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# A DELAYED MODEL OF UNEMPLOYMENT WITH A GENERAL NONLINEAR RECRUITMENT RATE

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**Abstract.** This article examines unemployment phenomenon using delay differential equations. Our objective consists in evaluating the influence of the delay and recruitment function on the stability of the equilibrium. We propose a nonlinear function of general type. The dynamics of this model is analyzed by constructing a Lyapunov function. First, we prove the existence and uniqueness of positive equilibrium. Next, we demonstrate the overall stability of this equilibrium. Finally, we present some numerical examples (bilinear function, with separate variables and general) to illustrate and compare our results in different situations.

**Keywords:** unemployment model; delay; global stability; Lyapunov function; LaSalle's principle.

**2010 AMS Subject Classification:** 34K20, 34K25.

## 1. Introduction

Unemployment occurs when people who are willing and able to work at prevailing wages cannot find jobs. It has a considerable negative impact on the economic and social development of the country. Its effects are not limited to a particular person but affect the whole family, especially the younger generation. Although they are full of life and dreams, they are depressed by a long period of unemployment. which leads to frustration and, as a result, the unemployed

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get involved in activities that hinder the normal life of society. Recently, there has been growing interest in unemployment models with time delay, $\tau$ [19, 20]. The introduction of this delay is dictated by the observed time lag between the determination of recruitment needs and the vacancies creation. Generally, these advances have mainly taken place within the framework of the analysis of the stability of the labor market equilibrium.

In [2017, [18]], Pathan et al. proposed the following delayed model :

$$(1)\qquad \begin{cases} \frac{dU}{dt} = \Lambda - \mu U - f(U,V) + \beta E, \\ \frac{dE}{dt} = f(U,V) - (\alpha + \beta)E, \\ \frac{dV}{dt} = C(U_\tau) - \delta V, \end{cases}$$

where $\Lambda$ is the rate of increase in the number of unemployed people, $k$ is the rate of change of the number of unemployed people becoming employed, $\beta$ is the rate of employed people resigned, being fired or dismissed from their jobs, $\mu$ is the rate of migration as well as death of unemployed people, $\alpha$ is the rate of migration, retirement or death of employed people, $\sigma$ is the rate of creating new vacancies, $\delta$ is the diminution rate of available vacancies due to lack of government funds.

The first equation of system (2) represents the dynamics of the unemployed population, $U$,composed of a term $\Lambda - \mu U$ which denotes the growth of $U$ and the term $f(U,V) = kUV$ called the functional response, which corresponds to the number of vacancies V occupied per unit time by $U$ and the term $\beta E$ of employed persons who quit, are fired or dismissed from their jobs. fired from their jobs. The second equation of the system (2) discusses the dynamics of the employed labor force, $E$. The variation of $E$ depending on the function $f(U,V)$ from which is subtracted retirement, loss of service and death $(\alpha + \beta)E$. The third equation gives the evolution of the creation of new vacancies according to $C(U) = \sigma U$ and the diminution rate of available vacancies due to lack of government funds.

A recent paper by Al-Sheikh et al. (2021) proposes the following ordinary differential model[1, 5]:

$$(2)\qquad \begin{cases} \frac{dU}{dt} = \Lambda - f(U,V) + \beta E - \mu U, \\ \frac{dE}{dt} = f(U,V) - \beta E - \alpha E, \\ \frac{dV}{dt} = C(U) - \delta V, \end{cases}$$

The authors assumed that $C(U) = \sigma U$ in [5] and in [1], they proposed

$$C(U) = \begin{cases} \sigma U & \text{if } 0 < U \leq U_m, \\ \sigma U_m & \text{if } U > U_m. \end{cases}$$

They obtained that the unique positive equilibrium $(U^*, E^*, V^*)$ of System (2) is globally asymptotically stable when $U^*E^* \leq \frac{\Lambda}{k}$.

Several questions can be raised about this model; in particular, the rate at which the active population fills open jobs in all the models cited is assumed to be proportional to the size of the unemployed population. This linear relationship seemed unrealistic, also the sufficient condition for the global asymptotic stability stability of the equilibrium position.

In [[12], 2018] Maalwi et al. proposed the following ordinary differential model:

$$\begin{cases} \frac{dU}{dt} = \Lambda - \mu U - f(U,V) + \beta E, \\ \frac{dE}{dt} = f(U,V) - \beta E - \alpha E, \\ \frac{dV}{dt} = \alpha E - \delta V, \end{cases} \tag{3}$$

The authors have found that the model (3) has two equilibria: the employment free equilibrium and the positive equilibrium. They proved that when the basic reproduction number is greater than unity, the employment free equilibrium is unstable and the second equilibrium is locally and globally asymptotically stable.

More recently [[23], 2021], Petaratip et al. took over the model (3) and they incorporated time delay in creating new vacancies. They obtained conditions for locally and globally asymptotically stable, and they indicated that the Hopf bifurcation phenomenon cannot exist under these conditions.

In this paper, we propose the following model of unemployment with time delay and a general nonlinear recruitment function $f(U,V) = f(U)V$:

$$\begin{cases} \frac{dU}{dt} = \Lambda - \mu U - f(U)V + \beta E, \\ \frac{dE}{dt} = f(U)V - (\alpha + \beta)E, \\ \frac{dV}{dt} = \sigma E_\tau - \delta V, \end{cases} \tag{4}$$

where $f$ is the rate of change of the number of unemployed people becoming employed and $\tau$ is the observed time lag between the determination of recruitment needs and the vacancies creation.

We assume that the function $f$ is a continuously differentiable function on $]0,+\infty[$ satisfying:

**$(H_0)$:** $f(0)=0$ ;

**$(H_1)$:** $f'(U)\geq 0$.

The rest of the paper is organized as follows: In Section 2, we prove the existence and uniqueness of the endemic equilibrium and by the Lyapunov-LaSalle invariance principle to prove the global stability of the disease-free equilibrium. In Section 3, we present some application of the main result with particular functional response. Finally, we present some concluding remarks.

## 2. Global Stability of the Trivial Equilibrium

We define the basic reproduction number of our model (4)by

$$R_0=\frac{\sigma f(\frac{A}{\mu})}{\delta(\alpha+\beta)}.$$

The following theorem presents the existence and uniqueness of equilibria.

**Proposition 1.** *System (4) always has a trivial equilibrium $P_0=(\frac{\Lambda}{\mu},0,0)$ which exists for all parameter values.*
*We have the following theorem on the global asymptotic stability of the trivial equilibrium $P_0$ of (4).*

**Proposition 2.** **(i):** *If $R_0<1$, then the disease free equilibrium $P_0$ is globally asymptotically stable.*

**(ii):** *If $R_0>1$, then the disease free equilibrium $P_0$ is unstable.*

*Proof.* We consider the following Lyapunov functional [12],

$$L_1(t) = U-U_0-U_0\ln(\frac{U}{U_0})+\frac{\beta}{2(\mu+\alpha)}(U-U_0+E)^2+E+pV. \tag{5}$$

Here $U_0 = \frac{A}{\mu}$ and $p$ is a positive constant to be determined.

The time derivative of $L_1$ satisfies

$$\frac{dL_1(t)}{dt} = (1 - \frac{U_0}{U})\dot{U} + \frac{\beta}{\mu+\alpha}(U - U_0 + E)(\dot{U} + \dot{E}) + \dot{E} + p\dot{V}.$$

By applying the derivative of the variables in system (4) we obtain

$$\begin{aligned}
\frac{dL_1(t)}{dt} &= (1 - \frac{U_0}{U})(\Lambda - f(U)V + \beta E - \mu U) + \frac{\beta}{\mu+\alpha}(U - U_0 + E)(\Lambda - \mu U - \alpha E) \\
&+ f(U)V - (\alpha+\beta)E + p(\sigma E - \delta V) \\
&= -[\tfrac{\mu}{U} + \tfrac{\beta E}{UU_0} + \tfrac{\mu\beta}{(\mu+\alpha)U_0}](U_0 - U)^2 - \tfrac{\alpha\beta}{(\mu+\alpha)U_0}E^2 + \sigma(p - \tfrac{\alpha+\beta}{\sigma})E + \delta(\tfrac{f(U_0)}{\delta} - p)V
\end{aligned}$$

Since $R_0 < 1$, we have $\frac{f(U_0)}{\delta} < \frac{\alpha+\beta}{\sigma}$; then, we can choose $p > 0$ such that $\frac{f(U_0)}{\delta} < p < \frac{\alpha+\beta}{\sigma}$. Hence, we have that $\frac{dL_1(t)}{dt} < 0$· Then, $P_0$ is globally asymptotically stable.

On the other hand, the Jacobian of model (4) at the trivial equilibrium $P_0$ is given by

$$J(P_0) = \begin{pmatrix} -\mu & \beta & -f(U_0) \\ 0 & -\alpha-\beta & f(U_0) \\ 0 & \sigma & -\delta \end{pmatrix}$$

Obviously, $J(P_0)$ has an eigenvalue $\lambda_1 = -\mu < 0$, and the other two eigenvalues $\lambda_2$ and $\lambda_3$ satisfy $\lambda_2 + \lambda_3 = -(\alpha+\beta)\delta(1 - R_0)$ Then, when $R_0 > 1$, $J(P_0)$ has a positive eigenvalue and $P_0$ is unstable. The proof of Proposition (2)is complete. □

## 3. Positive Equilibrium and Its Global Stability Analysis

**3.1. Equilibrium.** In the following proposition, we prove the existence of the unique positive equilibrium.

**Proposition 3.** *Let the hypotheses* $(H_0)$ *and* $(H_1)$. *If* $R_0 > 1$, *then the system* (4) *admits a unique endemic equilibrium* $P^* = (U^*, E^*, V^*)$, *with* $E^* = \frac{\Lambda - \mu U^*}{\alpha}$, $V^* = \frac{\sigma U^*}{\delta}$, *and* $U^*$ *is the unique solution of the following equation:*

$$f(U, \frac{\sigma(\Lambda - \mu U)}{\alpha\delta}) - \frac{\alpha+\beta}{\alpha}(\Lambda - \mu U) = 0.$$

*Proof.* If $E \neq 0$, then we have

$$
\begin{cases} U = \frac{A}{\mu} - \frac{\alpha E}{\mu}, \\ V = \frac{\sigma E}{\delta}, \\ \frac{\sigma}{\delta} f(\frac{A}{\mu} - \frac{\alpha E}{\mu}) - (\alpha + \beta) = 0. \end{cases} \tag{6}
$$

We consider the following function

$$
h(E) := \frac{\sigma}{\delta} f(\frac{\Lambda}{\mu} - \frac{\alpha E}{\mu}) - (\alpha + \beta).
$$

Since the function $h$ is continuous on $E$, $h'(E) = \frac{-\alpha\sigma}{\mu\delta} f'(\frac{A}{\mu} - \frac{\alpha E}{\mu}) < 0$, $f(\frac{\Lambda}{\mu}) = -(\alpha + \beta) < 0$, and $h(0) = \frac{\sigma}{\delta} f(\frac{\Lambda}{\mu}) - (\alpha + \beta) > 0$, if $R_0 > 1$. Hence, there exist unique positive $E^*$ such that $h(E^*) = 0$. Consequently there exist unique equilibrium $P = (U^*, E^*, V^*)$ of (4). The proof of Theorem 1 is completed. □

**3.2. Global stability analysis of the positive equilibrium.** Now, we discuss the global stability of the positive equilibrium $P^*$ of system (4).

**Proposition 4.** *Let the hypotheses $(H_0)$ and $(H_1)$ and $R_0 > 1$. If $\beta$ is close enough to zero, then the positive equilibrium $P^*$ is globally asymptotically stable.*

*Proof.* Define a Lyapunov functional

$$
L(t) = W_1(t) + W_2(t) + W_3(t) + W_4(t),
$$

where

$$
W_1(t) = \int_{U^*}^{U} (1 - \frac{f(U^*, V^*)}{f(\omega, V^*)}) d\omega,
$$

$$
W_2(t) = E - E^* - E^* \ln \frac{E}{E^*}, \qquad W_3(t) = \frac{f(U^*, V^*)}{\sigma E^*} (V - V^* - V^* \ln \frac{V}{V^*}),
$$

and

$$
W_4(t) = f(U^*, V^*) \int_0^{\tau} (\frac{E(t-\theta)}{E^*} - 1 - \ln \frac{E(t-\theta)}{E^*}) d\theta.
$$

We will show that $\frac{dL(t)}{dt} \leq 0$ for all $t \geq 0$. We have :

$$\begin{aligned}
\frac{dL(t)}{dt} &= (1-\frac{f(U^*,V^*)}{f(U,V^*)})\dot{U} + (1-\frac{E^*}{E})\dot{E} + \frac{f(U^*,V^*)}{\sigma E^*}(1-\frac{V^*}{V})\dot{V} \\
&\quad + f(U^*,V^*)(-\frac{E_\tau}{E^*} + \frac{E}{E^*} + \ln\frac{E_\tau}{E}) \\
&= (1-\frac{f(U^*,V^*)}{f(U,V^*)})(\Lambda - f(U,V) + \beta E - \mu U) + (1-\frac{E^*}{E})(f(U,V) - (\alpha+\beta)E) \\
&\quad + \frac{f(U^*,V^*)}{\sigma E^*}(1-\frac{V^*}{V})[\sigma E_\tau - \delta V] + f(U^*,V^*)(-\frac{E_\tau}{E^*} + \frac{E}{E^*} + \ln\frac{E_\tau}{E})
\end{aligned}$$

Using the relations

$$\Lambda = \mu U^* + f(U^*,V^*) - \beta E^*, \qquad (\alpha+\beta)E^* = f(U^*,V^*),$$

$$\sigma U^* = \delta V^*, \qquad \Lambda = \mu U^* + \alpha E^*,$$

the time derivative function $\frac{dL(t)}{dt}$ becomes

$$\begin{aligned}
\frac{dL(t)}{dt} &= (1-\frac{f(U^*,V^*)}{f(U,V^*)})(\mu U^* + f(U^*,V^*) - \beta E^* - f(U,V) + \beta E - \mu U) \\
&\quad + (1-\frac{E^*}{E})(f(U,V) - f(U^*,V^*)\frac{E}{E^*}) + \frac{f(U^*,V^*)}{\sigma E^*}(\sigma E_\tau - \delta V - \frac{\sigma U V^*}{V} + \delta V^*) \\
&\quad + f(U^*,V^*)(-\frac{E_\tau}{E^*} + \frac{E}{E^*} + \ln\frac{E_\tau}{E})
\end{aligned}$$

Then

$$\begin{aligned}
\frac{dL(t)}{dt} &= -\mu(1-\frac{f(U^*,V^*)}{f(U,V^*)})(U-U^*) + \beta(1-\frac{f(U^*,V^*)}{f(U,V^*)})(E-E^*) + f(U^*,V^*)\ln\frac{E_\tau}{E} \\
&\quad + f(U^*,V^*)[3 - \frac{f(U^*,V^*)}{f(U,V^*)} - \frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)} - \frac{V^*}{V}\frac{E_\tau}{E}] \\
&\quad + f(U^*,V^*)(-\frac{V^*}{V} + \frac{f(U,V)}{f(U,V^*)})
\end{aligned}$$

By the proprieties of a logarithm function, we have that

$$\ln\frac{E_\tau}{E} = \ln\frac{E_\tau}{E^*}\frac{V^*}{V} + \ln\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)} + \ln\frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)} + \ln\frac{f(U^*,V^*)}{f(U,V^*)},$$

and we obtain

$$
\begin{aligned}
\frac{dL(t)}{dt} &= -\mu(1-\frac{f(U^*,V^*)}{f(U,V^*)})(U-U^*)+\beta(1-\frac{f(U^*,V^*)}{f(U,V^*)})(E-E^*)\\
&+f(U^*,V^*)[1-\frac{f(U^*,V^*)}{f(U,V^*)}+\ln\frac{f(U^*,V^*)}{f(U,V^*)}]\\
&+f(U^*,V^*)[1-\frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)}+\ln\frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)}]\\
&+f(U^*,V^*)[1-\frac{V^*}{V}\frac{E_\tau}{E}+\ln\frac{V^*}{V}\frac{E_\tau}{E}]\\
&+f(U^*,V^*)[1-\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}+\ln\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}]\\
&+f(U^*,V^*)[1-\frac{V}{V^*}+\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}+\ln\frac{V}{V^*}+\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}].
\end{aligned}
$$

By factoring the last term, we have

$$
\begin{aligned}
\frac{dL(t)}{dt} &= -\mu(1-\frac{f(U^*,V^*)}{f(U,V^*)})(U-U^*)+\beta(1-\frac{f(U^*,V^*)}{f(U,V^*)})(E-E^*)\\
&+f(U^*,V^*)[1-\frac{f(U^*,V^*)}{f(U,V^*)}+\ln\frac{f(U^*,V^*)}{f(U,V^*)}]\\
&+f(U^*,V^*)[1-\frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)}+\ln\frac{E^*}{E}\frac{f(U,V)}{f(U^*,V^*)}]\\
&+f(U^*,V^*)[1-\frac{V^*}{V}\frac{E_\tau}{E}+\ln\frac{V^*}{V}\frac{E_\tau}{E}]\\
&+f(U^*,V^*)[1-\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}+\ln\frac{V}{V^*}\frac{f(U,V^*)}{f(U,V)}]\\
&+f(U^*,V^*)(\frac{V}{V^*}-\frac{f(U,V)}{f(U,V^*)})(\frac{f(U,V^*)}{f(U,V)}-1).
\end{aligned}
$$

From the monotonicity of the function $f$ on $U$, the following inequality holds:

$$\frac{f(U^*,V^*)}{f(U,V^*)})(U-U^*)\geq 0.$$

Furthermore, since $f(U,V)=f(U)V$, the following equality holds:

$$(\frac{V}{V^*}-\frac{f(U,V)}{f(U,V^*)})(\frac{f(U,V^*)}{f(U,V)}-1)=0.$$

Note that the function $1-b+\ln b\leq 0$ for any $b>0$, where equality holds if and only if $b=1$. Therefore, if $\beta$ is close enough to zero, then $\frac{dL(t)}{dt}\leq 0$. Since $\frac{dL(t)}{dt}=0$ if and only if $U=U^*$, $E=$

$E^*$ and $V = V^*$, by LaSalle invariance principle, the equilibrium $P^*$ is globally asymptotically stable. This completes the proof. □

## 4. Numerical Simulations

In this section, we give some numerical simulations to illustrate the theoretical analysis. Let

$$f(U,V) = \frac{k_1 UV}{1 + k_0 U}.$$

We take the parameters of the system (4) as follows:

$$\Lambda = 10, \quad \alpha = 0.9, \quad \mu = 0.005, \quad \delta = 0.02, \quad \beta = 0.1 \textit{ and } \quad \sigma = 0.5.$$

By Proposition 4, we have $P^*$ is globally asymptotically stable (see Figure 1).

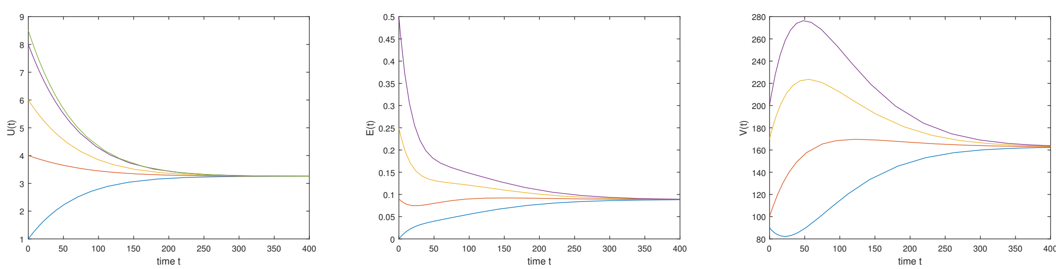


FIGURE 1. Solutions $(U,E,V)$ of model (4) are globally asymptotically stable and converge to the positive equilibrium $P^*$

## 5. Concluding Remarks

In this work, we have proposed a generalization of the unemployment model. Our contribution consists to consider this model with a generalized delayed nonlinear recruitment function, $f(U,V)$.

By the Lyapunov theorem, we have proved that the proposed model is globally asymptotically stable. Also, we observed that the time delay, between the vacancies creation and the occupation of these vacancies, has no effect on the stability of unemployment model. This result gives an improvement of the sufficient condition of the global stability proposed in the works [1, 5, 20], in the particular case: $(f(U,V) = kUV)$.

## Conflict of Interests

The author(s) declare that there is no conflict of interests.

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