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MULTIPLE CRITERIA DECISION MAKING BASED ON BIPOLAR FUZZY SETS APPLICATION TO FUZZY TOPSIS

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Abstract. In the present study, we developed a new method for multiple criteria decision making (MCDM) based on bipolar value sets. The proposed method is to develop a new methodology name BFS-TOPSIS, which considers the positive and negative decisions of the decision-maker. The BFS-TOPSIS is a combination of bipolar value sets with the fuzzy TOPSIS method and uses a score function to select the best alternative. We have shown the design algorithms BFS-TOPSIS and illustrate our proposed methods with examples.

Keywords: fuzzy TOPSIS; MCDM; bipolar fuzzy sets; score function; BFS-TOPSIS.

2010 AMS Subject Classification: 94D05, 62C86

1. INTRODUCTION

Multiple criteria decision-making problem is very important in the 21st century, especially in regard to Big Data, the terminology for the vast and growing volume of data available on the Internet and in the cloud, so models for decision analysis are also very important. A study of the model theory of set theory and the actual problems encountered in many forms of decision-making [1], [2], found that there are many types of data. MCDM refers to the screening, prioritizing, ranking, or selecting of a set of alternatives from usually independent, incommensurable,

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or conflicting attributes. One popular approach used in MCDM is the technique known as TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), a multi-criteria decision analysis method originally published in 1981 by Hwang and Yoo [3]. Theoretically, for solving MCDM problems, TOPSIS is based on the idea of the selection of the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). The concept of TOPSIS is applicable to the fuzzy environment that is usual in MCDM. In that approach, the rating of each alternative, and the weight of each criterion, are described in linguistic terms by Chen [4]. Other researchers are interested in the concept of TOPSIS and have applied it in various fields [5], [7], [6], [8]. However, data used in decision-making contains both numerical data and linguistic variables, and some information has both bipolarity, and positive and negative values that must be considered. So bipolar information, and these other considerations, affect the effectiveness and efficiency of decision making.

Bipolar information is frequently to be found in daily life, in , organizational decision making, economic analysis, project performance reviews, development projects evaluation, risk analysis; indeed, in most decision making activities where contributing factors and outcomes are unclear. Bipolarity is important to separate positive and negative information responses, which positive information presents possible considerations, while negative information presents as forbidden or surely false considerations. If we consider that both may be viewed as both positive and negative information, we should therefore consider information being both advantageous and disadvantageous which leads to better decision making. Thus the concept of bipolar fuzzy sets is more of interest in Mathematics. In 1994, Zhang [9] initiated the bipolar fuzzy set as bipolar fuzzy logic. This has been widely applied to solve many real-world problems. In 1998, the notions of bipolar fuzziness and interval-based bipolar fuzzy logic were generalized as a real-valued bipolar fuzzy logic [10]. Based on these notions, bipolar fuzzy set theory and its applications were developed [11], [12],[13]. A membership degree range of bipolar fuzzy sets is in $[0, 1]$ and $[-1, 0]$. The membership degree 0 of an element means that the elements are irrelevant to the corresponding property. The membership degree in $(0, 1]$ and $[-1, 0)$ of an element indicates that the element somewhat satisfies the property and implicit counter-property, respectively. These mathematical concepts are a development to improve Mathematics. In

2018, Alghamdi M. A. et al.[14] introduced the idea of multi-criteria decision-making methods in a bipolar fuzzy environment. In 2020, Muhammad Akram et al.[15] proposed Bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I methods. In concept, the score function improved the algorithm for use in MCDM problems [16] [17],[18], [19].

In this paper, we present a new bipolar fuzzy (BF) set in TOPSIS, which we entitle the BFS-TOPSIS method, for solving MCDM problems that are equipped with bipolar fuzzy information. The outline of the paper is as follows: First, the definition of importance is described and then we show the design of the algorithms applied in the BFS-TOPSIS method. Finally, we illustrate our proposed methods with examples.

2. PRELIMINARIES

In this section, we review the important definitions of fuzzy sets and bipolar fuzzy sets to be used in MCDM models.

Definition 2.1. [20] Let X be a non-empty set. An *fuzzy subset* μ of X is a function $\mu : X \rightarrow [0, 1]$.

Definition 2.2. Let X be a non-empty set. A bipolar fuzzy set (BF set) μ on X is an object having the form

$$\mu := \{(u, \mu^p(u), \mu^n(u)) \mid u \in X\},$$

where $\mu^p : X \rightarrow [0, 1]$ and $\mu^n : X \rightarrow [-1, 0]$.

Next, we introduce the concept of score function.

Definition 2.3. Let ψ , then score function is real valued function define and denoted as,

$$\mathcal{S}(X_i) = \mu_{X_i}^p + \mu_{X_i}^n$$

where $\psi(X_i) \in [-1, 1]$ and $\psi(X_i) = 0$ if and only if $\mu_{X_i}^p = \mu_{X_i}^n$.

Definition 2.4. Improve score function is real valued function on ψ define and denoted as,

$$\mathcal{S}(X_i) = (\mu_{X_i}^p)^2 \mathcal{S}(X_i) + (\mu_{X_i}^n)^2 \mathcal{S}(X_i) - (\mu_{X_i}^p \mu_{X_i}^n) \mathcal{S}(X_i).$$

Definition 2.5. Double improve score function is real valued function on ψ define and denoted as,

$$\mathcal{D}(X_i) = \mathcal{S}(X_i) + \mathcal{I}(X_i).$$

3. ALGORITHM FOR BFS-TOPSIS

An algorithm of fuzzy TOPSIS is a technique that emphasizes the character of the criteria which is suitable for decision-making. Accordingly, the patterns of decision-making have to be fixed. Furthermore, to enhance the efficiency of the decision-making process, the model should be determined by the decision-maker, drawing from different topic areas (brain storming) to receive diverse decision patterns leading to various comments on the complexity of the problems being analyzed, which are subsequently merged together. The resulting model will yield higher probability in selection.

Next, we applied the bipolar fuzzy set on the score function to the TOPSIS method (BFS-TOPSIS method) to solve some problems in the real world. To elaborate, let

- (1) $D = \{D_1, D_2, \dots, D_k\}$ is a set of decision-maker where $k = 1, 2, \dots, K$,
- (2) $A = \{A_1, A_2, \dots, A_i\}$ is a set of assessing alternatives where $i = 1, 2, \dots, m$, and
- (3) $C = \{C_1, C_2, \dots, C_j\}$ is a set of criteria where $j = 1, 2, \dots, n$.

The criteria are selected according to the outcomes of the investigation of the decision-maker. Furthermore, assume that each alternative's suitability rating and criteria are assigned to bipolar fuzzy values and the weights. The steps of the BFS-TOPSIS method are as follows:

(i) The weights W of the BF set are assigned to each criterion by the decision maker that satisfy the condition of normality, that is, for

$$W = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix},$$

where $w_j = \frac{\bar{w}_j}{w}$, $\bar{w}_j = \frac{1}{k}(w_j^1 + w_j^2 + \dots + w_j^k)$, $w = w_1 + w_2 + \dots + w_n$ and $\sum_{j=1}^n w_j = 1$.

(ii) Each alternative A_i is evaluated with respect to n criteria from the k decision-maker. All the values assigned to the alternatives for each criterion form a decision matrix D as

$$D = [x_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

where $x_{ij} = (x_{ij}^p, x_{ij}^n) = \frac{1}{k}((x_{ij}^p, x_{ij}^n)^1 + (x_{ij}^p, x_{ij}^n)^2 + \dots + (x_{ij}^p, x_{ij}^n)^k)$. For each entry $x_{ij} = (x_{ij}^p, x_{ij}^n)$, $x_{ij}^p \in [0, 1]$ represents the degree of satisfaction of alternative i under criteria j and $x_{ij}^n \in [-1, 0]$ represents the degree of dissatisfaction of the alternative i under criteria j .

Note that, if $x_{ij}^p = 1$, then the alternative i shows the maximum satisfaction behavior of the criteria j and in the case of $x_{ij}^n = -1$, we say that alternative i represents the maximum degree of dissatisfaction of the criteria j .

(iii) We can construct the BF set weighted fuzzy decision matrix $V = [v_{ij}]_{m \times n}$ as

$$\text{where } v_{ij} = (v_{ij}^p, v_{ij}^n) = (w_j x_{ij}^p, w_j x_{ij}^n).$$

(iv) We can define the positive part of the BF set of positive-ideal solution \dot{A}^p and negative-ideal solution \check{A}^p from v_{ij}^p as

$$\dot{A}^p = (\dot{v}_1^p, \dot{v}_2^p, \dots, \dot{v}_n^p) \text{ and } \check{A}^p = (\check{v}_1^p, \check{v}_2^p, \dots, \check{v}_n^p)$$

$$\text{where } \dot{v}_j^p = \max_j v_{ij}^p \text{ and } \check{v}_j^p = \min_j v_{ij}^p, j = 1, 2, 3, \dots, n.$$

The negative part of BF set of positive-ideal solution \dot{A}^n and negative-ideal solution \check{A}^n from v_{ij}^n as

$$\dot{A}^n = (\dot{v}_1^n, \dot{v}_2^n, \dots, \dot{v}_n^n) \text{ and } \check{A}^n = (\check{v}_1^n, \check{v}_2^n, \dots, \check{v}_n^n)$$

$$\text{where } \dot{v}_j^n = \min_j v_{ij}^n \text{ and } \check{v}_j^n = \max_j v_{ij}^n, j = 1, 2, 3, \dots, n.$$

(v) The distance $d = (d^p, d^n)$ of each alternative from \dot{A}^p and \dot{A}^n can be currently calculated as

$$d^p = (\dot{d}^p, \check{d}^p) = \left(\sum_{j=1}^n d(v_{ij}^p, \dot{v}_j^p), \sum_{j=1}^n d(v_{ij}^p, \check{v}_j^p) \right), i = 1, 2, \dots, m,$$

and

$$d^n = (\dot{d}^n, \check{d}^n) = \left(\sum_{j=1}^n d(v_{ij}^n, \dot{v}_j^n), \sum_{j=1}^n d(v_{ij}^n, \check{v}_j^n) \right), i = 1, 2, \dots, m,$$

where $d(\cdot, \cdot)$ is the distance measurement between two fuzzy sets.

(vi) A closeness coefficient BF set C_i of each alternative A_i , as

$$C_i = (C_i^p, C_i^n) = \left(\frac{\ddot{d}_i^p}{d_i^p + \ddot{d}_i^p}, -\frac{\ddot{d}_i^n}{d_i^n + \ddot{d}_i^n} \right).$$

(vii) Calculate the score function as,

$$\mathcal{S}(C_i) = C_i^p + C_i^n.$$

Improve the score function as,

$$\mathcal{I}(C_i) = (C_i^p)^2 \mathcal{S}(C_i) + (C_i^n)^2 \mathcal{S}(C_i) - (C_i^p C_i^n) \mathcal{S}(C_i),$$

and the Double Improve score function as,

$$\mathcal{D}(C_i) = \mathcal{S}(C_i) + \mathcal{I}(C_i).$$

After obtaining these score function values, $\mathcal{D}(C_i)$ select the best alternative which is given by,

$$\mathcal{B}(A_i) = \max\{\mathcal{D}(C_i) | i = 1, 2, \dots, m\}.$$

4. A NUMERICAL EXAMPLE AND SOLUTION OF DECISION

In this section, we simulate the best project proposal scenarios for project funding. Suppose there are six projects $\{A_i | i = 1, 2, 3, 4, 5, 6\}$ submitted for consideration and designation of three decision-makers $\{D_i | i = 1, 2, 3\}$. The decision-makers brainstorm ideas to create criteria for selecting the best project proposals to receive funding, These criteria encompass quality C_1 , feasibility C_2 , modernity C_3 , utilization C_4 , and cost C_5 . The proposed method is applied to solve this problem, and the computational procedure is summarized as follows:

Step 1: Decision makers determine the weighted W of the criteria for assessing the quality of the project as follows:

TABLE 1. The importance weight of the criteria

| | C_1 | C_2 | C_3 | C_4 | C_5 |
|-------|-------|-------|-------|-------|-------|
| D_1 | 0.8 | 0.6 | 0.4 | 0.9 | 0.9 |
| D_2 | 0.7 | 0.6 | 0.7 | 0.8 | 0.8 |
| D_3 | 0.8 | 0.7 | 0.5 | 0.8 | 0.7 |

Step 2: Score a project proposal evaluation using a bipolar series and calculate the following weighted average in Table 2 and 3:

TABLE 2. Bipolar fuzzy decision matrix

| | | C_1 | C_2 | C_3 | C_4 | C_5 |
|-------|-------|-------------|-------------|-------------|-------------|-------------|
| D_1 | A_1 | (0.8, -0.2) | (0.5, -0.5) | (0.0, -1.0) | (0.6, -0.5) | (0.4, -0.6) |
| | A_2 | (0.5, -0.5) | (0.6, -0.5) | (0.6, -0.4) | (0.8, -0.2) | (1.0, 0.0) |
| | A_3 | (1.0, 0.0) | (0.4, -0.6) | (0.2, -0.6) | (0.6, -0.5) | (0.4, -0.6) |
| | A_4 | (0.5, -0.5) | (0.6, -0.4) | (0.5, -0.5) | (0.5, -0.5) | (0.6, -0.4) |
| | A_5 | (1.0, 0.0) | (0.5, -0.5) | (0.2, -0.6) | (0.4, -0.6) | (1.0, 0.0) |
| | A_6 | (0.5, -0.5) | (0.0, -0.4) | (0.6, -0.5) | (0.4, -0.5) | (0.6, -0.4) |
| D_2 | A_1 | (0.5, -0.5) | (0.6, -0.6) | (0.8, -0.2) | (1.0, -0.4) | (0.5, -0.2) |
| | A_2 | (0.3, -0.5) | (0.2, -0.4) | (0.6, 0.0) | (0.4, -0.6) | (0.8, 0.0) |
| | A_3 | (0.4, -0.6) | (0.5, 0.0) | (0.5, -0.5) | (0.6, -0.5) | (1.0, -0.6) |
| | A_4 | (0.6, -0.5) | (0.5, -1.0) | (0.6, -0.5) | (1.0, -0.5) | (0.5, -0.4) |
| | A_5 | (0.5, -0.5) | (0.4, -0.4) | (1.0, -0.2) | (0.6, -0.5) | (0.6, -0.4) |
| | A_6 | (0.4, -0.6) | (0.5, -0.6) | (0.6, -0.5) | (0.5, -0.6) | (0.5, 0.0) |
| D_3 | A_1 | (0.4, -0.4) | (0.5, -0.5) | (0.5, -0.5) | (0.2, -0.5) | (0.4, -0.4) |
| | A_2 | (0.6, -0.5) | (0.5, -0.6) | (0.6, -0.2) | (0.6, -0.4) | (0.4, -0.6) |
| | A_3 | (0.5, -0.5) | (0.4, -0.4) | (1.0, 0.0) | (0.8, -0.6) | (1.0, -0.5) |
| | A_4 | (0.4, -0.5) | (0.5, -0.4) | (0.6, 0.0) | (0.6, -0.5) | (0.4, -0.4) |
| | A_5 | (0.5, -0.6) | (0.2, 0.0) | (0.4, -0.5) | (1.0, 0.0) | (0.6, -0.6) |
| | A_6 | (0.3, -0.5) | (0.6, -0.4) | (0.6, 0.0) | (0.8, -0.4) | (1.0, -0.6) |

TABLE 3. Average of Bipolar fuzzy decision matrix

| | C_1 | C_2 | C_3 | C_4 | C_5 |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A_1 | (0.567, -0.367) | (0.533, -0.533) | (0.433, -0.567) | (0.600, -0.467) | (0.433, -0.400) |
| A_2 | (0.467, -0.500) | (0.433, -0.500) | (0.600, -0.200) | (0.600, -0.400) | (0.733, -0.200) |
| A_3 | (0.633, -0.367) | (0.433, -0.333) | (0.567, -0.367) | (0.667, -0.533) | (0.800, -0.567) |
| A_4 | (0.500, -0.500) | (0.533, -0.600) | (0.567, -0.333) | (0.700, -0.500) | (0.500, -0.400) |
| A_5 | (0.667, -0.367) | (0.367, -0.300) | (0.533, -0.433) | (0.667, -0.367) | (0.733, -0.333) |
| A_6 | (0.400, -0.533) | (0.367, -0.467) | (0.600, -0.333) | (0.567, -0.500) | (0.700, -0.333) |
| W | 0.184 | 0.152 | 0.120 | 0.200 | 0.192 |

Step 3: Calculate the weighted bipolar fuzzy decision matrix in Table 4 as follows.

TABLE 4. Weighted bipolar fuzzy decision matrix

| | C_1 | C_2 | C_3 | C_4 | C_5 |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A_1 | (0.123, -0.080) | (0.096, -0.096) | (0.061, -0.080) | (0.142, -0.110) | (0.098, -0.091) |
| A_2 | (0.101, -0.108) | (0.078, -0.090) | (0.085, -0.028) | (0.142, -0.094) | (0.166, -0.045) |
| A_3 | (0.137, -0.080) | (0.078, -0.060) | (0.080, -0.052) | (0.157, -0.126) | (0.181, -0.128) |
| A_4 | (0.108, -0.108) | (0.096, -0.108) | (0.080, -0.047) | (0.165, -0.118) | (0.113, -0.091) |
| A_5 | (0.145, -0.080) | (0.066, -0.054) | (0.075, -0.061) | (0.157, -0.086) | (0.166, -0.075) |
| A_6 | (0.087, -0.116) | (0.066, -0.084) | (0.085, -0.047) | (0.134, -0.118) | (0.158, -0.075) |

Step 4: Determine the positive and negative part BF set and the distance $d = (d^p, d^n)$ of each alternative from \dot{A}^p and \dot{A}^n in Table 5.

TABLE 5. Calculate the distance of each candidates

| Candidates | Distance | | | |
|------------|-------------|--------------|-------------|--------------|
| | \dot{d}^p | \ddot{d}^p | \dot{d}^n | \ddot{d}^n |
| A_1 | 0.110 | 0.072 | 0.060 | 0.036 |
| A_2 | 0.102 | 0.098 | 0.117 | 0.070 |
| A_3 | 0.101 | 0.129 | 0.094 | 0.075 |
| A_4 | 0.096 | 0.086 | 0.074 | 0.065 |
| A_5 | 0.108 | 0.123 | 0.103 | 0.033 |
| A_6 | 0.116 | 0.088 | 0.086 | 0.062 |

Step 5: Calculate the score function in Table 6.

TABLE 6. The score function and selected best alternative

| Candidates | C_i^p | C_i^n | $\mathcal{S}(C_i)$ | $\mathcal{I}(C_i)$ | $\mathcal{D}(C_i)$ | $\mathcal{B}(A_i)$ |
|------------|---------|---------|--------------------|--------------------|--------------------|--------------------|
| A_1 | 0.396 | -0.375 | 0.021 | 0.009 | 0.031 | 4 |
| A_2 | 0.490 | -0.375 | 0.115 | 0.065 | 0.179 | 3 |
| A_3 | 0.560 | -0.446 | 0.115 | 0.088 | 0.202 | 2 |
| A_4 | 0.472 | -0.469 | 0.003 | 0.002 | 0.005 | 6 |
| A_5 | 0.533 | -0.241 | 0.292 | 0.138 | 0.430 | 1 |
| A_6 | 0.432 | -0.420 | 0.012 | 0.006 | 0.018 | 5 |

After obtaining the score function, $\mathcal{D}(C_i)$ the best alternative is given by, $\mathcal{B}(A_i) = \max\{\mathcal{D}(C_i) | i = 1, 2, \dots, 4\}$, where $\mathcal{B}(A_i)$ is given in Table 6. So, selecting among the six available stocks, the best stock is A_5 .

5. CONCLUSION

The BFS-TOPSIS algorithm is a tool applicable to situations that require bipolar decision-making. For maximum efficiency, it is necessary to consider both positive data and negative simultaneously, which impliedly is normal behavior of natural decision making, enabling clear decision making. Therefore, the advanced tools we developed for multi-criterion decision theory can be effectively applied to enable better decision making in situations that are advantageous for the user.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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