.

(2) Clearly $\overline{\varpi} \sqsubseteq \overline{\varpi} \sqcup \overline{\boldsymbol{\varpi}}$. By (1) we have $\overline{\varpi} \sqcup \overline{\boldsymbol{\varpi}}$ is an IVF almost (m,n)-bi-ideal of G.

□

Note that for a subset L of G, define $L^0 := G$.

Lemma 3.3. *Let L be a non-empty subset of a semigroup G and $m \in \mathbb{Z}$. Then* $(\overline{\chi}_L)^m = \overline{\chi}^m_L$.

Theorem 3.4. *Let L is a non-empty subset of a semigroup G. Then L is an almost (m,n)-bi-ideal of G if and only if the characteristic function $\overline{\chi}_L$ is an IVF almost (m,n)-bi-ideal of G for all $m,n \in \mathbb{Z}$.*

Proof. Suppose that L is an almost (m,n)-bi-ideal of G. Then L is a subsemigroup of G. Thus by Theorem 2.11, $\overline{\chi}_L$ is an IVF subsemigroup of G. Let $d \in G$. Then by assumtion, there exists $e \in (L^m G L^n) \cap L$ such that $[(\overline{\chi}^m_L \circ \overline{G} \circ \overline{\chi}^n_L) \sqcap \overline{\chi}_L](e) \neq \overline{0}$. By Lemma 3.3,

$$[((\overline{\chi}_L)^m \circ \overline{G} \circ (\overline{\chi}_L)^n) \sqcap \overline{\chi}_L](e) \neq \overline{0}.$$

Thus $\overline{\chi}_L$ is an IVF almost (m,n)-bi-ideal of G.

Conversely, suppose that $\overline{\chi}_L$ is an IVF almost (m,n)-bi-ideal of G. Then $\overline{\chi}_L$ is an IVF subsemigroup. Thus by Theorem 2.11, L is a subsemigroup of G. Let $d \in G$. Then

$$[((\overline{\chi}_L)^m \circ \overline{G} \circ (\overline{\chi}_L)^n) \sqcap \overline{\chi}_L] \neq \overline{0}.$$

Thus there exists $e \in G$ such that $[((\overline{\chi}_L)^m \circ \overline{G} \circ (\overline{\chi}_L)^n) \sqcap \overline{\chi}_L](e) \neq \overline{0}$. By Lemma 3.3,

$$[(\overline{\chi}^m_L \circ \overline{G} \circ \overline{\chi}^n_L) \sqcap \overline{\chi}_L](e) \neq \overline{0}.$$

Thus $e \in L^m G L^n \cap L$. Hence $L^m G L^n \cap L \neq \emptyset$.
We conclude that L is an almost (m,n)-bi-ideal of G. □

For IVF set $\overline{\omega}$ of a semigroup G, defined $\text{supp}(\overline{\omega}) := \{e \in G \mid \overline{\omega}(e) \neq \overline{0}\}$.

Theorem 3.5. *Let $\overline{\omega}$ be an IVF set of a semigroup G. Then $\overline{\omega}$ is an almost (m,n)-bi-ideal of G if and only if sup$(\overline{\omega})$ is an IVF almost (m,n)-bi-ideal of G for all $m,n \in \mathbb{Z}$.*

Proof. Suppose that $\overline{\omega}$ is an almost (m,n)-bi-ideal of G. Then $\overline{\omega}$ is a subsemigroup of G. Thus by Theorem 2.9, $\text{sup}(\overline{\omega})$ is an IVF subsemigroup of G. Let $d \in G$. Then there exists $e \in G$ such that $[(\overline{\omega}^m \circ \overline{G} \circ \overline{\omega}^n) \sqcap \overline{\omega}](e) \neq \overline{0}$. Thus $\overline{\omega}(r) \neq \overline{0}$ and $e = c_1 c_2 \ldots c_m d e_1 e_2 \ldots e_n$ for some $c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n \in G$ such that

$$\overline{\omega}(c_1) \neq \overline{0},\ \overline{\omega}(c_2) \neq \overline{0}, \ldots, \overline{\omega}(c_m) \neq \overline{0},\ \overline{\omega}(e_1) \neq \overline{0},\ \overline{\omega}(e_2) \neq \overline{0}, \ldots, \overline{\omega}(e_n) \neq \overline{0}.$$

So $c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n, d \in$ supp$(\overline{\varpi})$. It implies that

$$[(\overline{\chi}_{supp(\omega)})^m \circ \overline{G} \circ (\overline{\chi}_{supp(\omega)})^n](e) \neq \overline{0} \text{ and } \overline{\chi}_{supp(f)}(e) \neq \overline{0}.$$

Hence $[(\overline{\chi}_{supp(\omega)})^m \circ \overline{G} \circ (\overline{\chi}_{supp(\overline{\varpi})})^n \sqcap \overline{\chi}_{supp(\overline{\varpi})}](e) \neq \overline{0}$. Thus $\overline{\chi}_{supp(\overline{\varpi})}$ is an IVF almost (m,n)-bi-ideal of G. By Theorem 3.4, supp$(\overline{\varpi})$ is an almost (m,n)-bi-ideal of G.

Conversely, suppose that supp$(\overline{\varpi})$ is an IVF almost (m,n)-bi-ideal of G. Then $(\overline{\varpi})$ is an IVF subsemigroup of G. Thus by Theorem 2.9, $(\overline{\varpi})$ is a subsemigroup of G. Let $r \in G$. Then by Theorem 3.4, $\overline{\chi}_{supp(\overline{\varpi})}$ is an IVF almost (m,n)-bi-ideal of G. Thus

$$[(\overline{\chi}_{supp(\overline{\varpi})})^m \circ \overline{G} \circ (\overline{\chi}_{supp(\overline{\varpi})})^n \sqcap \overline{\chi}_{supp(\overline{\varpi})}] \neq \overline{0}.$$

So there exists $e \in S$ such that $[(\overline{\chi}_{supp(f)})^m \circ \overline{G} \circ (\overline{\chi}_{supp(\overline{\varpi})})^n \sqcap \overline{\chi}_{supp(\overline{\varpi})}](r) \neq \overline{0}$.
Hence $(\overline{\chi}_{supp(\overline{\varpi})})^m \circ \overline{G} \circ (\overline{\chi}_{supp(\overline{\varpi})})^n(e) \neq \overline{0}$ and $\overline{\chi}_{supp(\overline{\varpi})}(e) \neq \overline{0}$.
Thus there exist $c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n, d \in G$ supp$(\overline{\varpi})$ and $e = c_1 c_2 \ldots c_m d e_1 e_2 \ldots e_n$. So $\overline{\varpi}(c_1) \neq \overline{0}$, $\overline{\varpi}(c_2) \neq \overline{0}, \ldots, \overline{\varpi}(c_m) \neq \overline{0}$, $\overline{\varpi}(e_1) \neq \overline{0}$, $\overline{\varpi}(e_2) \neq \overline{0}, \ldots, \overline{\varpi}(e_n) \neq \overline{0}$.
Hence $[(\overline{\varpi}^m \circ \overline{G} \circ \overline{\varpi}^n)](e) \neq \overline{0}$ implies $[(\overline{\varpi}^m \circ \overline{G} \circ \overline{\varpi}^n) \sqcap \overline{\varpi}](e) \neq \overline{0}$.
Therefore $\overline{\varpi}$ is an almost (m,n)-bi-ideal of G. □

Definition 3.6. An IVF almost bi-ideal $\overline{\varpi}$ is called *minimal* if for all nonzero IVF almost bi-ideals $\overline{\boldsymbol{\varpi}}$ of a semigroup G such that $\overline{\boldsymbol{\varpi}} \sqsubseteq \overline{\varpi}$ implies supp$(\overline{\varpi})$ =supp$(\overline{\boldsymbol{\varpi}})$.

Definition 3.7. An IVF almost (m,n)-bi-ideal $\overline{\varpi}$ is called *minimal* if for all nonzero IVF almost (m,n)-bi-ideals $\overline{\boldsymbol{\varpi}}$ of a semigroup G such that $\overline{\boldsymbol{\varpi}} \sqsubseteq \overline{\varpi}$ implies supp$(\overline{\varpi})$ =supp$(\overline{\boldsymbol{\varpi}})$.

Theorem 3.8. *Let L be a non-empty subset of a semigroup G. Then L is a minimal almost (m,n)-bi-ideal of G if and only if $\overline{\chi}_L$ is a minimal IVF almost (m,n)-bi-ideal of G.*

Proof. Suppose that L is a minimal almost (m,n)-bi-ideal of G. Then by Theorem 3.4, $\overline{\chi}_L$ is an IVF almost (m,n)-bi-ideal of G. Let $\overline{\varpi}$ be an IVF almost (m,n)-bi-ideal of S such that $\overline{\varpi} \sqsubseteq \overline{\chi}_L$. Then supp$(\overline{\varpi}) \sqsubseteq$ supp$(\overline{\chi_L}) = L$. By Theorem 3.5, sup$(\overline{\varpi})$ is an almost (m,n)-bi-ideal of G. By supposition, supp$(\overline{\varpi}) = L =$ supp$(\overline{\chi_L})$. Hence $\overline{\chi}_L$ is a minimal IVF almost (m,n)-bi-ideal of G.

Conversely, suppose that $\overline{\chi}_L$ is a minimal IVF almost (m,n)-bi-ideal of G and let D be an almost (m,n)-bi-ideal of G such that $D \subseteq L$. Then $\overline{\chi}_D$ is an IVF almost (m,n)-bi-ideal of G

such that $\overline{\chi}_D \sqsubseteq \overline{\chi}_L$. Thus $D = \text{supp}(\overline{\chi}_B) = \text{supp}(\overline{\chi}_L) = L$. Therefore L is a minimal almost (m,n)-bi-ideal of G. □

Corollary 3.9. *Let G has no proper almost (m,n)-bi-ideal if and only if for all IVF almost (m,n)-bi-ideal $\overline{\omega}$ of G, supp$(\overline{\omega}) = G$.*

Proof. It follows from Theorem 3.8. □

ACKNOWLEDGEMENTS

The author are grateful to School of Science, University of Phayao for grant support.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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