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NUMERICAL SOLUTION OF FREE STREAM MHD FLOW WITH THE EFFECT OF VELOCITY SLIP CONDITION FROM AN INCLINED POROUS PLATE

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Abstract: This article is concerned with Magnetohydrodynamic forced convective boundary layer flow past an inclined plate in a porous medium. Additionally, the effect of heat generation and velocity parameters are considered. The velocity slip and thermal slip effects on velocity and temperature profiles are taken into the account to address the phenomenon of heat transfer. The fundamental system of partial differential equations with necessary boundary conditions are converted into nonlinear ordinary differential equations. The reduced ordinary differential equations are solved numerically by the Finite difference method. It is observed that the results are significantly influenced by the various dimensionless parameters such as magnetic parameter, grashof number, Heat generation parameter, Prandtl number, velocity slip parameter. The impact of pertinent physical quantities on velocity and temperature profile is presented through graphs and tables. The results depicted that Magnetic parameter and permeability parameter have propensity to decrease the velocity profile.

Keywords: velocity slip condition; free stream; thermal slip condition; MHD flow; porous medium.

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1. INTRODUCTION

Magnetohydrodynamics free convection nanofluid flow in porous medium has gained remarkable interest among researchers. MHD free convection is used in numerous industrial applications, particularly in chemical engineering, electronic devices, nuclear reactors, polymer processing, and many more applications. Nanofluids have made major contribution to the boundary layer problems, as nanoparticles enhance the thermal conductivity of the fluid. In the present study Sakiadis et al. [1] was first to present the behavior of the boundary layer flow problem towards the flat surface. Alam et al. [2] presented the convective flow of Newtonian fluid in the presence of the magnetic field. Ramya et al. [3] studied the two-dimensional fluid flow with no-slip boundary conditions. Sulochana et al. [4] elaborated the MHD flow due to an inclined porous plate with a chemical reaction. Mondal et al. [5] explored the heat and mass transfer of the fluid by the analysis of symmetry in the presence of the porous medium. Rasool et al. [6] examined the effect of Brownian motion and thermophoresis parameters over an inclined plate. maleki et al. [7] discussed heat generation and absorption effect on nanofluid flow. Bhuvanewari et al. [8] obtained a symmetric solution of incompressible boundary layer fluid flow. Srinivasacharya et al. [9] considered free stream temperature over an inclined plate. Reddy et al. [10] discussed the mixed convection nanofluid flow in porous medium. Sharma et al. [11] presented the effect of porosity in fluid flow due to horizontal channel. Mandal et al. [12] studied the magnetohydrodynamic fluid flow through a porous medium with buoyancy force effect. Rafique et al. [13] analyzed the thermophoretic and Brownian motion effect on micropolar nanofluid flow. Srinivasacharya et al. [14] investigated fluid flow resulting from the mixed convection under the influence of heat generation/absorption. Fanaee et al. [15] investigated the heat and mass transfer towards the micro-channels. Ahmmed et al. [16] observed the characteristics of free convection flow of nanofluid due to thermal conductivity and heat generation. chamkha et al. [17] obtained the numerical solution of the power-law model with suction injection effect at the boundary layer. Kaushik et al. [18] presented the carreau fluid flow towards the stagnation point over a stretching sheet. Mishra et al. [19] observed the incompressible viscous fluid flow along with variable thickness conditions due

to the stretching sheet. Kaushik et al. [20] elaborated the stagnation point flow and heat transfer of electrically conducting fluid. Swain et al. [21] obtained homotopy solution of the MHD flow problem with the joule heating effect. Mallikarjuna et al. [22] studied the two-phase flow with the effect of velocity slip condition. Gbadeyan et al. [23] explored the impact of magnetic field on fluid flow in the presence of porous medium. Ahmad et al. [24] investigated the impact of viscous dissipation on two distinct fluid flow. Zafar et al. [25] studied heat transfer with chemical reaction over an inclined plate. Qiang et al. [26] discussed the dufour effect on MHD flow, owing to the parallel plate. The comparison of the results, obtained by the analytical and numerical methods was done in this study. Shah et al. [27] analyzed the influence of velocity slip on heat transfer. The effect of the porous medium was also included in the study. Ramya et al. [28] obtained the numerical solution of boundary layer nanofluid flow over a nonlinear stretching sheet. Imran et al. [29] proposed a Newtonian model with homogeneous- heterogeneous conditions. Ahmad et al. [30] addressed the boundary layer flow with the influence of Brownian motion and thermophoresis effect on heat transfer through an inclined surface.

The novelty of the present investigation is to address the combined study of two-dimensional free stream convection flow and heat transfer of nanofluid due to inclined plate. The velocity slip and thermal slip conditions are also considered in this study. The effect of pertinent flow parameters on velocity and temperature profile including skin friction coefficient and Nusselt number are analyzed. The current findings were compared to previously published work, and they were determined to be in good accord.

2. PROBLEM FORMULATION

In the present analysis, we considered the convective boundary layer flow of electrically conducting fluid along with an inclined plate in a porous medium. The plate is inclined from vertical with an inclination angle α . x coordinate taken in the direction of the plate and the y coordinate is perpendicular to the plate. The motion of the fluid is affected by a uniform magnetic field which is considered along y -axis with the strength $B = \frac{B_0}{\sqrt{x}}$. Where B_0 is constant. T_w is

surface temperature. T_∞ is ambient temperature. The value of T_w is considered greater than T_∞ .

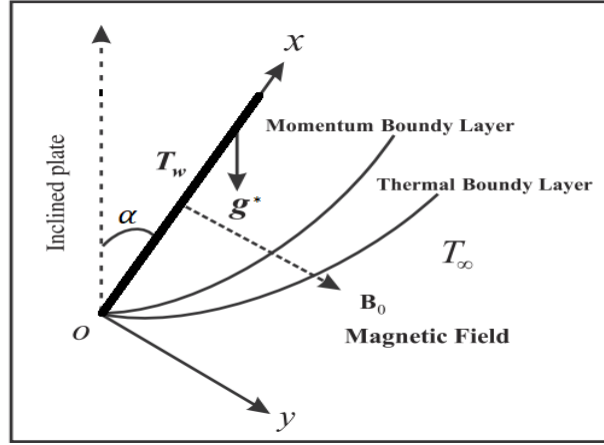


Figure1: Physical model and coordinates system of the flow

u, v are velocity components along x and y coordinates respectively. T is the temperature of the fluid.

The governing equations of the flow problem with appropriate boundary conditions can be stated as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B^2}{\rho} (u - U_\infty) + g^* \beta_t (T - T_\infty) \cos \alpha + \frac{\nu}{k_1} (u - U_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{Q_0}{\rho c_p} (T - T_\infty) + \sigma \frac{B^2}{\rho c_p} u^2 \quad (3)$$

Associated boundary conditions are:

$$u = N\nu_f \frac{\partial u}{\partial y}, \quad T = T_w + D \frac{\partial T}{\partial y}, \quad v = 0 \quad \text{at } y = 0 \quad (4)$$

$$u = U_\infty, T = T_\infty \quad \text{as } y \rightarrow \infty \quad (5)$$

u is the velocity of the fluid along x -direction and v is the velocity of the fluid in y -direction. T

is the fluid temperature. Where $N = N_1 x^{\frac{-n-1}{2}}$ is the velocity slip factor, $D = D_1 x^{\frac{-n-1}{2}}$ is the thermal slip factor. ν is kinematic viscosity. $U_\infty = U_f(x + b)^n$ is the free stream velocity, U_f is the constant, T_∞ is the free stream temperature, g^* is the acceleration due to gravity, β_t is coefficient of thermal expansion, μ is Dynamic coefficient of viscosity, α is the angle of

inclination, ρ is the density of the fluid, c_p is specific heat, λ is thermal conductivity, Q_0 is heat absorption coefficient, σ is the electrical conductivity of the fluid.

3. SIMILARITY TRANSFORMATION

$$\eta = y \sqrt{\frac{U_\infty}{2\nu x}}, \quad \xi = \sqrt{2\nu x U_\infty} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

where η similarity is variable, $f(\eta)$ is non-dimensional stream function, $\theta(\eta)$ is non-dimensional temperature.

Substituting similarity transformation into equations (1)-(3) including boundary conditions (4)-(5) obtained equations are:

$$f'''' + ff'' - Gr_t \theta \cos \alpha + \frac{2n}{n+1} \omega^2 f' + 2M(\omega - f') - Kf' = 0 \quad (7)$$

$$\theta'' + Pr f \theta' + Ec Pr (f''^2 + M f'^2) + Pr Q \theta = 0 \quad (8)$$

$\omega = \frac{U_f}{U_\infty}$ is the velocity parameter, $Gr_t = \frac{g^* \beta_t (T_w - T_\infty) 2x}{U_\infty^2}$ is local temperature grashof number, n is velocity power index, $M = \frac{\sigma B_0^2 2x}{\rho U_\infty}$ is magnetic parameter, $K = \frac{\nu}{k_1}$ is permeability parameter. $Ec = \frac{U_\infty^2}{c_p (T_w - T_\infty)}$ is the Eckert number, $Pr = \frac{\nu c_p}{k}$ is Prandtl number, $Q = \frac{Q_0 2x}{\rho c_p U_\infty}$ is the local heat generation parameter.

With boundary conditions:

$$f'(0) = 1 + \delta_1 f''(0), \quad \theta(0) = 1 + \delta_2 \theta'(0), \quad f(0) = 0. \quad \text{at } \eta = 0 \quad (9)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (10)$$

$\delta_1 = N_1 \sqrt{\frac{av(n+1)}{2}}$ is velocity slip parameter, $\delta_2 = D_1 \sqrt{\frac{a(n+1)}{2\nu}}$ is thermal slip parameter.

The skin friction coefficient

$$C_f = \frac{\tau_w}{\rho U_\infty^2}, \quad \text{where } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (11)$$

The Nusselt number

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \text{ where } q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (12)$$

$$C_f Re_x^{1/2} = f''(0) \quad (13)$$

$$Nu_x Re_x^{-1/2} = -\frac{1}{2} \theta'(0) \quad (14)$$

Where $Re_x = \frac{U_\infty 2x}{\nu}$ is the Local Reynolds number.

4. NUMERICAL METHOD

In order to solve the corresponding nonlinear differential equations with appropriate boundary conditions, it is required to convert BVP into the system of first-order IVP. Finite difference method is used to solve the equations (7)-(8) with boundary conditions (9)-(10).

$$f' = p, \theta' = s, \quad \theta'' = s' \quad (15)$$

$$f'' = p' = q$$

$$q' = -fq + Gr_t \theta \cos \alpha - \frac{2n}{n+1} \omega^2 p - 2M(\omega - p) + Kp \quad (16)$$

$$s' = -Prfs - EcPr(q^2 + Mp^2) - PrQ\theta \quad (17)$$

Under the boundary condition

$$p(0) = 1 + \delta_1 q(0), \theta(0) = 1 + \delta_2 s(0), f(0) = 0. \quad \text{at } \eta = 0 \quad (18)$$

$$p(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (19)$$

Appropriate initial guesses are taken for the iterative process. In the computational procedure, to satisfy the boundary conditions at the infinity, we have taken $\eta_{max} = 15$ or $\eta_\infty = 15$. The step size for the calculation was taken as $\nabla = 0.01$.

5. RESULTS AND DISCUSSION

In this flow problem, the effect of relevant parameters such as magnetic parameter (M), permeability parameter (K), Local temperature grashof number (Gr_t), Eckert number (Ec), Prandtl number (Pr), Heat generation parameter (Q). Angle of inclination (α), on velocity and temperature profile have been carried out. The results are displayed through graphs and tables. The initial guesses for the pertinent parameters were taken as $Gr_t = 6, n = -0.65, M = 0.2, \omega = 0.5, \alpha = \frac{\pi}{6}, K = 1, Pr = 0.7, Ec = 0.2, \delta_1 = 0.1, \delta_2 = 0.1$. These values are taken as common except variation in respective figures and tables.

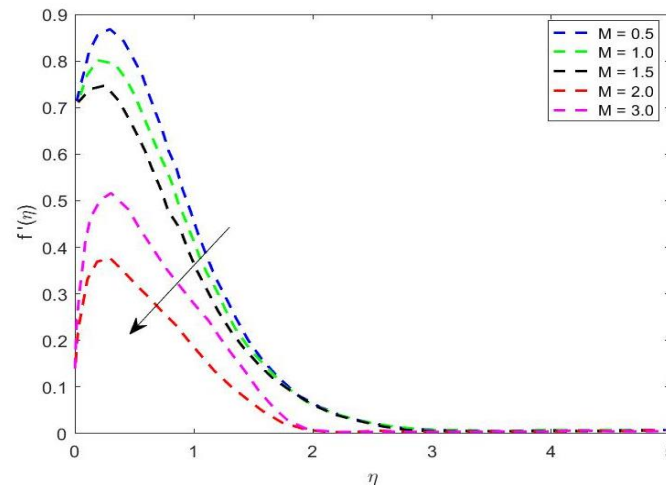


Figure2: Velocity profile versus M

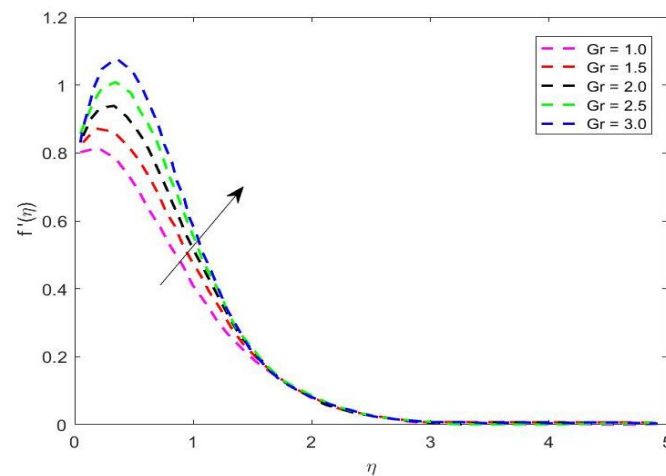


Figure3: Velocity profile versus Gr_t

Fig2 displays to scrutinize the variation in velocity profile due to magnetic parameter M . It is observed when magnetic field strength increases, the fluid velocity drops substantially. Due to the presence of magnetic field a resistive force occurs against the fluid motion, which causes the reduction in velocity of the fluid. **Fig3** Illustrate the behavior of velocity distribution for different values of local temperature grashof number (Gr_t). It is noticed that velocity enhances with the rising value of temperature grashof number (Gr_t). This phenomenon occurs by thermal buoyancy force which enhances the fluid velocity. The highest velocity observed near the boundary layer of the porous plate but slowly lessens to the free stream velocity.

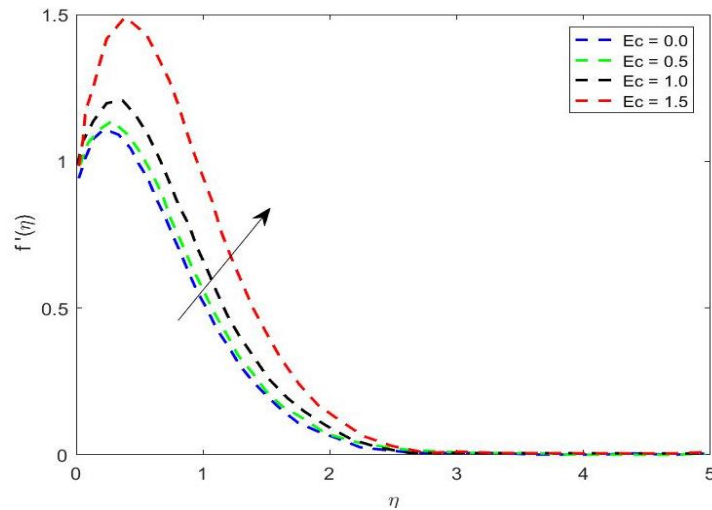


Figure4: Velocity profile versus Ec

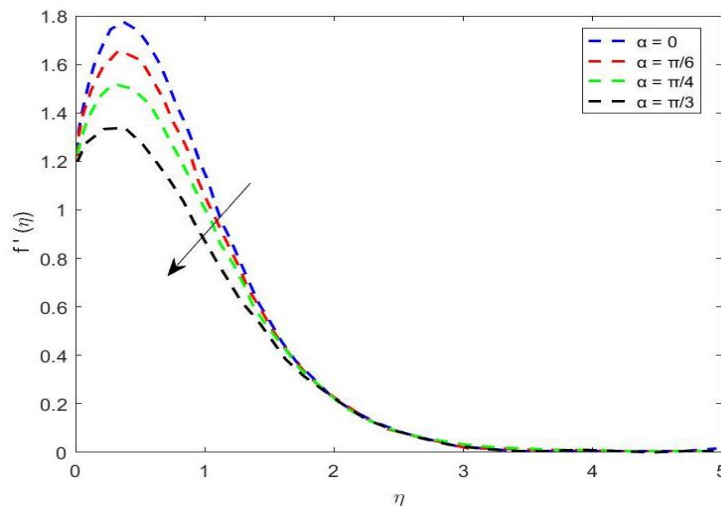


Figure5: Velocity profile versus α

Fig4 Drawn to analyze the influence of Eckert number (Ec). Ec is the mathematical relationship between the kinetic energy and fluid enthalpy. The kinetic energy is transformed into internal energy, which increases the fluid's velocity. **Fig5** demonstrate the velocity against the angle of the inclination α . Velocity profile is decreasing function for the rising value of α . Higher value of inclination angle maximize the impact of buoyancy force, therefore motion of the fluid decreases, as a result velocity profile also decreases. For $\alpha = 0$, buoyancy force is maximum that causes the higher free stream velocity.

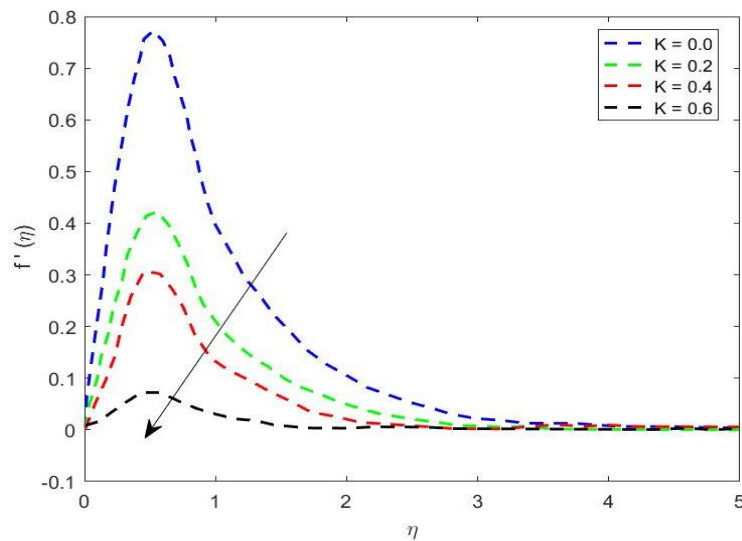


Figure6: Velocity profile versus K

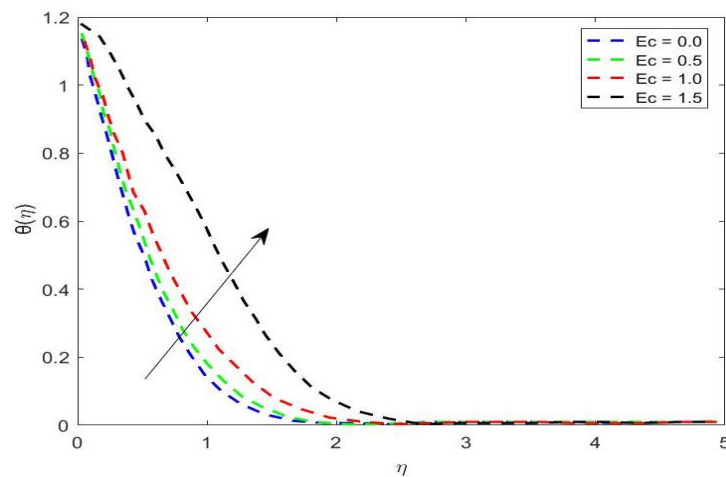


Figure7: Temperature profile versus Ec

Fig6 represents the influence of permeability parameter (K), resistance of porous medium decreases for the larger value of permeability parameter(K),which decreases the velocity profile.

Fig7 Depicts the variation in the temperature profile for various value of Eckert number (Ec). As seen escalating value of Ec enhances the temperature profile.

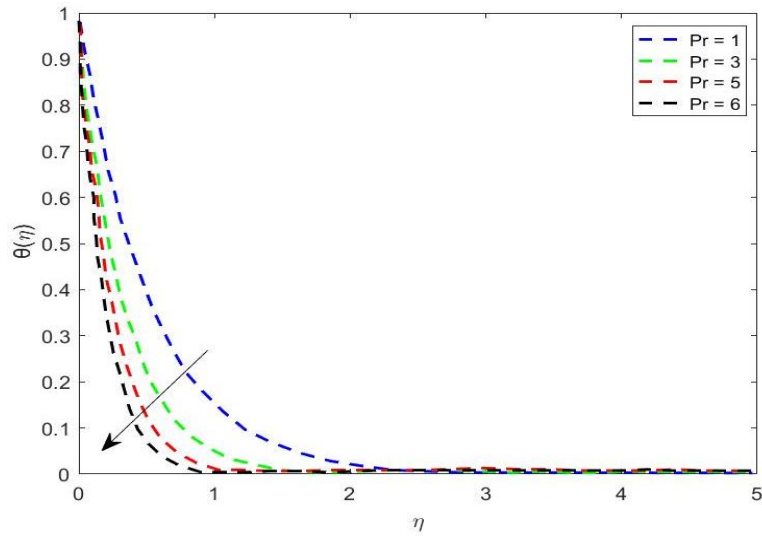


Figure 8: Temperature profile versus Pr

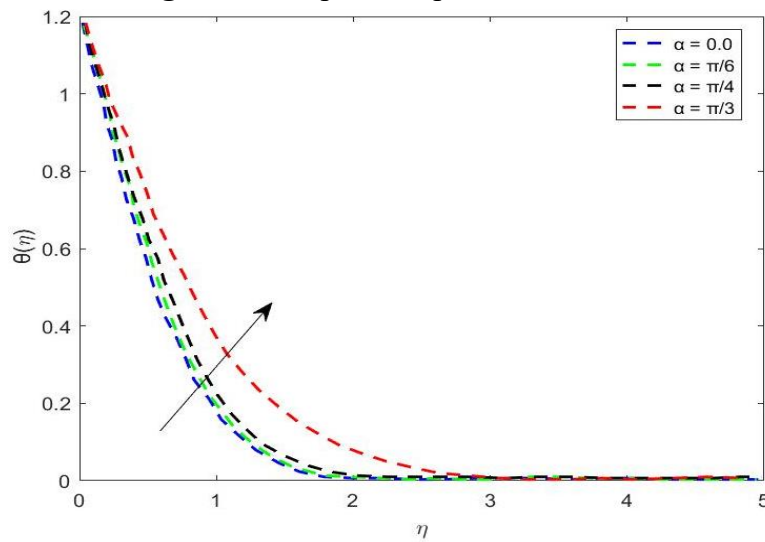


Figure9: Temperature profile versus α

Table1: Comparison of numerical values of local Nusselt number and skin fraction coefficient $Gr_t = 6$, $n = -0.65$, $\omega = 0.5$, $K = 1$, $Pr = 0.7$, $Ec = 0.2$.

					Present Result		Mandal et al.[5]	
M	Pr	δ_1	δ_2	α	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$
0.5	0.75	0.3	0.2	$\pi/6$	0.902920322	0.824503112	0.90292	0.82450
1	0.75	0.6	0.5	$\pi/6$	0.899965271	0.87451521	0.89996	0.87451
1.5	0.75	0.9	1.0	$\pi/6$	0.895731412	0.955973142	0.89573	0.95597

Fig8 plotted to observe the changes in temperature profile due to variation in Prandtl number (Pr).temperature profile depleted as the value of Pr increases, An increase in Pr indicates a decrease in thermal conductivity. As a result, we have noticed a decrease in the temperature profile.**Fig9** represents the effect of angle of inclination α on temperature profile. the temperature profile leads to increase as the angle of inclination increases. **Table 1** shows that the Local Nusselt number lessens for the enhancing value of velocity slip parameter but the Skin friction coefficient increases for higher value of Thermal slip.

6. CONCLUSION

The heat transfer and free stream boundary layer flow of nanofluid along an inclined plate is considered in the present study.Following conclusions are drawn from this study:

- Velocity profile decreases with the increasing value of Magnetic parameter (M).
- Velocity profile decreases as the inclination angle (α) increases but reverse phenomenon is observed in the Temperature profile.
- Enhancing the value of Eckert number (Ec) causes an increment in the temperature profile.
- Temperature profile lessens with the rising value of Prandtl number (Pr).

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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