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A RELIABLE TAYLOR SERIES SOLUTION TO THE NONLINEAR REACTION-DIFFUSION MODEL REPRESENTING THE STEADY-STATE BEHAVIOUR OF A CATIONIC GLUCOSE-SENSITIVE MEMBRANE

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Abstract: The nonlinear reaction-diffusion model, which represents the steady-state behaviour of a cationic glucose-sensitive membrane with consideration of oxygen limitation and swelling-dependent diffusivities of involved species inside the membrane, is discussed. Analytical expressions of substrate concentration of oxygen, glucose, and gluconic acid in planar coordinates at steady-state conditions are derived for all kinetic parameters, and hence the effect of various factors on the responsiveness of the membrane is analysed. Efficient approaches based on the hyperbolic function and Taylor's series methods are used to derive the approximate analytical solutions of the nonlinear boundary value problem. A numerical simulation was generated by highly accurate and widely used computer generated routines.

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The derived analytical expressions are shown to be in strong agreements with the numerical results established in the literature. It is concluded that each method is a powerful tool for solving high-order boundary value problem in engineering and science.

Keywords: boundary value problem; cationic glucose; hyperbolic function; reaction-diffusion equation; sensitive membrane; Taylor's series.

2010 AMS Subject Classification: 34A34, 41A58.

1. INTRODUCTION

Insulin-dependent diabetes has been a leading health concern worldwide because of the serious complications that are associated with it, such as retinopathy, neuropathy, and vascular disease. As the ultimate goal of having a self-regulated insulin delivery system requires substantial time and effort to be fulfilled, it has not been fully achieved. However, there have been some remarkable developed strategies to reproduce the usual pattern of insulin kinetics. Testing whether a good metabolic control can prevent the long-term complications of diabetes includes intensified conventional therapy with multiple daily injections and continuous subcutaneous insulin infusion with external or implanted pumps [1].

Glucose-sensitive membranes are made using immobilized glucose oxidase (GOD) in pH-sensitive polymers [2], where in the presence of glucose; they swell and become more permeable to insulin. Glucose-sensitive membranes have been employed in insulin delivery systems as they deliver insulin in response to glucose [3–5]. In the presence of glucose and oxygen, these systems produces gluconic acid and hydrogen peroxide inside the copolymer [6–10]. At elevated glucose levels, cationic hydrogels, which consists of pH and amino groups decrease [7]. Several researchers studied the behavior of cationic hydrogels made from dimethyaminoethyl methacrylate, diethyaminoethyl methacrylate, 2-hydroxyethyl methacrylate and poly (ethylene glycol) grafts [8–17].

As it is unlikely to attain optimal design of Glucose-sensitive membranes without setting up proper mathematical models for which approximate analytical solutions are determined, there has been

some remarkable advances in theoretical modeling. Abdekhodaie and Wu [7] presented a theory describing the steady state behavior of a cationic glucose-sensitive membrane while taking into account oxygen limitation and swelling-dependent diffusivities of species inside the membrane. Albin et al. [5] developed a mathematical model describing the steady-state behavior of two types of glucose sensitive membranes that are both synthetic hydrogels containing immobilized glucose oxidase enzyme. Leypoldt et al. [17]developed a model of two-substrate enzyme electrode for glucose. Klumb et al. [18] proposed a theoretical model to evaluate possible designs for an insulin delivery system based upon a glucose sensitive hydrogel containing immobilized glucose oxidase and catalase. Other theoretical models discussions can be found in [19,20].

Analytical solutions for nonlinear boundary value problems are more desired than numerical solutions because they provide a more accurate sensitive analysis of kinetic parameters on the governing system and hence facilitate the development of optimized models. The nonlinear mathematical model discussed in this paper has been solved analytically using the homotopy analysis method, Genocchi Polynomials and Adomian decomposition method [21,22]. Other widely used methods that are prone to deliver accurate analytical results for solving this kind of system include Green's function iterative method [23,24], variational iteration method [25,26], and homotopy perturbation method [27–29].

In this communication, we present efficient and reliable approaches to analytically solve a system of nonlinear differential equation in the cationic glucose-sensitive membrane. The simplicity and efficiency of the proposed approaches stem from the fact that basic conceptual mathematics is being used. Therefore, these approaches are easily accessible to researchers for further investigation of the effect of kinetic parameters and possibly obtain an optimal glucose-sensitive membranes. The reliability of the proposed methods will be investigated by direct comparison with numerical simulations from the literature and software built-in functions.

2. BOUNDARY VALE PROBLEM TO GLUCOSE-SENSITIVE MEMBRANE

The chemical reaction scheme inside a glucose-sensitive membrane is described by [7]

$$\text{Glucose} + O_2 \xrightarrow{\text{Glucose oxidase}} \text{Gluconicacid} + H_2O_2 \quad (1)$$

The incorporated catalase then implies the conversion

$$H_2O_2 \xrightarrow{\text{Catalase}} H_2O_2 + \frac{1}{2}O_2 . \quad (2)$$

When the excess of catalase is immobilized with glucose oxidase, the overall mechanism of the reaction is described by

$$\text{Glucose} + \frac{1}{2}O_2 \rightarrow \text{Gluconic acid} \quad (3)$$

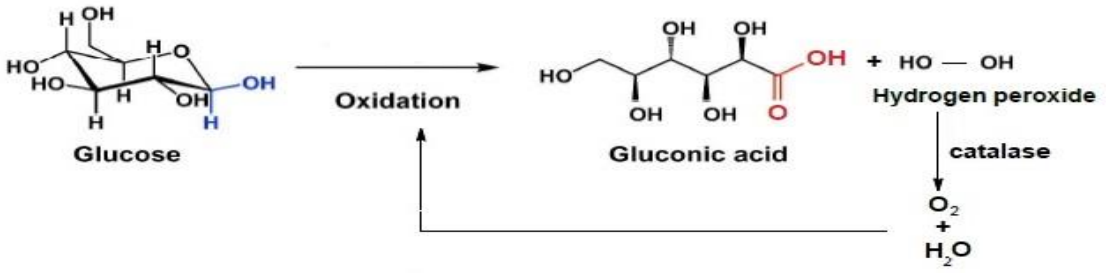

Figure 1. Schematic diagram illustrating the structure of glucose conversion to gluconic acid by glucose oxidase

It is evident from the reaction, that in the presence of catalase, only one-half of an oxygen molecule is consumed per molecule of glucose [7]. For the completion and self-consistency of the research, the derivation of the governing nonlinear differential equations in planar coordinates inside the cationic glucose-sensitive membrane is given in Appendix D of the supplementary material.

2.1 Relation between the concentrations of species

Algebraic manipulations of Eqs. (D6)–(D7) and Eqs. (D7)–(D8) lead to Eqs. (4) and (5), respectively (See Appendix D for details)

$$\frac{d^2}{d\chi^2}\left(\frac{2u(\chi)}{\mu_2} - \frac{\gamma\, v(\chi)}{\mu_1}\right) = 0, \quad (4)$$

$$\frac{d^2}{d\chi^2}(v(\chi) + w(\chi)) = 0. \quad (5)$$

Using boundary conditions (D9), concentrations of glucose and gluconic acid are obtained in terms of the concentration of oxygen

$$v\,(\chi) = 1 + \frac{2\mu_1(u\,(\chi)-1)}{\gamma\,\mu_2}, \tag{6}$$

$$w\,(\chi) = 1 - v\,(\chi) = -\frac{2\mu_1(u\,(\chi)-1)}{\gamma\,\mu_2}. \tag{7}$$

The main objective now is to obtain an analytical expression for the concentration profile $u(\chi)$ of oxygen, which will immediately lead to analytical expressions for the concentration profiles $v(\chi)$ and $w(\chi)$. The relation between glucose, oxygen and gluconic acid concentrations for all values of kinetics parameters is given by

$$u(\chi)\left(\frac{4\mu_1}{\gamma\mu_2}\right) - v(\chi) + w(\chi) = \left(\frac{4\mu_1}{\gamma\mu_2}\right) - 1. \tag{8}$$

3. Derivation of Analytical Expressions of Concentrations

In this section, we introduce simple, efficient and reliable techniques to derive analytical expressions of concentration for oxygen, which will immediately lead to the determination of analytical expressions for glucose and gluconic acid.

3.1 A modified hyperbolic function method

Special functions, in general, have always been used as an effective tool to solve nonlinear differential systems. For example, J. He utilized the exponential function to solve a nonlinear wave equation [30]. Furthermore, a recent research article used the gamma function to derive a semi-analytic solution to a small amplitude oscillator equation[31]. The modified hyperbolic function method, considered a special case of the exponential function method [30] , is reliable and highly accurate in obtaining semi-analytic solutions of nonlinear models [32,33].

Employing the hyperbolic function method to solve the boundary value problem (D6)–(D9) yields the analytical expression for the normalized concentration of oxygen (see details in Appendix B)

$$u(\chi) = \cosh(b\chi) - \tanh(b/2) \tag{9}$$

Using equations (6) and (7) lead to the following analytical expressions for the normalized concentration of glucose and gluconic acid

$$v(\chi) = 1 + \frac{2\mu_1}{\gamma\mu_2}\left(\cosh(b\chi) - \tanh\left(\frac{b}{2}\right)\sinh(b\chi) - 1\right), \tag{10}$$

$$w(\chi) = \frac{2\mu_1}{\gamma\mu_2}\left(1 - \cosh(b\chi) - \tanh\left(\frac{b}{2}\right)\sinh(b\chi)\right), \tag{11}$$

where the unknown parameter b satisfies the equation

$$b^2\left(1 + \frac{\alpha\,\mathrm{sech}(\beta/2}{\gamma\left(1 + 2\mu_1\left(\frac{\mathrm{sech}(b/2) - 1}{\gamma\mu_2}\right)\right)} + \frac{\beta\,\mathrm{sech}(\beta/2)}{\gamma}\right) = \frac{\mu_2}{\gamma}. \tag{12}$$

Numerical values of b for various values of the fundamental parameters can be obtained easily by using any computer algebra software. In Table S.1, the numerical values of b, which are used in Figures 2-6, are computed for various values of parameters.

3.2 Taylor series method

Taylor series method (TSM) is one of the simplest and most effective methods to solve nonlinear equations. Moreover, TSM is accessible to the broader research community because it requires no robust mathematical analysis background. Although some obstacles might emerge when using TSM, like in the case of strong nonlinear differential equations, the way around these obstacles is usually easy such as using more derivatives, and getting higher degreed polynomials or using Padé approximant. In recent years, TSM has been intensively employed to solve nonlinear ordinary and fractional differential equations such as Lane-Emden, third-order boundary value problems, fractional Bratu-type equations, and nonlinear oscillator problems [34–43].

Using Taylor series approach, analytical expressions for normalized concentrations are obtained for the general case (details in Appendix C). For example, the analytical expressions for normalized concentrations for the experimental values $\beta = 0.5, \gamma = 5, \mu_0 = \mu_1 = \mu_2 = 10, \alpha = 0.1$ and $p = -1.6749$ are given by

$$u(\chi) = 1 - 1.6749\chi + 2.2321\chi^2 - 1.1216\chi^3 + 0.6579\chi^4 - 0.0935\chi^5, \tag{13}$$

$$v(\chi) = 1 - \frac{2\mu_1}{\gamma\mu_2}(1.6749\chi + 2.2321\chi^2 - 1.1216\chi^3 + 0.6579\chi^4 - 0.0935\chi^5), \tag{14}$$

$$w(\chi) = \frac{2\mu_1}{\gamma\mu_2}(1.6749\chi + 2.2321\chi^2 - 1.1216\chi^3 + 0.6579\chi^4 - 0.0935\chi^5). \tag{15}$$

3.3 Previous analytical expression of concentrations

Sevukaperumal et al. [21] employed the homotopy analysis method (HAM) to derive the following

analytical expressions for the concentration of oxygen inside the cationic glucose-sensitive membrane

$$u(\chi) = \cosh(C\chi) + B\cosh(C\chi) + hD\cosh(C\chi) + D/\sinh(C), \tag{16}$$

where

$$C = \sqrt{\frac{\mu_2}{2}},$$

$$B = \frac{(1-\cosh(C))}{\sinh(C)}, \text{ and}$$

$$D = M_1\begin{bmatrix} 2B\sinh(2C) + (1+B^2)\cosh(2C) \\ +3(B^2-1) + 2(1-2B^2)\cosh(C) \end{bmatrix} + M_2\begin{bmatrix} B(3+B^2)\sinh(3C) \\ +(1+3B^2)(\cosh(3C)-\cosh(C)) \\ +12C(1-B^2)(\sinh(C)+B\cosh(C)) \end{bmatrix} \tag{17}$$

in which

$$M_1 = \frac{\gamma\mu_2(\alpha+\beta) - 2\beta\mu_1}{6\gamma(\gamma\mu_2 - 2\mu_1)} \text{ and } M_2 = \frac{\beta\mu_1}{16\gamma(\gamma\mu_2 - 2\mu_1)},$$

and h is the convergence control parameter. Analytical expressions for the concentration of glucose and gluconic acid are obtained by substituting Eq. (16) into Eqs. (6) and (7), respectively.

3.4 Determination of pH profile inside the membrane

The pH in gluconic acid is determined by the concentration of buffer ions and gluconic acid in the microsphere. Gluconic acid production with a concentration of C_{a} inside the membrane changes pH to pH_2 via [8]

$$\mathrm{pH}_2 = \mathrm{p}K + \log\left\{\frac{10^{\mathrm{pH}_1-\mathrm{p}K} - \frac{C_{\mathrm{a}}}{[\mathrm{buffer}]}(1+10^{\mathrm{pH}_1-\mathrm{p}K})}{1+\frac{C_{\mathrm{a}}}{[\mathrm{buffer}]}(1+10^{\mathrm{pH}_1-\mathrm{p}K})}\right\}. \tag{18}$$

From Eq. (D.5), we have $C_a = \frac{wC_a^*}{[\mathrm{buffer}]}$ and hence from Eq. (11) pH is determined by

$$\exp\left(\mathrm{pH}_2 - \mathrm{p}K\right) = \frac{10^{\mathrm{pH}_1-\mathrm{p}K} - \left(\left(\frac{2\mu_1(1-u(\chi))}{\gamma\mu_2}\right)\frac{C_a^*}{[\mathrm{buffer}]}\right)(1+10^{\mathrm{pH}_1-\mathrm{p}K})}{1+\left(\left(\frac{2\mu_1(1-u(\chi))}{\gamma\mu_2}\right)\frac{C_a^*}{[\mathrm{buffer}]}\right)(1+10^{\mathrm{pH}_1-\mathrm{p}K})} \tag{19}$$

3.5 Estimation of kinetics parameters

From Eq. (D2), the following is easily obtained

$$\frac{1}{R} = \frac{1}{v_{\max}} + \frac{1}{C_{\mathrm{g}}}\left(\frac{K_{\mathrm{g}}}{v_{\max}}\right) + \frac{1}{C_{\mathrm{ox}}}\left(\frac{K_{\mathrm{ox}}}{v_{\max}}\right) \tag{20}$$

where R, C_{g} and C_{ox} are known while the remaining parameters are unknown. Using multiple recursion analysis, we can obtain the kinetic parameter $v_{\max}$ and the Michaelis-Menten K_g and K_{ox}.

4. NUMERICAL SIMULATION

The nonlinear differential equations (D.6–D.9) are solved numerically using the MATLAB function pdex4 (see Appendix E in the supplementary material). The numerical solutions are compared to the derived analytical expressions and the results for the concentration of species are summarized in Figures 2-5 and Tables S.2-S.4. Strong agreements between analytical and numerical results are noted.

5. DISCUSSION

By employing a modified hyperbolic function method, we derived Eqs. (9–11) as approximate analytical expressions for the concentrations of oxygen, glucose and gluconic acid for all experimental values of parameters. Using the same experimental values of α, β, γ, μ_1 and μ_2 used in [8], the derived analytical results are shown to strongly agree with numerical results as depicted in Figure 2.

Figures 3–5 illustrate the concentration profiles of oxygen u, glucose v, and gluconic acid w for various values of the dimensionless parameters. In addition to strong agreement with numerical results, it is inferred from these figures that for all possible cases of Thiele modulus ($\mu_1 = \mu_2$, $\mu_1 > \mu_2$ and $\mu_1 < \mu_2$), the concentrations of oxygen $u(\chi)$ and glucose $v(\chi)$ decrease gradually before they start to increase when the dimensionless length reaches half of the thickness of the membrane. However, the behavior of the concentration of the gluconic acid is exactly

opposite to that of the concentration of oxygen and glucose as seen in Figure 5.

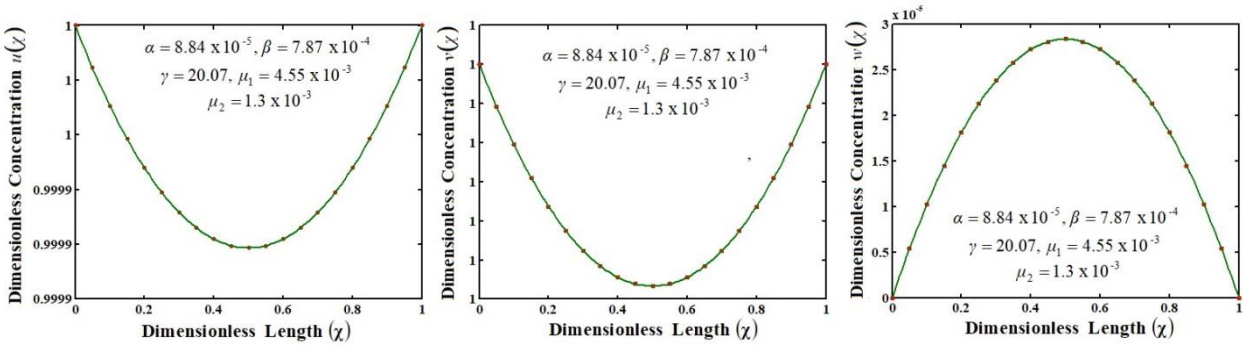

Figure 2. Dimensionless concentration profiles of oxygen u, glucose v and gluconic acid w against dimensionless distance. Solid line represent the analytical solution using Eq. (9-11) whereas dotted line represents the numerical solution in [8].

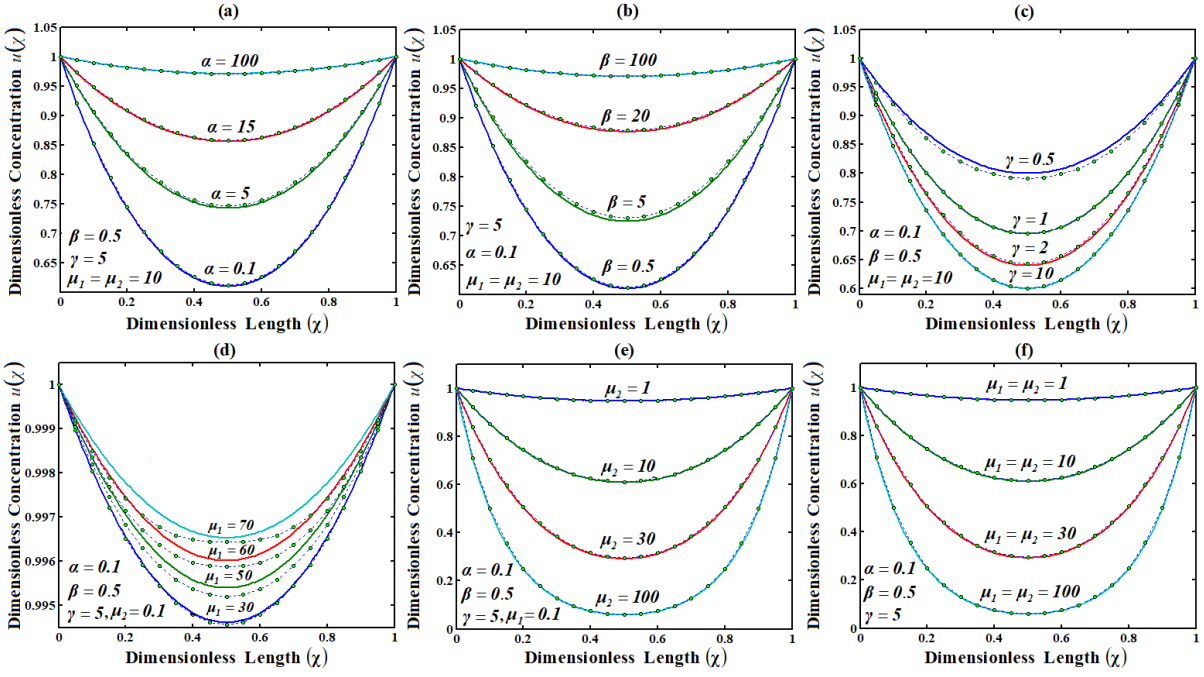

Figure 3. Dimensionless concentration profiles of oxygen u against the dimensionless distance χ using Eq. (9).

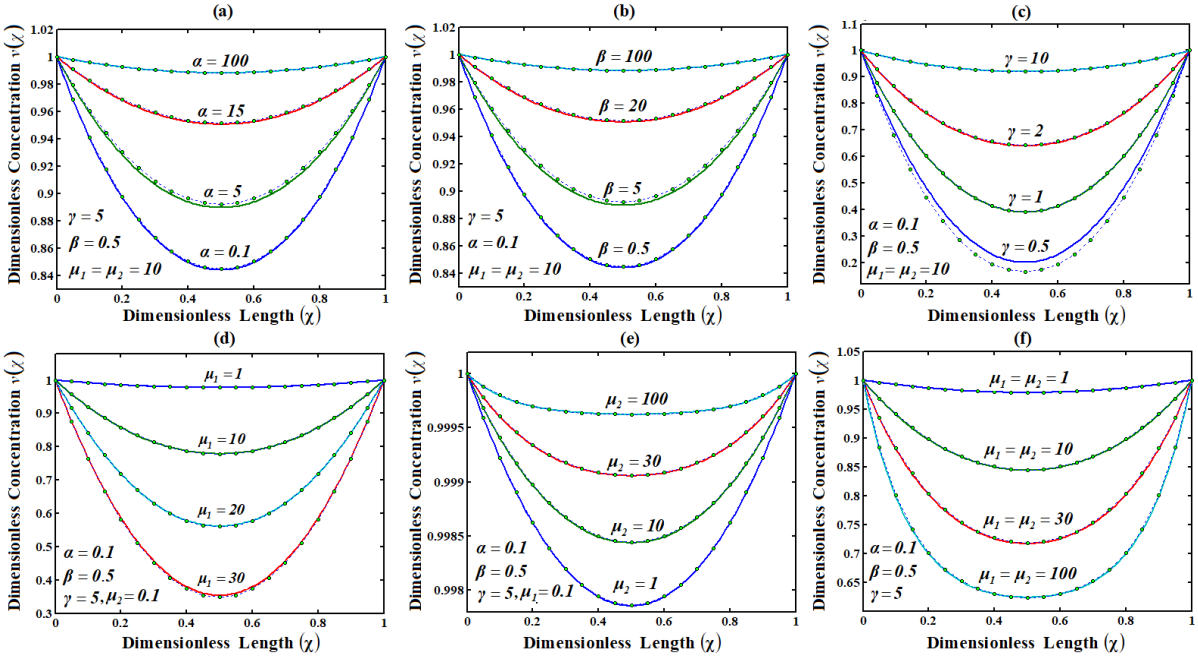

Figure 4. Dimensionless concentration profiles of glucose v against the dimensionless distance χ using Eq. (10).

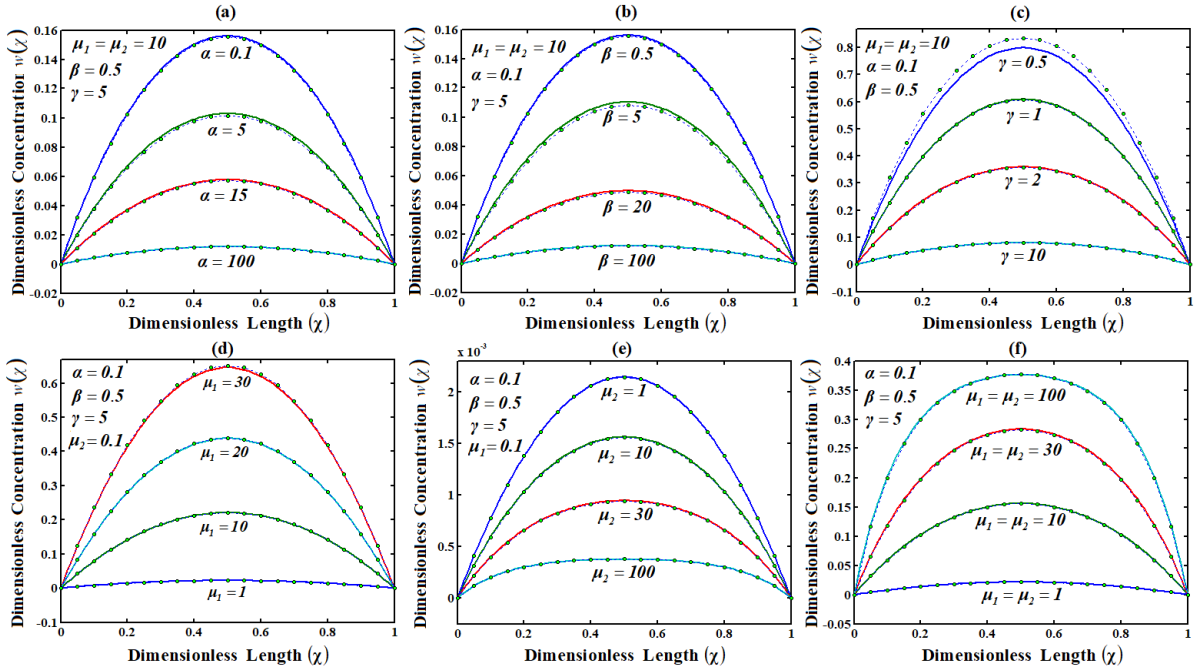

Figure 5. Dimensionless concentration profiles of gluconic acid w against the dimensionless distance χ using Eq. (11).

5.1 Effects of the constants α, β and γ

As mentioned in the supplementary material, the constant α is defined as the ratio of the Michaelis constants K_{g} and K_{ox}, while the constant β represents the ratio of the bulk concentration C_{g}^{*} and Michaelis constant K_{ox}. Figures 3(a) and 4(a) show that α (β) is directly proportional to the concentration of oxygen (glucose). However, the kinetic parameters α and β are inversely proportional to the concentration of gluconic acid as shown in Figures 5(a–b).

The kinetic parameter γ represents the ratio of the concentration of oxygen and glucose in external solution. From Figures 3(c), 4(c) and 5(c), it is inferred that as γ decreases, the gradients of the concentration profiles of oxygen and gluconic acid increase while the gradient of the concentration profile of glucose decreases.

5.2 Effects of membrane thickness (l), maximal reaction velocity (v_{max}) and diffusion coefficients (D and D_{ox})

As introduced in the supplementary material, μ_1 and μ_2 are referred to as Thiele modulus for the oxygen and glucose, respectively. They are both directly proportional to the product of thickness of membrane l and maximal reaction velocity v_{max}. But μ_1 is inversely proportional to diffusion parameter D and μ_2 is indirectly proportional to diffusion parameter D_{ox}. The influence of parameters μ_1 and μ_2 can be studied from Figures 3(d–f), 4(d–f) and 5(d–f). It is observed from Figure 3(d) that an increase in μ_1 (i.e. increase of the maximal reaction velocity, thickness of the membrane and/or decrease of the diffusion parameter) implies an increase in the concentration of oxygen. Figure 3(e) states that the effect of μ_2 on the concentration of oxygen is opposite to that of μ_1. Figure 3(f) points out that a decrease in the common value of both Thiele modulus μ_1 and μ_2 results in an increase in the concentration of oxygen. Figures 4(d–f) show that the effect of μ_1 and μ_2 on the concentration of glucose is exactly opposite to their effect on the concentration of oxygen. Figures 5(d–e) show that when μ_1 is increasing or μ_2 is decreasing, then the concentration of gluconic acid is increasing in its gradient at the center of the length of the membrane. When μ_1 and μ_2 are equal and increase, the concentration of gluconic acid increases as depicted in Figure 5(f). The effect of μ_1 on the concentration of gluconic acid is

evidently stronger than the effect of μ_2. In each enzymatic reaction, the maximal reaction velocity $v_{\max}$ is proportional to the concentration of the enzyme in the microsphere and the maximum reaction rate is determined by the overall kinetics.

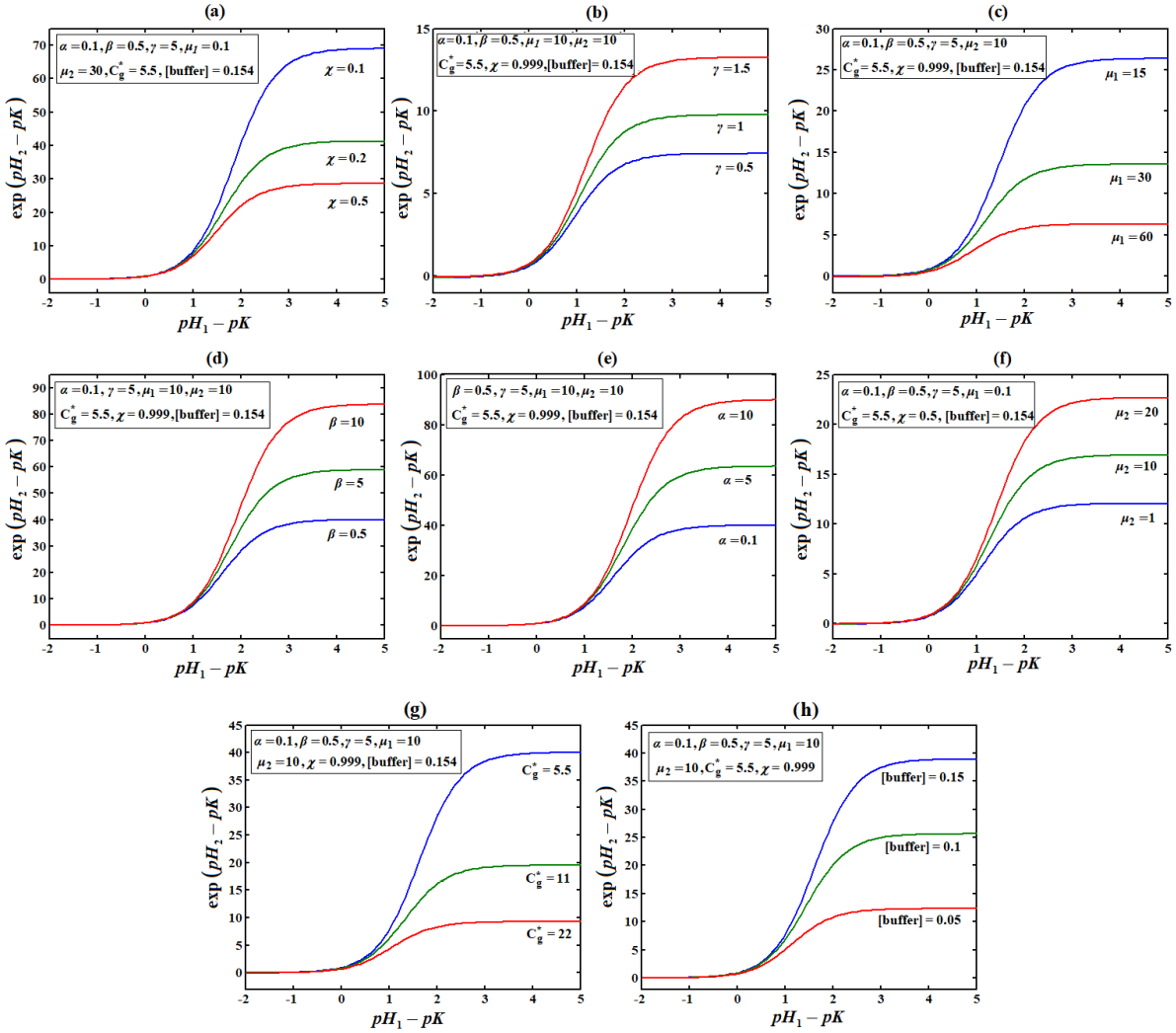

Figure 6. Plot of $\exp(\mathrm{pH}_2 - \mathrm{p}K)$ against $\mathrm{pH}_1 - \mathrm{p}K$ using Eq. (19)

5.3 Effect of kinetic parameters on the pH profile

Figure 6, which plots the curve $\exp(\mathrm{pH}_2 - \mathrm{p}K)$ against $\mathrm{pH}_1 - \mathrm{p}K$ shows that $\exp(\mathrm{pH}_2 - \mathrm{p}K)$ uniformly increases for increasing values of $\mathrm{pH}_1 - \mathrm{p}K$. From this figure, we observe that $\exp(\mathrm{pH}_2 - \mathrm{p}K)$ approaches zero when the pH of a buffer remains fixed in the presence or the

absence of gluconic acid. It is also observed that increasing the values of $\alpha, \beta, \gamma, \mu_2$ and [buffer] will result in increasing the gradient of $\exp(\mathrm{pH}_2 - \mathrm{p}K)$. Moreover, decreasing the thickness of the membrane, the value of the concentration of glucose in the external solution and μ_1 causes the $\exp(\mathrm{pH}_2 - \mathrm{p}K)$ to reach its maximal gradient.

6. CONCLUSIONS

Two efficient and reliable methods based on the modified hyperbolic function and Taylor's series were employed to derive analytical expressions, in terms of kinetic parameters, for the concentrations of oxygen, glucose, and gluconic acid. These analytical expressions where shown to be of high accuracy when compared to numerical solutions from the literature and numerical simulations generated by the widely used MATLAB function pdex4. The kinetic rate constants are determined and its effect on the concentration profiles were discussed. The derived analytical results can be further utilized to determine the effect of different parameters on the governing system and hence optimize the design of a glucose membrane. The simplicity of the approaches used in this research make them accessible to the wider natural science community to solve other nonlinear reaction-diffusion equations that arise in physical and chemical sciences.

NOMENCLATURE

Symbol	Definition and units
D_Y	Diffusion coefficient of catalyst $Y = OX, g, a$ (cm^2/s)
$C_Y(x)$	Concentration of the catalyst $Y = OX, g, a$ (mol/cm^3)
C_Y^*	Species concentration in the external medium $Y = OX, g$ (mM)
K_Y	Michaelis–Menten constant for $Y = OX, g$ (mol/cm3)
v_{max}	Maximal reaction velocity (cm/s)
x	The spatial coordinate (cm)
l	Thickness of the membrane (cm)
u, v and w	Dimensionless concentration of oxygen, glucose and gluconic acid
α, β and γ	Dimensionless constants
μ_1 and μ_2	Thiele modulus for the oxygen and glucose
u	Dimensionless concentration of glucose (None)
v	Dimensionless concentration of oxygen (None)
w	Dimensionless concentration of gluconic acid (None)
χ	Dimensionless distance (None)

APPENDIX A. BASIC CONCEPT OF THE MODIFIED HYPERBOLIC FUNCTION METHOD

We beging by expressing a general second order differential equation p_k in the following form [28]:

$$p_k: f(u_k, u_k', u_k'', a_k, b_k) = 0 \tag{A.1}$$

where p_k represents the nonlinear differential equation, $u_k = u_k(x, a_k, b_k, \cdots)$ in which a_k, b_k are given parameters, $k = 1, 2, \cdots, n$ and $L \le x \le U$.

The boudary conditions are defined by

$$\text{At } x = L, u_k(x) = u_{kL_0} \text{ or } u'_k(x) = u_{kL_1} \tag{A.2}$$

$$\text{At } x = U, u_k(x) = u_{kU_0} \text{ or } u'_k(x) = u_{kU_1} \tag{A.3}$$

Assume that the solution of equation (A.1) is a hyperbolic function of the form

$$u_k(x) = A_k \cosh(bx) + B_k \sinh(bx) \tag{A.4}$$

where the constant coefficients A_k and B_k are determined from the boundary conditions (A.2) and (A.3) as follows:

$$u_k(L) = A_k \cosh(bL) + B_k \sinh(bL) = u_{L_0} \text{ or } u'_k(L) = m(A_k \cosh(bL) + B_k \sinh(bL)) = u_{L_0} \tag{A.5}$$

$$u_k(U) = A_k \cosh(bU) + B_k \sinh(bU) = u_{U_0} \text{ or } u'_k(U) = m(A_k \cosh(bU) + B_k \sinh(bU)) = u_{U_0} \tag{A.6}$$

The unknown parameter b can be obtained by substituting Eq. (A.4) into Eq. (A.1) for any prescribed value $x = K$, where $L \le K \le U$. That is b is obtained by solving the equation

$$p_k: f\big(u_k(K, A_k, B_k, b), u_k'(K, A_k, B_k, b), u_k''(K, A_k, B_k, b)\big) = 0 \tag{A.7}$$

APPENDIX B. ANALYTICAL SOLUTION OF THE EQUATIONS (D6-D9) USING THE MODIFIED HYPERBOLIC FUNCTION METHOD

Consider the following dimensionless nonlinear boundary value problem (D.6) and (D.9), renamed here as (B.1) and (B.2), respectively

$$\frac{d^2u}{dx^2} - \frac{\mu_2}{2}\frac{u}{\left(1+\frac{\alpha u}{\gamma v}+\frac{\beta u}{\gamma}\right)} = 0 \tag{B.1}$$

$u(\chi) = 1$ for $\chi = 0$ and $\chi = 1$ $(B.2)$

As described in Appendix A, we assume that the approximate solution of Eqs. (B.1)–(B2) is given by

$$u(\chi) = A\cosh(b\chi) + B\sinh(b\chi) \quad (B.3)$$

where A, B and b are constants. From boundary conditions (B.2), it immediately follows that $A = 1$, $B = -\tanh(b/2)$ and hence

$$u(\chi) = \cosh(b\chi) - \tanh(b/2)\sinh(b\chi) \quad (B.4)$$

To find b, rewrite Eq. (B.1) as follows,

$$\mathbb{F}(\chi) = \frac{d^2y}{dx^2} - \frac{\mu_2}{2}\frac{u}{\left(1+\frac{\alpha u}{\gamma v}+\frac{\beta u}{\gamma}\right)} = 0 \quad (B.5)$$

Substituting (B.4) into (B.5) and then letting $\chi = 1/2$ gives

$$\mathbb{F}(\chi) = b^2\text{sech}(b/2) - \frac{\mu_2}{2}\frac{\text{sech}(b/2)}{\left(1+\left(\frac{\alpha}{\gamma+\frac{2\mu_1(\text{sech}(b/2)-1)}{\mu_2}}+\frac{\beta}{\gamma}\right)\text{sech}(b/2)\right)} = 0 \quad (B.6)$$

Simple algebraic manipulation implies the b can be computed for various values of the parameters $\alpha, \beta, \gamma,$ and μ from the implicit relation

$$b^2\left(1+\frac{\alpha\ \text{sech}(b/2)}{\gamma\left(1+\frac{2\mu_1(\text{sech}(b/2)-1)}{\gamma\mu_2}\right)}+\frac{\beta\ \text{sech}(b/2)}{\gamma}\right) = \frac{\mu_2}{2}. \quad (B.7)$$

Eqs. (B.4) and (B.7) yield a semi-analytical expression of the concentration of oxygen for all kinetic parameters. Semi-analytical expressions of concentrations of glucose and gluconic acid are readily obtained from Eqs. (10) and (11).

APPENDIX C. ANALYTICAL SOLUTION OF NONLINEAR EQUATIONS (D6-D9) USING TAYLOR SERIES METHOD

For Eq. (B.1), consider the Maclaurin series (Taylor's series at $\chi = 0$) for the dimensionless concentration of $u(\chi)$ given by

$$u(\chi) = \sum_{q=0}^{r} \left(\frac{d^q u}{d\chi^q}\Big|_{\chi=0} \right) \frac{\chi^q}{q!}. \qquad (C.1)$$

From boundary conditions (Eq. (D.9)), it is concluded that $u(0) = 0$. Now consider $u'(0) = p$, where p is a constant and let $\frac{d^q u}{d\chi^q}\Big|_{\xi=0} = A_q$ and $\frac{d^q v}{d\chi^q}\Big|_{\xi=0} = B_q$, then

$$u(\chi) = \sum_{q=0}^{r} A_q \frac{\chi^q}{q!}. \qquad (C.2)$$

As a natural consequence of Eq. (B.1), we obtain

$$A_0 = 1, A_1 = p, A_2 = \frac{1}{2}\frac{\mu_2}{a_0}, A_3 = A_2\left(p - \frac{a_1}{a_0}\right), A_4 = {A_2}^2 - 2A_3\frac{a_1}{a_0} - A_2\frac{a_2}{a_0}$$

$$A_5 = A_2\left(\begin{array}{l} A_3 + 6\frac{a_1}{a_0^2}\left(pa_1 + a_2 - \frac{a_1^2}{a_0} - \frac{1}{4}\mu_2\right) - 3\frac{a_2}{a_0} - \frac{A_3}{a_0}\frac{\alpha+\beta}{\gamma} \\ + \frac{\alpha}{a_0}\left(\frac{6{B_1}^3 - p{B_1}^2 + 3A_2B_1 - 6B_1B_2 + 3pB_2 + 6B_3}{\gamma}\right) \end{array}\right). \qquad (C.3)$$

Also, from Eq. (6), we obtain

$$B_1 = \frac{2p\mu_1}{\gamma\mu_2}, B_2 = \frac{\mu_1}{ra_0}, B_3 = \frac{p\mu_1}{{a_0}^2}\left(1 + \frac{2\alpha\mu_1}{\gamma^2\mu_2}\right), \qquad (C.5)$$

in which

$$a_0 = 1 + \frac{\alpha}{\gamma} + \frac{\beta}{\gamma}; a_1 = p\frac{(\alpha+\beta)}{\gamma} + \frac{\alpha B_1}{\gamma}; a_2 = A_2\left(\frac{\alpha+\beta}{\gamma}\right) + \alpha\left(\frac{2B_1(B_1-p) - B_2}{\gamma}\right). \qquad (C.6)$$

Using Eqs. (C.4–C.6) in Eq. (C.3) leads to the analytical expression

$$u(\chi) = 1 + p\chi + \frac{1}{4}\frac{\mu_2}{a_0}\chi^2 + \frac{A_2}{6}\left(p - \frac{a_1}{a_0}\right)\chi^3 + \frac{1}{24}\left({A_2}^2 - 2A_3\frac{a_1}{a_0} - A_2\frac{a_2}{a_0}\right)\chi^4$$
$$+ \left(\frac{A_2}{120}\left(\begin{array}{l} A_3 + 6\frac{a_1}{a_0^2}\left(pa_1 + a_2 - \frac{a_1^2}{a_0} - \frac{1}{4}\mu_2\right) - 3\frac{a_2}{a_0} - \frac{A_3}{a_0}\frac{\alpha+\beta}{\gamma} \\ + \frac{\alpha}{a_0}\left(\frac{6{B_1}^3 - p{B_1}^2 + 3A_2B_1 - 6B_1B_2 + 3pB_2 + 6B_3}{\gamma}\right) \end{array}\right)\right)\chi^5. \qquad (C.7)$$

The unknown constant p is easily computed from the boundary condition $u(1) = 1$. For example, using Eq. (C.7) we obtain that the analytical expressions for normalized concentrations using parameters $\beta = 0.5, \gamma = 5, \mu_0 = \mu_1 = \mu_2 = 10, \alpha = 0.1$ are given by

$$u(\chi) = 1 + p\chi + 2.2321\chi^2 + 0.6696p\chi^3 - (0.0319p^2 - 0.7474)\chi^4$$
$$+ (0.0024p^2 + 0.0491)p\chi^5, \qquad (C.8)$$

$$v(\chi) = 1 + \frac{2\mu_1}{\gamma\mu_2}(p\chi + 2.2321\chi^2 + 0.6696p\chi^3 - (0.0319p^2 - 0.7474)\chi^4 + (0.0024p^2 + 0.0491)p\chi^5), \quad (C.9)$$

$$w(\chi) = -\frac{2\mu_1}{\gamma\mu_2}(p\chi + 2.2321\chi^2 + 0.6696p\chi^3 - (0.0319p^2 - 0.7474)\chi^4 + (0.0024p^2 + 0.0491)p\chi^5). \quad (C.10)$$

Using boundary condition $\boldsymbol{u}(\mathbf{1}) = \mathbf{1}$, we obtain from Eq. (C.8) that $\boldsymbol{p}$ satisfies the equation

$$3.9795 + 1.7178p - 0.0319p^2 + 0.0024p^3 = 1, \quad (C.11)$$

whose real solution is $p = -1.6749$. Substituting this value of p in Eqs. (C.8–C.10) imply Eqs. (13-15).

APPENDIX-D. MATHEMATICAL FORMULATION OF THE PROBLEM.

The one-dimension steady- state nonlinear reaction-diffusion equations in cationic glucose-sensitive membrane have been analyzed by Abdekhodaie and Wu [9]. For the self-consistency, the steady-state nonlinear equations for the concentrations of glucose, oxygen and gluconic acid are given below.

$$D_{\mathrm{i}}\frac{d^2C_{\mathrm{i}}}{dx^2} + v_{\mathrm{i}}R = 0 \text{ where } \quad \mathrm{i} = \mathrm{g}, \mathrm{ox}, \mathrm{a} \quad \text{(D1)}$$

The terms 'g', 'ox' and 'a' provided to indicate glucose, oxygen and gluconic acid respectively, the stoichiometric coefficients, v_{i}, are -1, -1/2, 1 in turn for $\mathrm{i} = \mathrm{g}, \mathrm{ox}$ and a, C_{i} is the concentration, D_{i} the corresponding diffusion coefficient in the membrane, x is the spatial coordinate and R is the overall reaction rate of the form

$$R = \frac{v_{\max}C_{\mathrm{g}}C_{\mathrm{OX}}}{C_{\mathrm{OX}}(K_{\mathrm{g}} + C_{\mathrm{g}}) + C_{\mathrm{g}}K_{\mathrm{OX}}} \quad \text{(D2)}$$

where $v_{\max}$ is the maximum reaction rate, K_{g} and K_{ox} are Michaelis-Menten constant for the glucose and glucose oxidase respectively. For the case $C_{\mathrm{OX}}(K_{\mathrm{g}} + C_{\mathrm{g}}) < C_{\mathrm{g}}K_{\mathrm{OX}}$, overall reaction rate can be simplified to:

$$R = \frac{v_{max}}{K_{OX}} C_{OX} \tag{D3}$$

the boundary conditions for Eq. (D1) is given by assuming that the membrane is immersed in a well stirred external medium with a constant concentration of each species due to continuous flow of a fresh medium.

$$C_{OX} = C_{OX}^{*}; \quad C_g = C_g^{*}; \quad C_a = 0 \ \text{ at } \ x = 0, x = l \tag{D4}$$

where l is the thickness of the membrane and C_{OX}^{*} and C_g^{*} are the concentrations of oxygen and glucose in the external solution, respectively. We can assume that the diffusion coefficient of glucose and gluconic acid are equal ($D_g = D_a = D$).

The non-linear differential equations arise from Eq.(D1) for various concentrations are altered as dimensionless equations by using the dimensionless parameters mentioned in [38]

$$\chi = \frac{x}{l}; \ u = \frac{C_{OX}}{C_{OX}^{*}}; \ v = \frac{C_g}{C_g^{*}}; \ w = \frac{C_a}{C_g^{*}}; \ \alpha = \frac{k_g}{k_{OX}}; \beta = \frac{C_g^{*}}{k_{OX}}; \ \gamma = \frac{C_g^{*}}{C_{OX}^{*}}; \ \mu_1 = \frac{v_{max} l^2}{D k_{OX}}; \ \mu_2 = \frac{v_{max} l^2}{D_{OX} k_{OX}} \tag{D5}$$

Eq. (D1) reduced to dimensionless forms for various concentrations as follows:

$$\frac{d^2u}{d\chi^2} - \frac{\mu_2}{2} \frac{u}{\left(1 + \frac{\alpha u}{\gamma v} + \frac{\beta u}{\gamma}\right)} = 0 \tag{D6}$$

$$\frac{d^2v}{d\chi^2} - \frac{\mu_1}{\gamma} \frac{u}{\left(1 + \frac{\alpha u}{\gamma v} + \frac{\beta u}{\gamma}\right)} = 0 \tag{D7}$$

$$\frac{d^2w}{d\chi^2} + \frac{\mu_1}{\gamma} \frac{u}{\left(1 + \frac{\alpha u}{\gamma v} + \frac{\beta u}{\gamma}\right)} = 0 \tag{D8}$$

where u, v and w represent the dimensionless concentration of oxygen, glucose and gluconic acid. α, β and γ are dimensionless constant. μ_1 and μ_2 are the Thiele modulus for the oxygen and glucose. Now the boundary conditions reduced to

$$u(\chi) = 1; \quad v(\chi) = 1; \quad w(\chi) = 0 \ \text{ at } \ \chi = 0, \quad \chi = 1 \tag{D9}$$

APPENDIX E: NUMERICAL PROGRAM FOR THE SOLUTION OF SYSTEMS OF NON-LINEAR EQS. (D6)-(D8)

```
function pdex4dimenless
m = 0;
x = linspace(0,1,21);
t = linspace(0, 100000);
sol = pdepe(m, @pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1); u2 = sol(:,:,2); u3 = sol(:,:,3);
plot(x,u1(end))
% ---------------------------------------------------------
function [c,f,s]=pdex4pde(x,t,u,DuDx)
c = [1; 1; 1];
f = [1; 1; 1].* DuDx;
alpha = 0.1; beta    = 0.5; gamma = 5; mu2 = 10; mu1 = 10;
F = -mu2*u(1)/(2*(1+alpha/gamma*(u(1)/u(2))+beta*(1/gamma)*u(1)));
F1= -mu1*u(1)/(gamma*(1+alpha/gamma*(u(1)/u(2))+beta*(1/gamma)*u(1)));
F2=    mu1*u(1)/(gamma*(1+alpha/gamma*(u(1)/u(2))+beta*(1/gamma)*u(1)));
s = [F; F1 ;F2];
% ---------------------------------------------------------
function u0 = pdex4ic(x);
u0 = [0; 1; 0];
% ---------------------------------------------------------
function [pl,ql,pr,qr]=pdex4bc(xl,ul,xr,ur,t)
pl = [ul(1)-1; ul(2)-1; ul(3)];
ql = [0; 0; 0];
pr = [ur(1)-1; ur(2)-1; ur(3)];
qr = [0; 0; 0];
```

Table S.1: Numerical values of parameter ‘b’ (Eq. (12)) used in Figures for various values of kinetic parameters

$\beta=0.5,\ \gamma=5,\mu_1=10,\mu_2=10$		$\alpha=0.1,\ \gamma=5,\mu_1=10,\mu_2=10$		$\alpha=0.1,\beta=0.5,\mu_1=\mu_2=10$		$\alpha=0.1,\beta=0.5,\gamma=5,\mu_1=0.1$	
α	Numerical values of b	β	Numerical values of b	γ	Numerical values of b	μ_2	Numerical values of b
0.1	2.1562	0.5	2.1562	0.5	1.3860	1	0.6701
5	1.6214	5	1.6950	1	1.8100	10	2.1585
10	1.3267	10	1.3815	1.5	1.9600	20	3.0862
15	1.1456	20	1.0514	2	2.0328	30	3.8069
100	0.4909	100	0.4948	10	2.1960	100	7.0462
$\alpha=0.1,\beta=0.5,\gamma=5,\mu_2=0.1$		$\alpha=0.1,\beta=0.5,\gamma=5,\mu_2=0.1$		$\alpha=0.1,\beta=0.5,\gamma=5$		-	-
μ_1	Numerical values of b	μ_1	Numerical values of b	$\mu_1=\mu_2$	Numerical values of b	-	-
1	0.2113	30	0.2080	1	0.6699	-	-
10	0.2108	50	0.1923	10	2.1562	-	-
20	0.2099	60	0.1789	30	3.8025	-	-
30	0.2080	70	0.1668	100	7.0437	-	-

Table S.2. Comparison of dimensionless concentration u with simulation result for various values of α when $\beta = 0.5, \gamma = 5, \mu_1 = \mu_2 = 10$

χ	$\alpha = 0.1, b = 2.1562, p = -1.6749$					$\alpha = 15, b = 1.1456, p = -0.5779$					$\alpha = 100, b = 0.4910, p = -0.1175$				
	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (13)	% deviation Eq. (9)	% deviation Eq. (13)	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (13)	% deviation Eq. (9)	% deviation Eq. (13)	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (13)	% deviation Eq. (9)	% deviation Eq. (13)
0.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
0.2	0.7436	0.742	0.7463	0.2152	0.3631	0.9083	0.9068	0.9084	0.1651	0.0110	0.9812	0.9812	0.9812	0.0000	0.0000
0.4	0.6256	0.6241	0.6313	0.2398	0.9111	0.8631	0.8614	0.8632	0.1970	0.0116	0.9719	0.9718	0.9719	0.0103	0.0000
0.6	0.6256	0.6241	0.6343	0.2398	1.3907	0.8631	0.8614	0.8632	0.1970	0.0116	0.9719	0.9718	0.9719	0.0103	0.0000
0.8	0.7436	0.742	0.7532	0.2152	1.2910	0.9083	0.9068	0.9084	0.1651	0.0110	0.9812	0.9812	0.9812	0.0000	0.0000
1.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
	Average deviation			0.1516	0.6593	Average deviation			0.1207	0.0075	Average deviation			0.0034	0.0000

Table S.3. Comparison of dimensionless concentration u with simulation result for various values of β when $\alpha = 0.1, \gamma = 5, \mu_1 = \mu_2 = 10$

χ	$\beta = 0.5, p = -1.6749, b = 2.1562$					$\beta = 5, p = -0.4897, b = 1.695$					$\beta = 100, p = -0.1188, b = 0.4948$				
	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation Eq. (9)	% deviation Eq. (C.7)	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation Eq. (9)	% deviation Eq. (C.7)	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation Eq. (9)	% deviation Eq. (C.7)
0.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
0.2	0.7436	0.7420	0.7463	0.2152	0.3631	0.8253	0.8197	0.8239	0.6785	0.1696	0.9810	0.9809	0.9810	0.0102	0.0000
0.4	0.6256	0.6241	0.6313	0.2398	0.9111	0.7405	0.7345	0.7376	0.8103	0.3916	0.9715	0.9713	0.9715	0.0206	0.0000
0.6	0.6256	0.6241	0.6343	0.2398	1.3907	0.7405	0.7345	0.7360	0.8103	0.6077	0.9715	0.9713	0.9715	0.0206	0.0000
0.8	0.7436	0.7420	0.7532	0.2152	1.2910	0.8253	0.8197	0.8204	0.6785	0.5937	0.9810	0.9809	0.9810	0.0102	0.0000
1.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
	Average deviation			0.1516	0.6593	Average deviation			0.4963	0.2938	Average deviation			0.0103	0.0000

Table S.4. Comparison of dimensionless concentration u with simulation result for various values of γ when $\alpha = 0.1, \beta = 0.5, \mu_1 = \mu_2 = 10$

χ	$\gamma = 0.6, p = -0.9775, b = 1.5108$					$\gamma = 2, p = -1.5385, b = 2.196$					$\gamma = 10, p = -1.7121, b = 2.232$				
	Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation		Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation		Numerical Simulation	MHFM Eq. (9)	Taylor series Eq. (C.7)	% deviation	
				Eq. (9)	Eq. (C.7)				Eq. (9)	Eq. (C.7)				Eq. (9)	Eq. (C.7)
0.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
0.2	0.8433	0.8502	0.8520	0.8182	1.0317	0.7657	0.7627	0.7635	0.3918	0.2873	0.7360	0.7353	0.7430	0.0951	0.9511
0.4	0.7713	0.7785	0.7898	0.9335	2.3985	0.6561	0.6532	0.6513	0.4420	0.7316	0.6153	0.6148	0.6301	0.0813	2.4053
0.6	0.7713	0.7785	0.8005	0.9335	3.7858	0.6561	0.6532	0.6488	0.4420	1.1126	0.6153	0.6148	0.6381	0.0813	3.7055
0.8	0.8433	0.8502	0.8735	0.8182	3.5812	0.7657	0.7627	0.7582	0.3918	0.9795	0.7360	0.7353	0.7605	0.0951	3.3288
1.0	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
	Average deviation			0.5839	1.7995	Average deviation			0.2779	0.5185	Average deviation			0.0588	1.7318

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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