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QUANTITATIVE ANALYSIS OF TRANSMISSION DYNAMIC OF BOKO HARAM FANATIC THROUGH MASS MEDIA

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Abstract. A deterministic model for the proliferation of fanatic ideology is designed and used to qualitatively assess the role of recruitment and retention in the Boko Haram formation. Here, contacts between individuals extend beyond their physical meaning to include the spread of ideas through phone conversations, media sources, emails, and other similar ways. Our research indicates that loss of immunity to fanaticism and the rate at which previously convicted and jailed fanatics (Boko Haram) members revert to being susceptible to fanaticism could induce backward bifurcation when the associated fanatic reproduction number, $R_0 = 1$; hence annihilation of fanaticism in the community will be arduous. This work focuses on the dynamics of the birthing of interest ideology using Mass Media and hence we explore questions such as, how do fanatics initiate within a community and how do they sustain themselves? The model shows that regulating the activities of the Mass media, more especially the social media will ameliorate and abridge the activities of the extremists alongside other measures.

Keywords: mathematical model; extremism; transmission dynamics; backward bifurcation; stability.

2020 AMS Subject Classification: 91D30.

1. Introduction

There are numerous heinous occurrences being perpetrated in many developing countries; ranging from indigence, poor health facilities, malnutrition, corruption and insurgency etc.

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These and other social vices bring out the survival instinct of human, which may drive them into erratic ideas which sometimes is not suitable to the basic principle of a country. In recent history armed organizations have been rampaging in different countries and Boko Haram (BH) is among the deadliest and most dreaded armed organisation rampaging West Africa especially [1]. The Jihadist group became violent in 2009 when their supposed leader was killed and since then, the causalities has geometrically risen over 40,000 and the displaced population is over 2.4 million people around Nigeria, Cameroon, Niger, and Chad [2]. Boko Haram has strategically meted their violent undertakings towards the traditional rulers, Sufi and Salafi religious Movements, civilian population and Nigerian state which the Jihadist terms as infidels, corrupted and illegitimate [3, 1]; it declares its own "state among the states of Islam" and sworn allegiance to the Islamic state in March 2015 [4]. The appreciation of the transmission dynamics of such groups increases the knowledge of the mechanism behind the evolution, vis a vis its eradication. Most often anti – extremist, anti – fanatic policy makers tends to formulate policies that has the capacity of depriving fanatic groups from its recruitment sources as well as discouraging its current members which in general is the most effective counter-terrorist approach [5]. The most peremptory area for the existence of such groups of fanatics are the recruitment and retention, hence it is significant to understand and Mathematically formulate the mechanism by which Boko Haram group establish a critical mass. The knowledge about Boko Haram's internal structure, organization, and its mobility is not very pronounced [1, 32]. Efforts to control the proliferation Boko Haram and its operation has been mounted by the Government, unfortunately it somewhat has not yielded the expected result because of its peculiarities. In this work we explored epidemiological model with homogeneous mixing to study the menace.

The study of dynamic human behaviour has been developed by different authors; consequently [6] used epidemiological models to characterize information Cascades in Twitter resulting from both News and Rumors. They used SEIZ model which is an enhanced epidemic model that apparently considered skeptics to characterize eight events across the World and spanning a range of event types. Epidemiological model was applied in this regard because of its potency to provide classical approach to study how information diffuses. The spread of extreme ideology was also modeled by [7]; their interest was to give a simple Mathematical model that could become a scientific back – up before intervention to control the spread of the extreme ideology is deployed in a community. A Mathematical model has been used in many areas; emotional state of a bipolar II disorder patient [28], COVID - 19 [18] and also used to consider effect of the pressure of an extreme ideological group on a society [31, 8]. The later is centered on the dynamical mathematical study of the ideological evolution of the population in a canton where some groups/organization want to get political recognition through bloodletting and the likes. A thorough work was done by Erika [9] on the mathematical study of two models that quantifies some of the theories of co-development and co - existence of focused groups in the society; sciences which is an extension of the foundation study done by Castillo – Chavez and Song [5]; her result showed that when more efforts is directed towards recruitment and retention of members, the more effective it will be in transmitting the ideology to the vulnerable individuals and thus converting them to believers.

Examples abound where individuals influenced one another either by coercion or maybe they have not met in person. These form of influence can happen as a result of abduction, reorientation by virtue of cohabitation of enforcement, via phone conversation and other electronic means. These are the type of contact we assumed in our work and thereby venture into developing some qualitative framework that might enable us suggest some strategic ways of combating Boko Haram and related fanatic groups. On the ground of the secretive nature of Boko Haram, its internal structure remains unknown, whether it has a centralised authority or decentralised cells [10]. Using disagregated data from the Armed Conflict location and Event Data Projects [11] on political violence in the regions, Prieto et al. [1] developed an algorithm that has the ability to detect the fragmentation of Boko Haram into several cells; they assumed that travel costs and reduced familiarity with unknown locations limit the mobility of the organization.

We study the spread of Boko Haram ideology as an epidemiological contact process, where interest group include children, socially active, fanatic groups, terrorist, religious group, union, informal social groups, political attuned and the likes. The transmission of the idea is not necessary on individual – to – individual process but rather it is a part of group effort. Our study does not explicitly incorporate heterogeneity; we are considering a homogeneous population, the recruitment in this work is due to attraction and coercion. We present a new deterministic model that studies the dynamics of recruitment and retention of Boko Haram group. The model treats fanaticism as a disease, hence the model could fit well for an epidemiological setting. Instead of studying the spread of the extreme ideology, we looked into the retention and two ways of recruitment either by coercion: abduction, kidnapping etc or mass media. Our model allows for changes in human behaviour where someone who was previously not a fanatic can be come fanatic and the other round. The article is organized as follows: The model is formulated in section 2, basic properties of the model is studied in section 3, asymptotic analysis of the model is done in section 4 and the bifurcation analysis is presented in section 5. The numerical simulation and discussion is done in section 6 and finally, conclusion along side some recommendation is presented in section 7.

2. Mathematical Model Formulation

We consider five mutually – exclusive compartments namely: susceptible individuals $S(t)$; exposed individuals by means of physical contact by birth and coercion either by abduction or kidnapping etc $E_1(t)$; individuals exposed through mass media $E_2(t)$; Boko Haram fanatics $F(t)$, in other words individuals who have been exposed, conceptualized and are fully committed to Boko Haram ideology and members of BH group who have been incarcerated and undergoing rehabilitation in the prison $R(t)$.The total population at time t, is denoted by $T(t)$, so that

$$T(t) = S(t) + E_1(t) + E_2(t) + F(t) + R(t) \tag{1}$$

BH fanatics interactions in the population is modeled using standard incidence function. The population of persons who are susceptible to BH ideology $S(t)$ is increased by the recruitment of immigrants who are susceptible into the population at the rate, Λ, exposed individuals who had physical contact with BH fanatics but due to their exposure or/and government interventions through reorientation and tactical programmes become susceptible again at the rate of γ_1; persons who was exposed through mass media but because of government and other NGOs efforts through these mass media become susceptible again at the rate of γ_2; convicted individuals who leave the prison after series of counseling and reorientation at the rate of ν, where $\frac{1}{\nu}$ is the average period of imprisonment. It is assumed that a fraction θ of νR remains susceptible, but the remaining fraction $(1-\theta)$ recidivate and gets involved in BH activities again; BH fanatics who decide to drop the ideology because of self realization, tension exerted by joint task force, government promises of amnesty etc to become susceptible again at the rate γ_3; we are dealing with imperfect society where not all persons who profess BH ideology faces conviction. The susceptible population is reduce by BH fanatics at the rate λ (the BH fanatics force of infection), where

$$\lambda = \frac{\beta F}{T} \tag{2}$$

and β is the fanatic transmission rate. The susceptible population is additionally reduced by natural death (at a rate μ, we assume that natural removal occurs in all compartment at this rate), so that

$$\frac{dS}{dt} = \Lambda - \lambda S + \gamma_1 E_1 + \gamma_2 E_2 + \theta \nu R + \gamma_3 F - \mu S$$

The population of persons who are exposed to BH ideology by physical contact is increased by engaging in fanatic interaction that affects and subsequently, susceptible persons are partially infected a the rate λ. The fraction α of λS came in physical contact with BH fanatics while the remaining fraction $(1-\alpha)$ are exposed by their potency of accessing a given mass media that BH fanatics use to recruit electronically. This population is decreased by individuals that conceptualize and bought the ideology at the rate of k_1 and hence develop into the ability of infecting others; individuals who recover from the disease at the rate γ_1 and natural death. This gives

$$\frac{dE_1}{dt} = \alpha \lambda S - (k_1 + \gamma_1 + \mu) E_1.$$

The population of persons who are exposed by mass media is given as

$$\frac{dE_2}{dt} = (1-\alpha)\lambda S - (k_2 + \gamma_2 + \mu) E_2,$$

where k_2 is the rate at which they become BH fanatics.

The population of individuals who are committed to the BH ideology is given by

$$\frac{dF}{dt} = k_1 E_1 + k_2 E_2 + (1-\theta)\nu R - (\gamma_3 + \rho + \delta + \mu) F$$

where ρ is the mortality rate of BH fanatics due to in –fighting, desertion, demotivation, splintering, suicide etc. The population of BH fanatics who were caught and convicted $R(t)$ is increased at δ which is the incarceration rate and is decreased by the recidivate rate and natural death. This gives

$$\frac{dR}{dt} = \delta F - (\nu + \mu)R.$$

From the above formulations and assumptions, the model for the prolifilation of BH ideology in a population consists the following system of non – linear differential equations

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \lambda S + \gamma_1 E_1 + \gamma_2 E_2 + \theta \nu R + \gamma_3 F - \mu S \\
\frac{dE_1}{dt} &= \alpha \lambda S - (k_1 + \gamma_1 + \mu) E_1 \\
\frac{dE_2}{dt} &= (1-\alpha)\lambda S - (k_2 + \gamma_2 + \mu) E_2 \\
\frac{dF}{dt} &= k_1 E_1 + k_2 E_2 + (1-\theta)\nu R - (\gamma_3 + \rho + \delta + \mu) F \\
\frac{dR}{dt} &= \delta F - (\nu + \mu) R
\end{aligned} \tag{3}$$

with initial conditions

$S(0) = S_0 > 0, E_1(0) = E_{10} \geq 0, E_2(0) = E_{20} \geq 0, F(0) = F_0 \geq 0, R(0) = R_0 \geq 0$. All parameters that appear in the (3) take non - negative numerical values.

3. Basic Properties

3.1. Positivity of Solutions and well – posedness of model.

Lemma 3.1.

The closed set $D = \{(S, E_1, E_2, F, R) \in \mathbb{R}^5_+ : 0 \leq N \leq \frac{\Lambda}{\mu}\}$ is positively invariant and attract all positive solution of (3).

Proof. Let $S_0, E_{10}, E_{20}, F_0, R_0 \geq 0, S_0 + E_{10} + E_{20} + F_0 + R_0 = T_0$. Then there exists solutions $S(t), E_1(t), E_2(t), F(t), R(t)$ for the dynamical system (3), with initial value $S_0, E_{10}, E_{20}, F_0, R_0$ at time $t = 0$, that are defined for all $t \geq 0$. Clearly, $S(t), E_1(t), E_2(t), F(t)$, and $R(t)$ are nonnegative and $S(t) + E_1(t) + E_2(t) + F(t) + R(t) = T(t)$ for all t. Hence it follows that

$$\frac{dT}{dt} = \Lambda - \mu T - \rho F.$$

Therefore, $\frac{dT}{dt} \leq \Lambda - \mu T$, which gives that

$$Tt \leq T(0)e^{\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t}) \leq \frac{\Lambda}{\mu} + (T(0) - \frac{\Lambda}{\mu})e^{-\mu t}.$$

Since $T(0) \leq \frac{\Lambda}{\mu}$ then $T(t) \leq \frac{\Lambda}{\mu}$. Hence D is positively invariant and an attractor so that no solution path leaves the boundary. □

Lemma 3.2.

The region D is positively – invariant for the model (3) and , if $F_0 > 0$, then $S(t) > 0, E_1(0) > 0, E_2(0) > 0$ and $F(0) > 0$ for all $t > 0$ and R is bounded by $\hat{R} = \max\{R_0, \frac{\delta}{\upsilon+\mu}\}$.

Proof. The RHS of (3) is continuously differentiable and hence it is locally Lipschitz; hence there exists a unique solution $S(t), E_1(t), E_2(t), F(t), R(t)$ for system (3) with initial value $S_0, E_{10}, E_{20}, F_0, R_0$ that is defined on a maximal forward interval of existence. Consider the set $D \subset \mathbb{R}^5_+$, we have that $F(0) \geq 0$, in system (3) let $W_1 = k_1 + \gamma_1 + \mu, W_2 = k_2 + \gamma_2 + \mu, W_3 = \gamma_3 + \rho + \delta + \mu$ and $W_4 = \nu + \mu$ then we have that

$$E_1(t) = E_{10} \exp \int_0^{\infty} (\frac{\alpha\beta S}{T} - W_1) dt,$$

$$E_2(t) = E_{20} \exp \int_0^{\infty} (\frac{(1-\alpha)\beta S}{T} - W_2) dt,$$

$$F(t) = \Big[F_0 + \int_0^t (k_1 E_1(\tau) + k_2 E_2(\tau) + (1-\theta)\upsilon R(\tau)) e^{W_3(\tau - t_0)} d\tau\Big] e^{W_3(t_0 - t)},$$

$$R(t) = \Big[R_0 + \int_0^t \delta F(\tau) e^{W_4(\tau - t_0)} d\tau\Big] e^{W_4(t_0 - t)}.$$

So that $S(t) \geq 0, E_1(t) \geq 0, E_2(t) \geq 0, F(t) \geq 0, R(t) \geq 0$, for all $t > 0$. For boundedness of R, we show that if $R(t) \leq r, \forall t \geq 0$ and $r \geq \frac{\delta}{W_4}$, then $R_0 \leq r$. From (3) we have

$$\tfrac{dR(t_1)}{dt} = \delta F(t_1) - W_4 R(t_1) \leq \delta(F(t_1 - 1) \leq 0,$$

since $(F(t_1) - 1) < 0$ and $\delta > 0$. This is a contradiction which implies that $R(t) < r \forall t \geq 0$. Suppose that $R(0) > r \geq \frac{\delta}{W_4}$. To show that $Rt \leq R(0) \forall t \geq 0$, we assume that the last inequality does not hold. Hence there exists a $t_2 > 0$ such that $R(t_2 \geq R(0)$ and $R'(t_2) > 0$. However, since $R(t_2) > \frac{\delta}{W_4}$, then

$$\tfrac{dR(t_2)}{dt} = \delta F(t_1) - W_4 R(t_2) \leq \delta(F(t_2 - 1) \leq 0,$$

but $R'(t_2) > 0$. Hence we reach a contradiction and $R(t)$ is bounded above by $\hat{R}$ where $\hat{R} = \max\{R_0, \frac{\delta}{W_4}\}$. Which guarantees the positivity of the different groups making up the total population S, E_1, E_2, F, and R for all time $t \geq 0$. □

4. Asymptotic Analysis of the Model

4.1. Reproduction number of the menace. The fanatic – free equilibrium (FFE) of the model (3) is given as

$$E_0 = (S^0, E_1^0, E_2^0, F^0, R^0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right) \tag{4}$$

We use Next Generation Matrix [12] which comprises of two parts: F and V^{-1};

$$R_0 = \rho F V^{-1}) \tag{5}$$

where $F = \frac{\partial f_i(x_0)}{\partial x_j}, V = |\frac{\partial V_i(x_0)}{\partial x_j}|, \rho$ is the spectral radius. We define

$$f_i = \begin{bmatrix} \alpha\beta\frac{F}{T}S \\ (1-\alpha)\beta\frac{F}{T}S \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

and

$$V_i = \begin{bmatrix} W_1 E_1 \\ W_2 E_2 \\ -k_1 E_1 - k_2 E_2 - (1-\theta)\upsilon R + W_3 F \\ -\delta F + W_4 R. \end{bmatrix} \tag{7}$$

.

Differentiating (6) with respect to the outlined variables in (3), we have:

$$F = \begin{bmatrix} 0 & 0 & \alpha\beta\frac{S}{T} & 0 \\ 0 & 0 & (1-\alpha)\beta\frac{S}{T} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8}$$

and, performing the same derivative operation on (7), we have

$$V = \begin{bmatrix} W_1 & 0 & 0 & 0 \\ 0 & W_2 & 0 & 0 \\ -k_1 & -k_2 & W_3 & -(1-\theta)\upsilon \\ 0 & 0 & -\delta & W_4 \end{bmatrix}. \tag{9}$$

Hence we have that

$$V^{-1} = \begin{bmatrix} W_1^{-1} & 0 & 0 & 0 \\ 0 & W_2^{-1} & 0 & 0 \\ \frac{k_1 W_4}{W_1\left(-(1-\theta)\delta+W_3W_4\right)} & \frac{k_2 W_4}{W_2\left(-(1-\theta)\delta+W_3W_4\right)} & \frac{W_4}{-(1-\theta)\delta+W_3W_4} & \frac{1-\theta}{-(1-\theta)\delta+W_3W_4} \\ \frac{\delta k_1}{W_1\left(-(1-\theta)\delta+W_3W_4\right)} & \frac{\delta k_2}{W_2\left(-(1-\theta)\delta+W_3W_4\right)} & \frac{\delta}{-(1-\theta)\delta+W_3W_4} & \frac{W_3}{-(1-\theta)\delta+W_3W_4} \end{bmatrix}.$$

Therefore

(10)

$$FV^{-1} = \begin{bmatrix} \frac{\alpha\beta k_1 W_4 S}{W_2\left(-(1-\theta)\delta+W_3W_4\right)T} & \frac{\alpha\beta k_2 W_4 S}{W_2\left(-(1-\theta)\delta+W_3W_4\right)T} & \frac{\alpha\beta W_4 S}{-(1-\theta)\delta+W_3W_4T} & \frac{(1-\theta)\alpha\beta S}{-(1-\theta)\delta+W_3W_4T} \\ \frac{(1-\alpha)\beta k_1 W_4 S}{W_1\left(-(1-\theta)\delta+W_3W_4\right)T} & \frac{(1-\alpha)\beta k_2 W_4 S}{W_2\left(-(1-\theta)\delta+W_3W_4\right)T} & \frac{(1-\alpha)\beta W_4 S}{-(1-\theta)\delta+W_3W_4T} & \frac{(1-\alpha)(1-\theta)\beta S}{-(1-\theta)\delta+W_3W_4T} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

such that the spectral radius of (10) given as $\rho(FV^{-1}) = R_0$ is

$$R_0 = \frac{\left(\alpha k_1 W_2 + (1-\alpha)k_2 W_1\right)\beta W_4 S}{W_1 W_2\left(-(1-\theta)\delta + W_3 W_4\right)T}. \tag{11}$$

At Fanatic – Free Equilibrium, we have

$$R_0 = \frac{\left(k_2 W_1 + (k_1 W_2 - k_2 W_1)\alpha\right)\beta W_4}{W_1 W_2\left((\theta-1)\delta + W_3 W_4\right)}. \tag{12}$$

4.2. Local Stability of FFE.

Lemma 4.1. *Stability Of FFE*

The FFE (4) of the model (3) is locally asymptotical stable if $R_0 < 1$.

Proof. The Jacobian matrix of (3) at (4) is given as:

$$J(E_0) = \begin{bmatrix} -\mu & \gamma_1 & \gamma_2 & -\beta+\gamma_3 & \theta\upsilon \\ 0 & -W_2 & 0 & \alpha\beta & 0 \\ 0 & 0 & -W_2 & (1-\alpha)\beta & 0 \\ 0 & k_1 & k_2 & -W_3 & (1-\theta)\upsilon \\ 0 & 0 & 0 & \delta & -W_4 \end{bmatrix}. \tag{13}$$

To obtain the characteristic equation, we simplify

$$\left|J(E_0) - \lambda I\right| = 0, \tag{14}$$

which is equivalent to

$$\begin{bmatrix} -\mu-\lambda & \gamma_1 & \gamma_2 & -\beta+\gamma_3 & \theta\upsilon \\ 0 & -W_1-\lambda & 0 & \alpha\beta & 0 \\ 0 & 0 & -W_2-\lambda & (1-\alpha)\beta & 0 \\ 0 & k_1 & k_2 & -W_3-\lambda & (1-\theta)\upsilon \\ 0 & 0 & 0 & \delta & -W_4-\lambda \end{bmatrix} = 0.$$

Hence

$$(-\mu-\lambda)\Big[-\upsilon\delta(1-\theta)\Big(W_1W_2+(W_1+W_2)\lambda+\lambda^2\Big) \quad + \\ (-W_4-\lambda)\Big[(-W_2-\lambda)\Big(W_1W_3(W_1+W_3)\lambda-\alpha\beta k_1\Big) \quad + \quad k_2\beta(1-\alpha)(W_1+\lambda)\Big]\Big] = 0.$$

The first eigenvalue is $\lambda = -\mu$, the remaining can be obtained from

$$\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0, \tag{15}$$

where

$$A_1 = W_1+W_2+W_3+W_4,$$
$$A_2 = (W_1+W_2+W_3)W_4 + W_1(W_2+W_3) + W_2W_3 - \alpha\beta k_1 - (1-\alpha)\beta W_1 k_2 - \upsilon\delta(1-\theta),$$
$$A_3 = W_1(W_2W_4+W_3W_4+W_2W_3) - \alpha\beta k_1(W_4+W_2) - (1-\alpha)\beta k_2(W_4+W_1) + W_2W_3W_4$$
$$-\upsilon\delta(1-\theta)(W_1+W_2),$$
$$A_4 = W_4\Big(W_1W_2W_3 - \alpha\beta k_1 W_2 - (1-\alpha)\beta k_2 W_1\Big) - \upsilon W_1W_2\delta(1-\theta).$$

If all the remaining eigenvalues of (15) are negative, then we can say that (4) is asymptotically stable; to determine the stability of (15) we make use of the the theorem below.

Theorem 4.2.

Let $R_0 < 1$ if

(i.) $A_1 > 0, A_4 > 0$

(ii.) $A_1 A_2 - A_3 > 0$

(iii.) $A_3(A_1 A_2 - A_3) - A_1^2 A_4 > 0$

then FFE is locally asymptotically stable.

Proof. By Routh - Hurwitz criterion all roots of (15) have negative real parts if the following conditions hold

$$A_1 > 0, A_4 > 0, \begin{vmatrix} A_1 & 1 \\ A_3 & A_2 \end{vmatrix} > 0, \begin{vmatrix} A_1 & 1 & 0 \\ A_3 & A_2 & A_1 \\ 0 & A_4 & A_3 \end{vmatrix} > 0.$$

Since $W_1 > 0, W_2 > 0, W_3 > 0$ and $W_4 > 0$, then $A_1 > 0$.

$$\begin{aligned} A_4 &= W_4\Big(W_1 W_2 W_3 - \alpha\beta k_1 W_2 - (1-\alpha)\beta k_2 W_1\Big) - \upsilon W_1 W_2 \delta(1-\theta) \\ &= W_1 W_4\Big((W_2 W_3 + \alpha\beta k_2) - (\alpha\beta k_1 \tfrac{W_2}{W_1} + \beta k_2 + \upsilon \tfrac{W_2}{W_4)} \delta(1-\theta))\Big) \end{aligned}.$$

Observe that $0 < \frac{W_2}{W_1} < 0$ and $0 < \frac{W_2}{W_4} < 1$, so it is glaring that $(W_2 W_3 + \alpha\beta k_2) > (\alpha\beta k_1 \frac{W_2}{W_1} + \beta k_2 + \upsilon \frac{W_2}{W_4})$, hence $A_4 > 0$. Clearly $A_2 > 0$ and $A_3 > 0$. After long simplification, it is observed that $A_1 A_2 - A_3 > 0$ and $A_3(A_1 A_2 - A_3) - A_1^2 A_4 > 0$. Hence by Routhwitz criterion, all roots of (15) have negative real parts. □

Conclusively, all the eigenvalues of (13) are negative, hence (4) is locally asymptotically stable. □

4.3. Global Stability of FFE:.

We study the global stability of E_0 by construction of a Lyapunov function of (3).

Theorem 4.3.

The FFE of the model (3) is globally asymptotically stable if $R_0 < 1$.

Proof. we select the infected classes $E_1(t), E_2(t), F(t)$ of the population to construct a Lyapunov function $L(t)$ such that

$$L(t) = x_1E_1(t) + x_2E_2(t) + x_3F(t). \tag{16}$$

$L(t)$ is positive definite, hence is a justifiable Lyapunov function and $x_1 > 0, x_2 > 0$ and $x_3 > 0$. The time derivative of (16) gives

$$\dot{L}(t) = x_1\dot{E}_1(t) + x_2\dot{E}_2(t) + x_3\dot{F}(t);$$

then substituting appropriately we have

$$\begin{aligned} \dot{L}(t) &= x_1(\alpha\lambda S - W_1E_1) + x_2\Big((1-\alpha)\lambda S - W_2E_2\Big) + x_3(k_1E_1 - W_3F \\ &= \lambda S\Big(\alpha x_1 + (1-\alpha)x_2\Big) - E_1(x_1W_1 - k_1x_3) - E_2(x_2W_2 - k_2x_3) - X_3W_3F. \end{aligned} \tag{17}$$

We set coefficient of λS to the numerator of R_0, coefficient of F to the denominator of R_0 and coefficents of E_1 and E_2 is set to zero. Hence, we have that

$$\begin{aligned} \alpha x_1 + (1-\alpha)x_2 &= \Big(k_2W_1 + (k_1W_2 - k_2W_1)\alpha\Big)W_4 \\ x_3W_3 &= W_1W_2\Big((\theta-1)\delta + W_3W_4\Big) \\ x_1W_1 - k_1x_3 &= x_2W_2 - k_2x_3 = 0. \end{aligned}$$

Solving, we have

$$\begin{cases} x_3 = \dfrac{W_1W_2}{W_3}\Big((\theta-1)\delta + W_3W_4\Big), \\ x_2 = \frac{k_2W_1}{W_3}\Big((\theta-1)\delta + W_3W_4\Big), \\ x_1 = \frac{1}{\alpha}\Big[\Big(k_2W_1 + (k_1W_2 - k_2W_1)\alpha\Big) - (1-\alpha)\frac{k_2W_1}{W_3}\Big((\theta-1)\delta + W_3W_4\Big)\Big]. \end{cases} \tag{18}$$

Substituting (18) into (17), we have

$$\begin{aligned} \dot{L}(t) &= \lambda S\{\Big[\Big(k_2W_1 + (k_1W_2 - k_2W_1)\alpha\Big) - (1-\alpha)\frac{k_2W_1}{W_3}\Big((\theta-1)\delta + W_3W_4\Big)\Big] \\ &\quad + (1-\alpha)\frac{k_2W_1}{W_3}\Big((\theta-1)\delta + W_3W_4\Big)\} - W_1W_2\Big((\theta-1)\delta + W_3W_4\Big)F \\ &= \beta\frac{F}{T}S\Big(k_2W_1 + (k_1W_2 - k_2W_1)\alpha\Big)W_4 - W_1W_2\Big((\theta-1)\delta + W_3W_4\Big)F \\ &= F\Big[\beta W_4(k_2W_1 + (k_1W_2 - k_2W_1)\alpha) - W_1W_2\Big((\theta-1)\delta + W_3W_4\Big)\Big], \end{aligned}$$

since $S \leq N$ in the domain of the invariant set. Hence we have that

$$\dot{L}(t) = W_1W_2\Big((\theta-1)\delta + W_3W_4\Big)(R_0 - 1)F.$$

Therefore $\dot{L}(t)$ is semi – negative definite in the invariant set if $R_0 < 1$ on the set under consideration i.e. $\dot{L}(t) \leq 0$ for $R_0 < 1$. By LaSalle's Invariance Principle [13] E_0 is globally asymptotically stable. □

5. BACKWARD BIFURCATION ANALYSIS

The phenomenon of backward bifurcation occurs in models that have multiple endemic equilibria when $R_0 < 1$ [14, 15, 16] . In this case, the classical epidemiological prerequisite of having $R_0 < 1$, is necessary but no longer sufficient for effective disease management, control or elimination. Knowing the parameters that will cause bifurcation will go a long way in determining factors that could hinder efforts in tackling fanaticism in the population. We claim the following result. In order to study the existence of backward bifurcation of model (3), we need to introduce the following results which are given by [5] Consider the general system of ordinary differential equations with a parameter ϕ:

$$\frac{dx(t)}{dt} = f(x, \phi) \tag{19}$$

where function $f(x, \phi) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ with $f \in C^2(\mathbb{R}^n \times \mathbb{R})$. Assume that $x = 0$ is a steady state of system (19), ie $f(0, \phi) \equiv 0$ for all ϕ. Let $Q = \left(\frac{\partial f_i}{\partial x_j}(0,0)\right)$ be the Jacobian of $f(x, \phi)$ at $(0,0)$.

Lemma 5.1. *The model (3) exhibits backward bifurcation at $R_0 = 1$ if $c > 0$. If $c < 0$, then model (3) exhibits a forward bifurcation when $R_0 = 1$, where $c = \frac{k_1}{W_1} - \frac{k_2}{W_2}\left(1 - \dfrac{1}{\alpha\beta}\right)$.*

Proof. Let $E_* = (S^*, E_1^*, E_2^*, F^*, R^*)$ represents any arbitrary endemic equilibrium of (3). Using center manifold theory [5] we shall prove the existence of backward bifurcation. Let $x_1 = S, x_2 = E_1, x_3 = E_2, x_4 = F, x_5 = R$ and $\lambda = \frac{\beta x_4}{N}$. We can rewrite (3) in the form of $\frac{dX}{dt} = F(X)$, with $F = (f_1, f_2, f_3, f_4, f_5)^T$ and $X = (x_1, x_2, x_3, x_4, x_5)^T$; hence we have

$$\begin{aligned}
f_1 &\equiv x_1' = \Lambda - \lambda S + \gamma_1 E_1 + \gamma_2 E_2 + \theta \nu R + \gamma_3 F - \mu S,\\
f_2 &\equiv x_2' = \alpha \lambda S - W_1 E_1,\\
f_3 &\equiv x_3' = (1-\alpha)\lambda S - W_2 E_2,\\
f_4 &\equiv x_4' = k_1 E_1 + k_2 E_2 + (1-\theta)\nu R - W_3 F,\\
f_5 &\equiv x_5' = \delta F - W_4 R,\\
N &= x_1 + x_2 + x_3 + x_4 + x_5.
\end{aligned}$$

Choosing $\beta = \beta^*$ as a bifurcation parameter. Solving for $\beta = \beta^*$ from $R_0 = 1$ gives

$$\beta^* = \frac{W_1 W_2 \left[\left(\frac{\theta - 1}{W_4} \right) \delta + W_3 \right]}{k_2 W_1 + \alpha (k_1 W_2 - k_2 W_1)}.$$

The Jacobian matrix of the transformed system, evaluated at the fanatic - free equilibrium with $\beta = \beta^*$, is given by

$$J^* = J(E_0)\mid_{\beta=\beta^*} = \begin{bmatrix} -\mu & \gamma_1 & \gamma_2 & \beta^*\gamma_3 & \theta\upsilon \\ 0 & -W_1 & 0 & \alpha\beta^* & 0 \\ 0 & 0 & -W_2 & (1-\alpha)\beta^* & 0 \\ 0 & k_1 & k_2 & -W_3 & (1-\theta)\upsilon \\ 0 & 0 & 0 & \delta & -W_4 \end{bmatrix}.$$

The matrix J^* has a right eigenvector given by $w = (w_1, w_2, w_3, w_4, w_5)^T$ where

$$\begin{aligned} w_1 &= \tfrac{\gamma_1 w_2 + \gamma_2 w_3 + (\beta^* + \gamma_3) w_4 + \theta\upsilon w_5}{\mu}; \quad w_2 = \tfrac{\alpha\beta^* w_4}{W_1}; \\ w_3 &= \tfrac{(1-\alpha)\beta^* w_4}{W_2}; \quad w_4 = \tfrac{k_1 w_2 + k_2 w_3 + (1-\theta)\upsilon w_5}{W_3}; \quad w_5 = \tfrac{\delta w_4}{W_4}. \end{aligned} \tag{20}$$

We assume that $w_4 > 0$ for all nonnegative parameters, then w_1 can be transform in terms of w_4 to get

$$w_1 = \frac{1}{\mu}\Big[\Big(\frac{\gamma_1\alpha}{W_1} + \frac{\gamma_2(1-\alpha)}{W_2} + 1\Big)\beta^* + \gamma_3 + \frac{\theta\upsilon\delta}{W_4}\Big] w_4.$$

Moreover, the left eigenvector $v = (v_1, v_2, v_3, v_4, v_5)$ associated with the zero eigenvalue satisfying $w \cdot v = 1$ is given by

$$\begin{aligned} v_1 &= 0; \quad v_2 = \tfrac{k_1 v_4}{W_1}; \quad v_3 = \tfrac{k_2 v_4}{W_2}; \\ v_4 &= \tfrac{\alpha\beta^{ast} v_2 + (1-\alpha)\beta^* v_3 + \delta v_5}{W_3}; \quad v_5 = \tfrac{(1-\theta)\upsilon v_4}{W_4}. \end{aligned}$$

Assuming that v_4 is positive, we calculate the partial derivatives at FFE, we obtain

$$\begin{aligned} \tfrac{\partial^2 f_1}{\partial x_1 \partial x_4} &= \tfrac{\partial^2 f_1}{\partial x_4 \partial x_1} = -\beta, \quad \tfrac{\partial^2 f_2}{\partial x_1 \partial x_4} = \tfrac{\partial^2 f_2}{\partial x_4 \partial x_1} = \alpha\beta, \\ \tfrac{\partial^2 f_3}{\partial x_1 \partial x_4} &= \tfrac{\partial^2 f_3}{\partial x_4 \partial x_1} = (1-\alpha)\beta; \quad \tfrac{\partial^2 f_2}{\partial x_4 \partial \beta} = \alpha; \quad \tfrac{\partial^2 f_3}{\partial x_4 \partial \beta} = 1-\alpha; \end{aligned}$$

all the other second - order partial derivatives are equal to zero. Furthermore, to compute for the coefficients a and b we make use of only the nonzero and positive derivatives for the terms $\frac{\partial^2 f_k}{\partial x_i \partial x_j}(E_0)$ and $\frac{\partial^2 f_k}{\partial x_i \partial \beta}(E_0)$, it follows that

$$a = 2 v_2 w_1 w_4 \frac{\partial^2 f_2}{\partial x_1 \partial x_4}(E_0) + 2 v_3 w_1 w_4 \frac{\partial^2 f_3}{\partial x_1 \partial x_4}(E_0)$$

and

$$b = v_2 w_4 \frac{\partial^2 f_2}{\partial x_4 \partial \beta}(E_0) + v_3 w_4 \frac{\partial^2 f_3}{\partial x_4 \partial \beta}(E_0).$$

Hence, we have that

$$a = \frac{2\alpha\beta^* v_4 w_4^2}{\Lambda}\Big[\Big(\frac{\gamma_1\alpha}{W_1} + \frac{(1-\alpha)\gamma_2}{W_2} + 1\Big)\beta^* + \gamma_3 + \frac{\theta\upsilon\delta}{W_4}\Big]\Big[\frac{k_1}{W_1} - \frac{k_2}{W_2}\Big(1 - \frac{1}{\alpha\beta^*}\Big)\Big], \tag{21}$$

$$b = \alpha v_4 w_4 > 0. \tag{22}$$

Since the coefficient b is always positive so that, according to Lemma 1 in [15], model (3) undergoes a backward bifurcation if the coefficient a is positive (i.e., $c > 0$). In other hand, model (3) undergoes a forward bifurcation if the coefficient a is negative (i.e., $c < 0$). This completes the proof. □

5.1. Possibility of Existence and Stability of Fanatic Endemic State (FEE). Let $E_* = (E^*, E_1^*, E_2^*, F^*, R^*)$ represent any arbitrary FEE of model (3). The equations in (3) are solved in terms of the force of infection at equilibrium point to give

$$\begin{aligned} &S^* = \frac{W_1W_2}{\lambda}\left(\frac{\Lambda + (\frac{\theta\upsilon\delta}{W_4} + \gamma_3)F}{W_1W_2(1+\mu) - \alpha(W_2 - W_1) - W_1}\right); E_1^* = \alpha W_2\left(\frac{\Lambda + (\frac{\theta\upsilon\delta}{W_4} + \gamma_3)F}{W_1W_2(1+\mu) - \alpha(W_2 - W_1) - W_1}\right);\\ &E_2^* = (1-\alpha)W_1\left(\frac{\Lambda + (\frac{\theta\upsilon\delta}{W_4} + \gamma_3)}{W_1W_2(1+\mu) - \alpha(W_2 - W_1) - W_1}\right); R^* = \frac{\delta}{W_4}F;\\ &F^* = \frac{\Lambda\Big(\alpha(k_1W_2 - k_2W_1) + k_2W_1\Big)}{\Big(W_3 - (1+\theta)\frac{\upsilon\delta}{W_4}\Big)\Big(W_1W_2(1+\mu) - \alpha(W_2 - W_1) - W_1\Big) - \Big(\frac{\theta\upsilon\delta}{W_4} + \gamma_3\Big)\Big(\alpha(k_1W_2 - k_2W_1) + k_2W_1\Big)}. \end{aligned} \tag{23}$$

Substituting F^* and N^* into (3) we have

$$M\lambda^{*2} - N\lambda^* = 0, \tag{24}$$

where

$$\begin{aligned} &M = W_4D\Lambda + \Big(W_4DA + B(\delta + W_4)\Big)F^*, N = (\beta BW_4 - W_1W_2W_4A)F^* - W_1W_2W_4\Lambda,\\ &A = \frac{\theta\upsilon\delta}{W_4} + \gamma_3, B = W_1W_2(1+\mu) - \alpha W_2 + (1-\alpha)W_1, D = \alpha W_2 + (1-\alpha)W_1; \end{aligned}$$

therefore, by the Descartes rule of signs a unique Fanatic endemic equilibrium exist i.e. $\lambda^* > 0$, when $R_0 > 1$ in as much as $M > 0$. Clearly, if $k_1W_2 > k_2W_1$, then $M > 0$ and there exists a unique fanatic endemic equilibrium for this case. Thus, we establish the following result.

Lemma 5.2. *The model (3) has a unique endemic equilibrium when $m_k < 0$, whenever $R_0 > 1$, where $m_k = k_1W_2 - k_2W_1$.*

Proof. Choosing β as the bifurcation parameter. Then $R_0 = 1$ is equivalent to

$$\beta = \beta^* = \frac{W_1W_2\left(\frac{\theta-1}{W_4}\delta + W_3\right) + k_2W_1}{\alpha(k_1W_2 - k_2W_1)}.$$

By a similar calculation as in Lemma (5.1), we can compute the right eigenvector associated with the zero eigenvalue, denoted by $w = (w_1, w_2, w_3, w_4, w_5)^T$, is given as

$$w_1 = \frac{1}{\mu}\left[\left(\frac{\gamma_1\alpha}{W_1} + (1-\alpha)\frac{\gamma_2}{W_2}\right)\beta^* + \gamma_3 + \frac{\theta\upsilon\delta}{w_4}\right]; \quad w_2 = \alpha\beta^*\frac{w_4}{W_1}; w_3 = (1-\alpha)\beta^*\frac{w_4}{W_2}, w_4 > 0; w_5\frac{\delta w_4}{W4};$$

and the corresponding left eigenvector, given by $v = (0, v_2, v_3, v_4, v_5)$

$$v_2 = \frac{k_1v_4}{W_1}, v_3 = \frac{k_2v_4}{W_2}, v_4 > 0, v_5 = \frac{(1-\theta)\upsilon}{W_4}v_4.$$

Furthermore, we obtain that

$$a = \frac{2\alpha\beta^* v_4 w_4^2}{\Lambda}\left[\left(\frac{\gamma_1\alpha}{W_1} + \frac{(1-\alpha)^2}{W_2} + 1\right)\beta^* + \gamma_3 + \frac{\theta\upsilon\delta}{W_4}\right]\left[\frac{R_0}{W_4}\Big((\theta-1)\delta + W_3W_4\Big)\right]$$

and

$$b = \alpha v_4 w_4 > 0.$$

Hence the fanatic endemic equilibrium shall exist and persist as long as $m_k < 0$. □

6. Numerical Simulation

We used MATHEMATICA to get the numerical solution of our model by applying NDSolve that uses Adams- type predictor-corrector method and leaves our result as interpolating function objects.

TABLE 1. Parameter values in model.

Parameters	Values of Parameter yr^{-1}	Source
Λ	0.029	[19]
μ	0.18	[11]
α	0.2	[17]
ρ	0.001	[17]
β	0.3	[8]
γ_1	0.26	[9]
γ_2	0.27	[9]
θ	0.8	[17]
γ_3	0.03	[8]
k_1	0.08	[8]
k_2	0.3	Estimated
δ	0.01	[17]
v	0.1	[17]

TABLE 2. Table of initial values

Variable at initial Value	Value	Source
$S(0)$	10000	Esttimated based on data from [11]
$E_1(0)$	1200	Esttimated based on data from [11]
$E_2(0)$	2000	Esttimated based on data from [11]
$F(0)$	800	Esttimated based on data from [11]
$R(0)$	800	Esttimated based on data from [11]

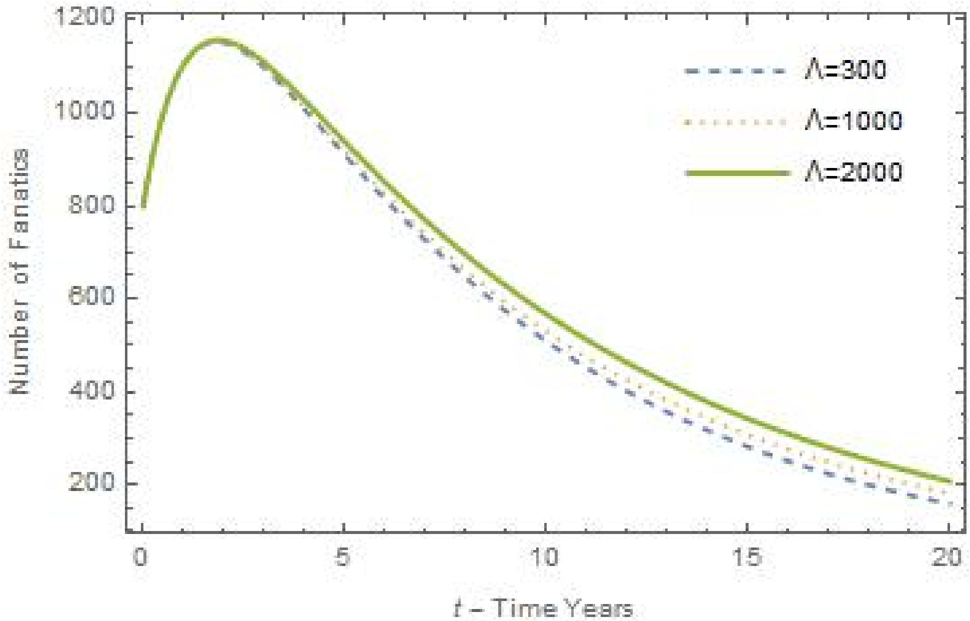

FIGURE 1. Variation in the population of fanatics for different values of Λ.

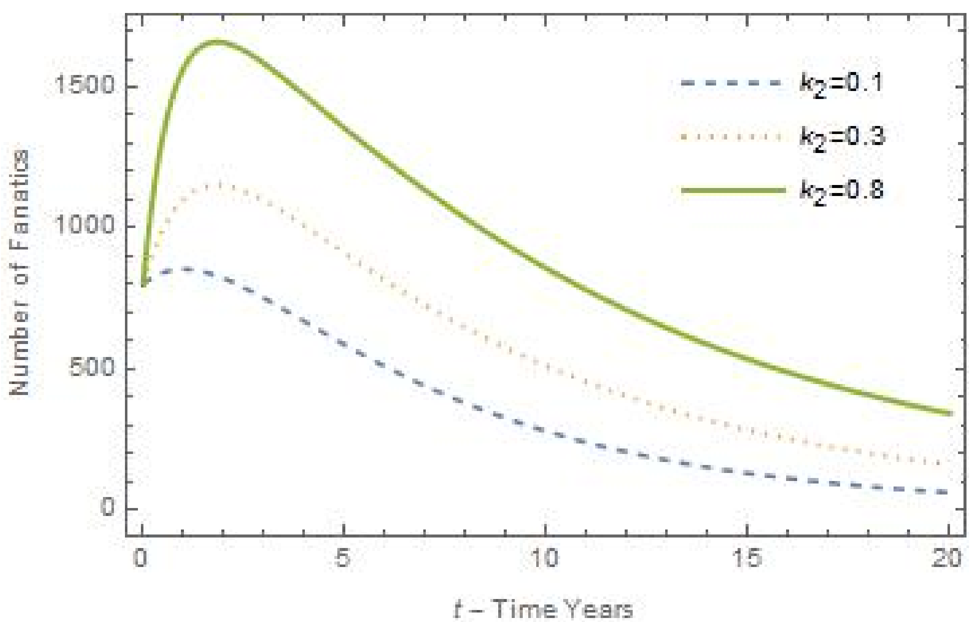

FIGURE 2. Variation in the population of fanatics for different values of k_2.

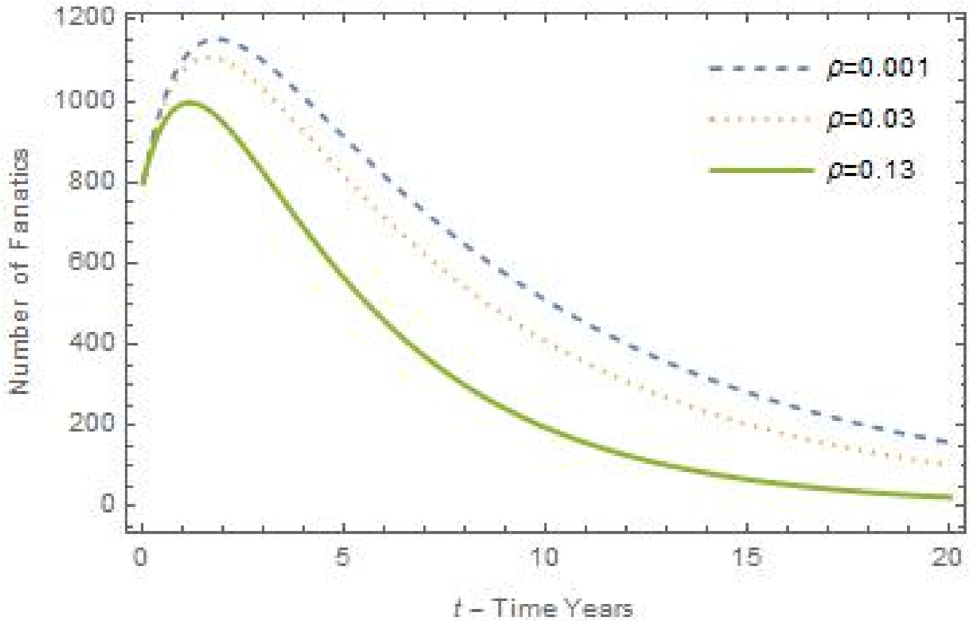

FIGURE 3. Variation in the population of fanatics for different values of ρ.

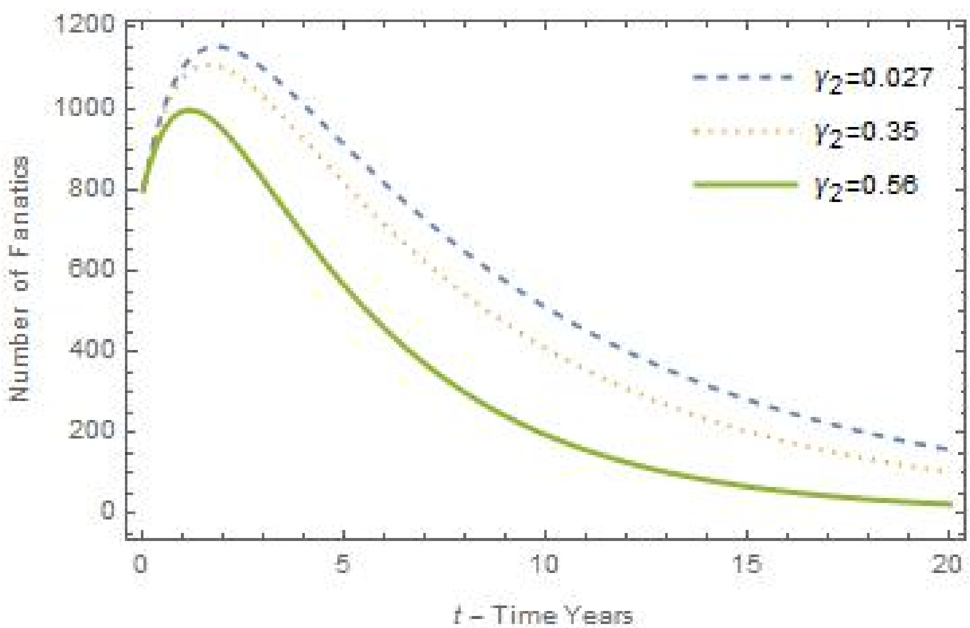

FIGURE 4. Variation in the population of fanatics for different values of γ_2.

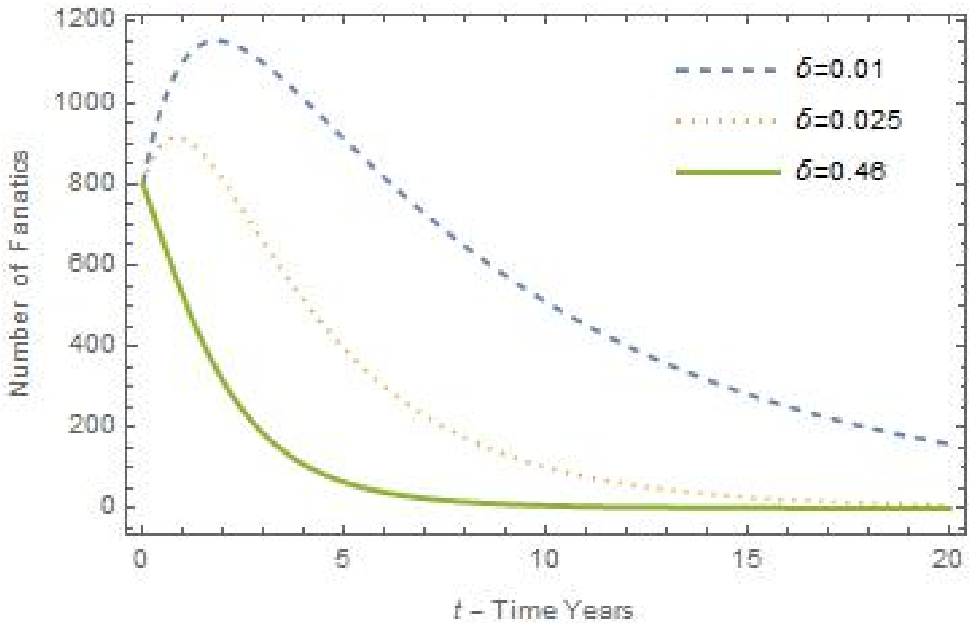

FIGURE 5. Variation in the population of fanatics for different values of δ.

Simulations with the parameters of the model using the values in 1 provides opportunities to obtain their impacts on the system variables and validate the model by restating the results previously established by other researchers. It is evident that when a region is attacked by extremists such as Boko Haram; the borders of such country need to be under strict surveillance such that the migration will be documented and monitored. From Figure 1 it is evident that unguarded borders will increase the number of extremists in a country, hence degenerating to number of extremist activities within the region; the result is similar to Okoye [20]. Over the years mass media has contributed greatly in dissemination of information from one quarter to another; but as product of post – modernity, mass media remain a twofold phenomena [21]. Mass media is tantamount to global thriving of politics especially the global yearning for democratization, international economy, and for peaceful coexistence but it also constitute avenue for extremist groups to spread their propaganda, manipulating of public perception of their activities, form forums for coordination, and eventually for recruitment. The modern ICT also provides shortcut to criminal victimization; it facilitates e– crime and e – terrorism [21]. In a society where the mass media is not monitored, Figure 2 shows explicitly that it has great stance on the multiplication and proliferation of fanatic activities which [22] supports in its special report 2014. The population of fanatics is relative to the cumulative density of government efforts in addressing the activities of insurgency. With regards to the evolving global norm of Responsibility to Protect (R2P), alongside the high human causalities, especially noncombatant members, use of policies of military involvements and the non-military deradicalisation cannot independently solve the insurgency rather, other approaches that will spore repentance of the already exposed population as in Figure 3 which is supported by [23]. At Present, the Nigerian military, along side with the civilian defense forces and Multinational Joint Task Force (MNJTF) has frustrated Boko Haram and hence, have pushed them to the Sambisa forest and Lake Chad region, and their operations restricted to Borno, Yobe and part of Adamawa states bordering Borno and Nigerian neighboring countries of Chad, Cameroon and Niger [24] as a result many members of Boko Haram has fallen on the course. From Figure 4 it is evident that as the growth of fanatic population decreases by reason of government efforts of improved combat and tactical units the society tends toward a serene environment. This reduction will also lead to decline in the cumulative density of government efforts because of mortality of the fanatic group members and incarceration of the ones caught, hence will lead to increase in the population of people in the correction centers which is shown in Figure 5.

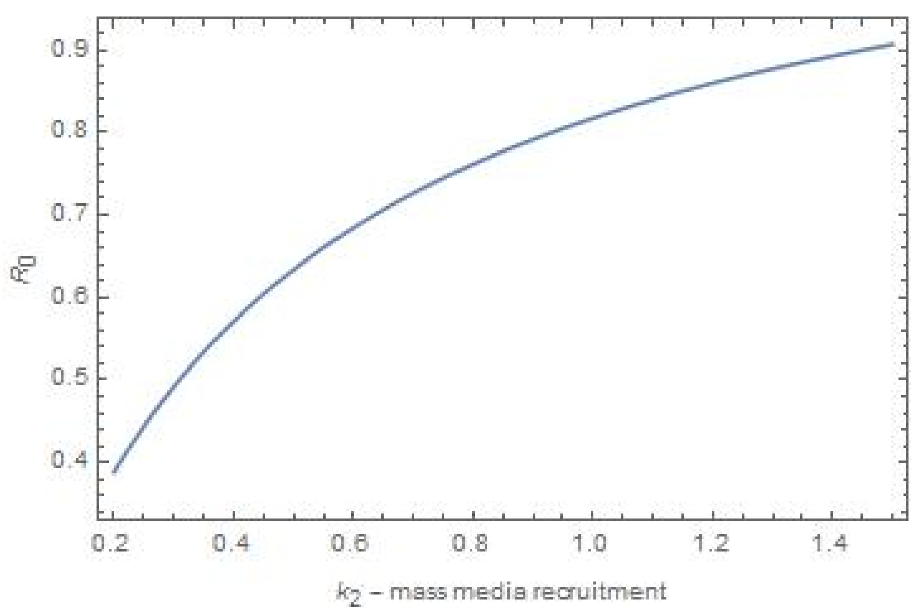

FIGURE 6. Sensitivity of recruitment of fanatics through Mass Media on R_0.

The contagiousness or transmissibility of fanaticism can studied using epidemiologic metric, basic reproduction number R_0; which is affected by biological, socio - behavioral and environmental factors that govern pathogen transmission. It is shown in Figure 6 that the basic reproduction number is sensitive to the effect of mass media in the recruitment of fanatics, in – fighting internal loss rate among extremists and the incarceration rate i.e. capturing rate of Boko Haram using government efforts. Figure 6 shows that mass media essential in the acculturation of fanaticism, it is used to mobilize, incite, and boost their constituency of actual and potential supporters and in so doing to increase recruitment, raise more funds and inspire further attacks [25].

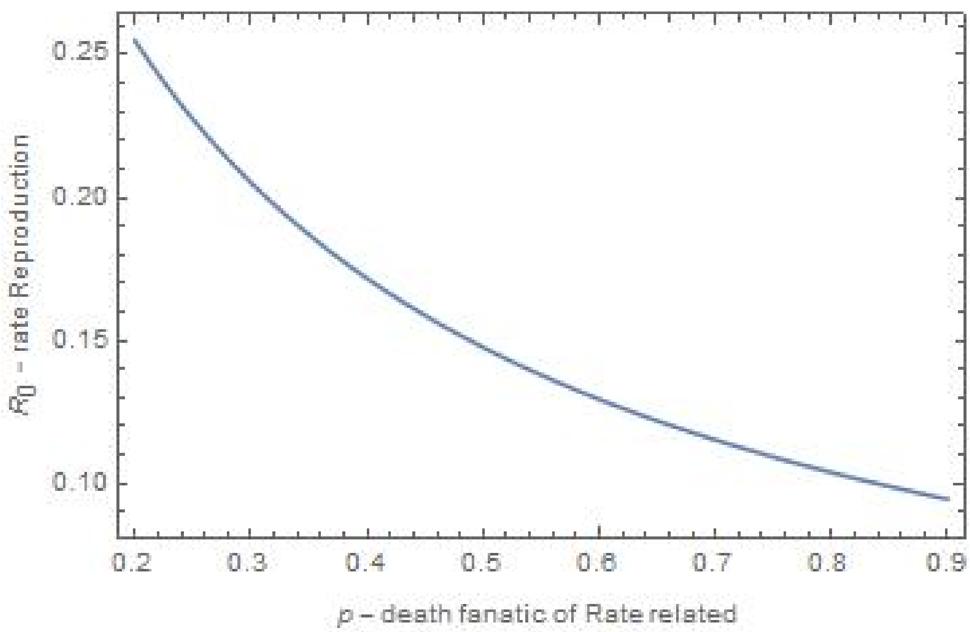

FIGURE 7. Sensitivity of ρ (death associated with fanaticism) on R_0.

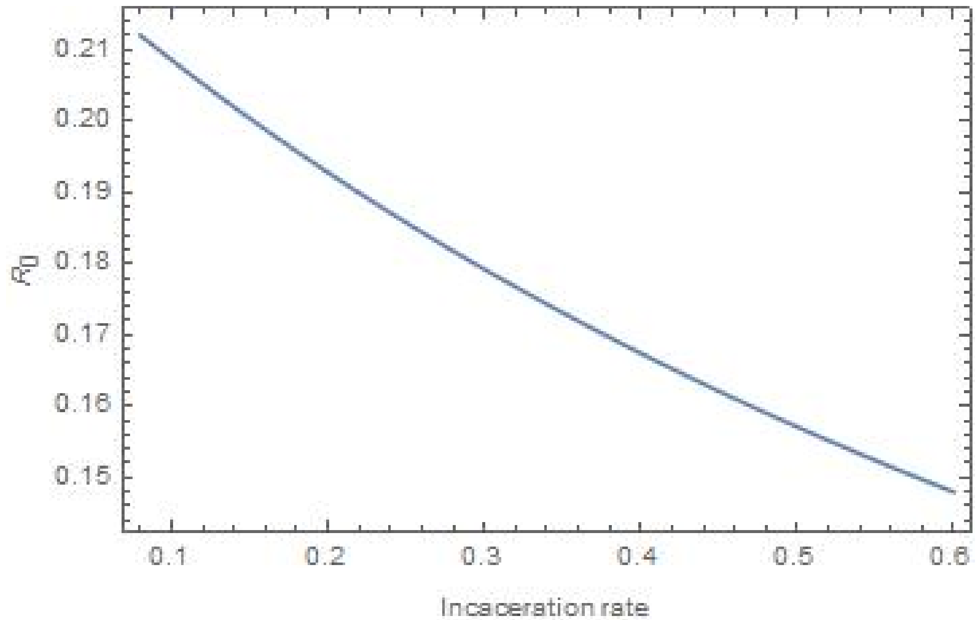

FIGURE 8. Sensitivity of incarceration rate, δ on R_0.

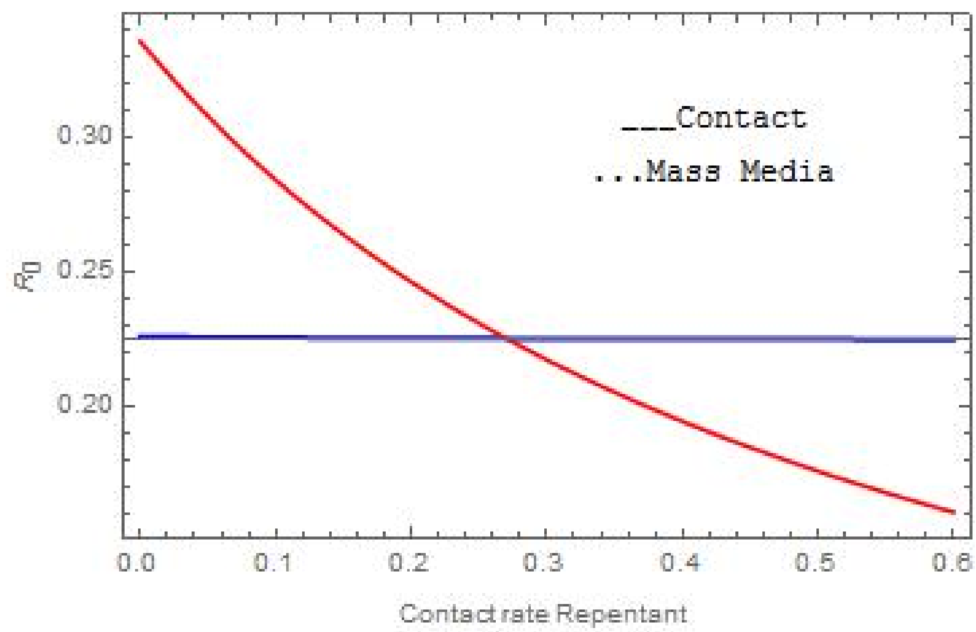

FIGURE 9. Rate of repentance γ_1, γ_2 of exposed population on R_0.

It is shown in Figure 7 that when the military engage the terrorist sects head - on, they will over ride most of their camps and in the course most of their kingpins will be exterminated and large amount of weapons and equipment which may include heavy artillery, tanks, and other armored and combat vehicles will be seized hence reducing the tendency of attacks and recruitment. From Figure 8, it is observed that when incarceration rate is increased the reproduction number is reduced. Information about the sponsors, sources of weapons and other links pertaining Boko Haram can be extracted from the members who are incarcerated, hence aiding in curtailing, alleviating and extermination of the sect and its practices. In addition, if journalists and other media workers engage in responsible journalism geared towards discouraging the act of insecurity in the nation and the government and other private and public sectors rise to the challenge of combating rising insecurity in the nation by embarking on communication – based approaches the reproduction number is reduced drastically as compared to physical engagements exemplified in Figure 9 which is slower in this regard.

7. Conclusion

In this paper, we present a nonlinear epidemiological model to analyze the spread of extremist ideology, being particular about the effect of Mass Media in the dynamics. The riddance of extremist: Boko Haram operations in communities shall be a relief to all concerned citizens. The model demonstrates that incarceration rate i.e. isolation of extremists has a substantial impact on the fight against Boko Haram and the likes, and should also be accompanied with government interventions through rehabilitation, reconstructive education against obnoxious ideologies, not limited to the rehabs but the entire population. From our analysis, it is observed that consistent in – fighting loss within the extremist results to reduction of the reproduction rate, consequently there shall be a decrease in the Boko Haram activities. This assertion is in line with [26], [17]. In addition, from the analysis, the model displays the fact that regulating the activities of Mass media, more especially the social media will ameliorate and abridge the activities of the extremists. Social media is a platform that Boko Haram use of recent in promoting their ideology and propaganda through videos, and transnationally appeal to individuals sympathetic to their course [27], [29], hence one of the best form of counter-terrorism is deoxidation of the means of recruitment [30]. We strongly recommend that border security should be guided strictly, Government should develop a clear – cut counter-terrorism strategy, a sufficient and effective technological and mutual trust between actors and locals in the management and utilization of intelligence. Hence state institutions should be able to coerce and convince citizens on its capacity to counter the danger of terrorism recruitment and expansion. Social media monitoring and regulation should be enacted to curtail online recruitment which is rampart and secretive in 21st century, and possibly will help in foiling most of the possible attacks. The debarment of extremist population is always difficult because of the appearance of the backward bifurcation at $R_0 = 1$. Journalists and other media workers should be occasionally trained on terrorism and conflict reporting so as to be attuned with modern techniques that could be used in ensuring effective use of the mass media in combating insecurity.

Availability of Data and Materials

We made use of data produced by the Armed Conflict Location & Event Data project (ACLED) (Raleigh et al. 2010) , which is available at their website https://www.acleddata.com/.

Authors' Contributions

MUE conceived the presented idea, and with EKO designed the schematic diagram to simplify and present the mathematical representation of the idea. The theoretical analysis of the model was done collectively by all concerned authors. Computation and numerical analysis was carried out by MUE, OAA. and OGA. The interpretation, verification and validation of the final result were carried out by MUE and OLO. All authors discussed the results and contributed to the final manuscript.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] R. Prieto Curiel, O. Walther, N. O'Clery, Uncovering the internal structure of Boko Haram through its mobility patterns, Appl. Netw. Sci. 5 (2020), 28.

[2] UN High Commissioner for Refugees, UNHCR Global Appeal 2018 - 2019. https://www.unhcr.org/media/unhcr-global-appeal-2018-2019-full-report.

[3] Z.P. Pieri, Boko Haram and the drivers of Islamist violence, Routledge, USA, 2019.

[4] H. Matfess, No home field advantage: the expansion of Boko Haram activity outside Nigeria in 2019, ACLED, UK, 2019.

[5] C. Castillo-Chavez, B. Song, 7. Models for the transmission dynamics of fanatic behaviors, in: H.T. Banks, C. Castillo-Chavez (Eds.), Bioterrorism: Mathematical Modeling Applications in Homeland Security, Society for Industrial and Applied Mathematics, 2003: pp. 155-172.

[6] F. Jin, E. Dougherty, P. Saraf, Y. Cao, N. Ramakrishnan, Epidemiological modeling of news and rumors on Twitter, Proceedings of the 7th Workshop on Social Network Mining and Analysis. (2013).

[7] D. Aldila, N. Nuraini, E. Soewono, Mathematical model for the spread of extreme ideology, AIP Conf. Proc. 1651 (2015), 33.

[8] F.J. Santonja, A.C. Tarazona, R.J. Villanueva, A mathematical model of the pressure of an extreme ideology on a society, Computers Math. Appl. 56 (2008), 836-846.

[9] E.T. Camacho, The development and interaction of terrorist and fanatic groups, Commun. Nonlinear Sci. Numer. Simul. 18 (2013), 3086-3097.

[10] E.E. Anugwom, The Boko Haram insurgence in Nigeria: Perspectives from within, Palgrave Macmillan, New York, 2018.

[11] C. Raleigh, rew Linke, H. Hegre, J. Karlsen, Introducing ACLED: An armed conflict location and event dataset, J. Peace Res. 47 (2010), 651-660.

[12] P. van den Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Math. Biosci. 180 (2002), 29-48.

[13] J. LaSalle, S. Lefschetz, The stability of dynamical systems, SIAM, Philadelphia, 1976.

[14] D. Greenhalgh, O. Diekmann, M.C.M. de Jong, Subcritical endemic steady states in mathematical models for animal infections with incomplete immunity, Math. Biosci. 165 (2000), 1-25.

[15] D. Greenhalgh, M. Griffiths. Backward bifurcation, equilibrium and stability of phenomenon in a three-stage extended brsv epidemic model, J. Math. Biol. (2009), 59 (1): 1–36.

[16] R. Anguelov, S.M. Garba, S. Usaini, Backward bifurcation analysis of epidemiological model with partial immunity, Computers Math. Appl. 68 (2014), 931-940.

[17] A. Goyal, J.B. Shukla, A.K. Misra, A. Shukla, Modeling the role of government efforts in controlling extremism in a society, Math. Meth. Appl. Sci. 38 (2014), 4300-4316.

[18] U.E. Michael, L.O. Omenyi, K.O. Elebute, A.A. Offia, Local and global stability analysis of a COVID-19 model dynamics with healthy diet as control, J. Math. Comput. Sci. 12 (2022), 176.

[19] C.P. Bhunu, S. Mushayabasa, H. Kojouharov, J.M. Tchuenche, Mathematical analysis of an HIV/AIDS model: impact of educational programs and abstinence in Sub-Saharan Africa, J. Math. Model Algor. 10 (2010), 31–55.

[20] C. Okoye, O.C. Collins, G.C.E. Mbah, Mathematical approach to the analysis of terrorism dynamics, in: J.T. Omenma, I.E. Onyishi, A.-M. Okolie (Eds.), Ten Years of Boko Haram in Nigeria, Springer Nature Switzerland, Cham, 2023: pp. 95-106.

[21] I. Bello, I.M. Jamilu, Media influence on terrorism: Analysis of Boko Haram as a product of postmodernism, J. Law Policy Glob. 59 (2017), 139-149.

[22] F.C. Onuoha, Why do youths join Boko Haram, special report 348, 2020.

[23] A.A. Akubo, B.I. Okolo, Boko Haram insurgency in Nigeria, Accord, AJCR (2018), 2.

[24] J.T. Omenma, I.E. Onyishi, A.M. Okolie, A decade of Boko Haram activities: the attacks responses and challenges ahead, Sccur J. 33 (2020), 337-356.

[25] Aiyesimoju, A. Boluwatife. Curbing Insurgencies in Nigeria: Roles for the media, Develop. Country Stud. 5 (2015), 18-29.

[26] A. Gutfrained, Understanding terrorist organizations with a dynamic model, Stud. Conflict Terrorism. 32 (2009), 45-59.

[27] M. Ette, S. Joe, 'Rival visions of reality': An analysis of the framing of Boko Haram in Nigerian newspapers and Twitter, Media, War Conflict. 11 (2018), 392-406.

[28] U.E. Michael, M.O. Oyesanya, Relating the effect of HPA axis to emotional state of bipolar II disorder patient, Asian Res. J. Math. 11 (2018), 1-19.

[29] United Nations, Better governance of underfunded, poorly managed Lake Chad Basin key to resolving conflict, suffering across region. Speaker Tell Security Council, United Nations Security Council, Briefing No. SC/13259 March 22, 2018.

[30] J. Slutzker, The Online Frontline: Inside Boko Haram's social media and movement to push back, Creative, 2018.

[31] J.R. Hoehn, N.S. Smagh, CRS in focus defense capabilities: joint all domain command and control, https://sgp.fas.org/crs/natsec/IF11493.pdf. (2021).

[32] N. Joseph, K.J. Chukwu, T.D. Ani, Thobejane. Digital Media reportage and the Boko Haram insurgency, African J. Dev Stud. 2021 (2021), 197.