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AVAILABILITY AND COST ANALYSIS OF A DETERIORATING SYSTEM

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Abstract: Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. The objective of this paper is to present the effect of minor and major maintenance rates, and perfect repair rate on system availability and profit generated. We analyzed the system by using Kolmogorov's forward equations method. Results have shown that perfect repair increases system availability and profit generated than minor and major maintenance. Through combine action of minor maintenance, major maintenance and perfect repair action the system availability and profit generated are improved. Models presented in this paper are important to engineers, maintenance managers, and plant management for proper maintenance analysis, decision and for safety of the system as a whole.

Keywords: Availability, profit, perfect repair, Minor maintenance, deterioration

2000 AMS Subject Classification: 90B25

1. INTRODUCTION

During operation, the strengths of systems are gradually deteriorated, until some point of deterioration failure, or other types of failures. As the age of equipment increases, the equipment slowly deteriorates correspondingly. Deterioration failure is still the inevitable fate of the equipment. In many manufacturing situation, the condition of the system has significant impact on the quantity and quality of the unit produced. However,

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availability/profit of an industrial system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. Improving the reliability and availability of system/subsystem, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieve be maintaining reliability and availability at highest order. To achieve high production and profit, the system should remain operative for maximum possible duration. It is important to consider profit as well as the quality requirement. Most of these systems are subjected to random deterioration which can results in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong equipment’s service life. Many maintenance models have been developed to describe a deterioration process and give optimal solutions to decrease the cost and increase the system reliability of such systems. Zuo *el al* [8] studied a warranty policy for a multi-state deteriorating and repairable products/systems where the system is repair or replaced based on two parameters, the degree of deterioration and the length of the residual warranty period. Optimal of these two parameters to minimize the expected warranty servicing cost per item sold have been examined. Grall *el al* [2] analyzed a condition- based maintenance policy for stochastically deteriorating systems where replacement threshold and inspection schedule are used to determine the long run expected maintenance cost per unit time. Wang [4] presents an excellent survey of maintenance policies of deteriorating system. Tang and Lam [5] presents a δ - shock maintenance model for a deteriorating system. A replacement policy N is adopted where the system is replaced by an identical new one at the *Nth* failure. Yusuf and Bala [7] deal with stochastic modeling of two unit parallel system under two types of failures, where the system works in normal mode, deterioration (slow, mild, or fast) in model I and normal and failure modes in model II, Marcous et al [3] deal with the modeling bridge deterioration, Wirahadikusumah et al [6] model deterioration of combined sewers.

The problem considered in this paper is different from the work of Yusuf and Bala [7]. In this study, we consider a system consisting of two identical units operating simultaneously. The system fails when both units fail. The system operates with full capacity initially which later deteriorates with age. The deterioration could be minor or major. Minor and major maintenance are employed as the system continues to deteriorate with major maintenance at significant deterioration levels and perfect repair at failure.

2. MATERIALS AND METHOD

In this study we consider a system consisting of two identical units operating simultaneously. The system fails when both units fail. The system operates with full capacity initially which later deteriorates with age. The deterioration could be minor (S_2) or major (S_3) with failure state (S_4).

2.1 *Assumptions***:**

1. Condition of the system: Perfect (S_1), Minor deterioration (S_2), Major deterioration (S_3), Failure (S_4)
2. At any given time the system is either in the operating state, deteriorating state or in the failed state.
3. The state of the system changes as time progresses.
4. The transition of the system from one state to the other takes place instantaneously.
5. The failure and repair rates are constant.
6. The units operate simultaneously

2.2 *Notations and Nomenclature:*

$\lambda_{12}, \lambda_{13}, \lambda_{23}$: Deterioration rate

λ_{14}: Failure rate of the system while in S_1

λ_{24}: Failure rate of the system while in S_2

λ_{34}: Failure rate of the system while in S_3

μ_R: Constant perfect repair action rate

μ_{21}, μ_{32}: Constant minor maintenance action rate

μ_{31}: Major maintenance action rate

Table 1 Transition rates table

	S_1	S_2	S_3	S_4
S_1		λ_{12}	λ_{13}	λ_{14}
S_2	μ_{21}		μ_{23}	λ_{24}
S_3	μ_{31}	μ_{32}		λ_{34}
S_4	μ_R			

3. FORMULATION OF THE MODEL

3.1 System Availability

Let $P_i(t)$ be the probability that the system at time $t \geq 0$ in state S_i, $i = 1,2,3,4$ and let $P(t)$ be the row vector probability at time t with initial conditions

$$P(0) = \left[P_1(0), P_2(0), P_3(0), P_4(0)\right] = \left[1,0,0,0\right]$$

We obtain the differential equations below using table 1 above:

$$\frac{dP_1}{dt} = -\left(\lambda_{12} + \lambda_{13} + \lambda_{14}\right)P_1(t) + \mu_{21}P_2(t) + \mu_{31}P_3(t) + \mu_R P_4(t)$$

$$\frac{dP_2}{dt} = -(\lambda_{23} + \lambda_{24} + \mu_{21})P_2(t) + \lambda_{12}P_1(t) + \mu_{32}P_3(t)$$

$$\frac{dP_3}{dt} = -\left(\lambda_{34} + \mu_{31} + \mu_{32}\right)P_3(t) + \lambda_{13}P_1(t) + \lambda_{23}P_2(t) \qquad (1)$$

$$\frac{dP_4}{dt} = -\mu_R P_4(t) + \lambda_{14}P_1(t) + \lambda_{24}P_2(t) + \lambda_{34}P_3(t)$$

The D.E in (1) above can be transform into matrix $\dot{P} = AP$ (2)

Where

$$A = \begin{bmatrix} -\left(\lambda_{12}+\lambda_{13}+\lambda_{14}\right) & \mu_{21} & \mu_{31} & \mu_R \\ \lambda_{12} & -\left(\lambda_{23}+\lambda_{24}+\mu_{21}\right) & \mu_{32} & 0 \\ \lambda_{13} & \lambda_{23} & -\left(\lambda_{34}+\mu_{31}+\mu_{32}\right) & 0 \\ \lambda_{14} & \lambda_{24} & \lambda_{34} & -\mu_R \end{bmatrix}$$

To compute the system availability, we maintain the initial condition in above:

$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0]$

Where the differential equations in (1) can be transform as:

$$\dot{P} = AP \qquad (6)$$

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \end{bmatrix} = \begin{bmatrix} -(\lambda_{12}+\lambda_{13}+\lambda_{14}) & \mu_{21} & \mu_{31} & \mu_R \\ \lambda_{12} & -(\lambda_{23}+\lambda_{24}+\mu_{21}) & \mu_{32} & 0 \\ \lambda_{13} & \lambda_{23} & -(\lambda_{34}+\mu_{31}+\mu_{32}) & 0 \\ \lambda_{14} & \lambda_{24} & \lambda_{34} & -\mu_R \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

In the steady-state, the derivatives of the state probabilities become zero so that

$$AP = 0 \qquad (7)$$

Setting the left (derivatives), the steady state probability is

$$\begin{bmatrix} -(\lambda_{12}+\lambda_{13}+\lambda_{14}) & \mu_{21} & \mu_{31} & \mu_R \\ \lambda_{12} & -(\lambda_{23}+\lambda_{24}+\mu_{21}) & \mu_{32} & 0 \\ \lambda_{13} & \lambda_{23} & -(\lambda_{34}+\mu_{31}+\mu_{32}) & 0 \\ \lambda_{14} & \lambda_{24} & \lambda_{34} & -\mu_R \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

With normalizing condition $P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) = 1$ (8)

Substituting (8) in one of the redundant rows of (7) we obtained the following system of linear equations in matrix form below

$$\begin{bmatrix} -(\lambda_{12}+\lambda_{13}+\lambda_{14}) & \mu_{21} & \mu_{31} & \mu_R \\ \lambda_{12} & -(\lambda_{23}+\lambda_{24}+\mu_{21}) & \mu_{32} & 0 \\ \lambda_{13} & \lambda_{23} & -(\lambda_{34}+\mu_{31}+\mu_{32}) & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady Steady-state availability is given by

$$A(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) = \frac{N_1}{D_1} \qquad (9)$$

$$N_1 = \mu_R(\lambda_{23}\lambda_{34} + \mu_{31}\lambda_{23} + \lambda_{24}\lambda_{34} + \mu_{31}\lambda_{24} + \mu_{32}\lambda_{24} + \mu_{21}\lambda_{34} + \mu_{21}\mu_{31} + \mu_{21}\mu_{32})$$
$$+\mu_R(\lambda_{12}\lambda_{34} + \mu_{31}\lambda_{12} + \mu_{32}\lambda_{12} + \mu_{32}\lambda_{13}) + \mu_R(\lambda_{12}\lambda_{23} + \lambda_{13}\lambda_{23} + \lambda_{13}\lambda_{24} + \mu_{21}\lambda_{13})$$

$$D_1 = \mu_R\mu_{31}\lambda_{12} + \mu_{21}\mu_{32}\lambda_{34} + \mu_{21}\mu_{31}\lambda_{34} + \mu_R\lambda_{23}\lambda_{34} + \mu_R\mu_{31}\lambda_{23} + \lambda_{23}\lambda_{34}^2 + \lambda_{24}\lambda_{34}^2$$
$$+\mu_{21}\lambda_{34}^2 + \mu_R\lambda_{24}\lambda_{34} + \mu_R\mu_{31}\lambda_{24} + \mu_R\mu_{32}\lambda_{24} + \mu_R\mu_{21}\lambda_{34} + \mu_R\mu_{21}\mu_{31} + \mu_R\mu_{21}\mu_{32} + \lambda_{12}\lambda_{23}\lambda_{34}$$
$$+\lambda_{12}\lambda_{24}\lambda_{34} + \mu_{31}\lambda_{12}\lambda_{24} + \mu_{32}\lambda_{12}\lambda_{24} + \lambda_{13}\lambda_{23}\lambda_{34} + \lambda_{13}\lambda_{24}\lambda_{34} + \mu_{32}\lambda_{13}\lambda_{24} + \mu_{21}\lambda_{13}\lambda_{34}$$
$$+\mu_{31}\lambda_{23}\lambda_{34} + \mu_{31}\lambda_{24}\lambda_{34} + \mu_{32}\lambda_{24}\lambda_{34} + \mu_R\lambda_{12}\lambda_{23} + \mu_R\lambda_{12}\lambda_{34} + \mu_R\mu_{32}\lambda_{12} + \mu_R\lambda_{13}\lambda_{23} + \mu_R\lambda_{13}\lambda_{24}$$
$$+\mu_R\mu_{21}\lambda_{13} + \mu_R\mu_{32}\lambda_{13}$$

3.2 Busy Period Analysis

Here we use the same initial condition as in subsection 3.1. From table 1, in state 2 to 4 the repairman is busy in those states performing minor, major maintenance and repairing the failed units therefore we use (7) and (8) to calculate busy period as follows:

$$B(\infty) = P_2(\infty) + P_3(\infty) + P_4(\infty) = \frac{N_2}{D_1} \tag{10}$$

$$N_2 = \mu_R(\lambda_{12}\lambda_{34} + \lambda_{12}\mu_{31} + \lambda_{12}\mu_{32} + \lambda_{13}\mu_{32}) + \mu_R(\lambda_{12}\lambda_{23} + \lambda_{13}\lambda_{23} + \lambda_{13}\lambda_{24} + \lambda_{13}\mu_{21}) + (\lambda_{34}\mu_{21}\mu_{32} + \lambda_{34}\mu_{21}\mu_{31} +$$
$$\lambda_{23}\lambda_{34}^2 + \lambda_{24}\lambda_{34}^2 + \mu_{21}\lambda_{34}^2 + \lambda_{12}\lambda_{23}\lambda_{34} + \lambda_{12}\lambda_{24}\lambda_{34} + \lambda_{12}\lambda_{24}\mu_{31} + \lambda_{12}\lambda_{24}\mu_{32} + \lambda_{13}\lambda_{23}\lambda_{34} + \lambda_{13}\lambda_{24}\lambda_{34}$$
$$+\lambda_{13}\lambda_{24}\mu_{32} + \lambda_{13}\lambda_{34}\mu_{21} + \lambda_{34}\lambda_{23}\mu_{31} + \lambda_{34}\lambda_{24}\mu_{31} + \lambda_{34}\lambda_{24}\mu_{32})$$

3.3 Profit Analysis

The system is subjected to minor, major maintenance and perfect repair as can be observed in states 2, 3and 4 of Let C_0 and C_1 be the revenue generated when the system is in working state and no income when in failed state, cost of minor, major maintenance and perfect repair respectively. Following El said [1] the expected total profit per unit time incurred to the system in the steady-state is

Profit=total revenue generated – cost incurred by the repair man due to maintenance

$$PF = C_0A(\infty) - C_1B(\infty) \tag{11}$$

where PF is the profit incurred to the system.

4. Results and Discussion

In this section, we numerically obtain the results for availability and profit function for the developed models using the following set of parameter values:

$\lambda_{12} = 0.5,\ \lambda_{13} = 0.01,\ \lambda_{14} = 0.3,\ \lambda_{23} = 0.5,\ \lambda_{24} = 0.82,$
$\lambda_{34} = 0.87,\ \mu_{12} = 0.5,\ \mu_{31} = 0.1,\ \mu_{32} = 0.3, C_0 = 1000, C_1 = 100$

For figures 1 to 8 with $\lambda_{24} = 0.9$ in Fig. 3

In each figure we vary the parameter in question and fix the rest for consistency. It is evident from figures 1 to 8 that the increase in minor or major maintenance or perfect rates induces increase in availability and profit of the system with perfect repair increases the system availability and profit than minor and major maintenance.

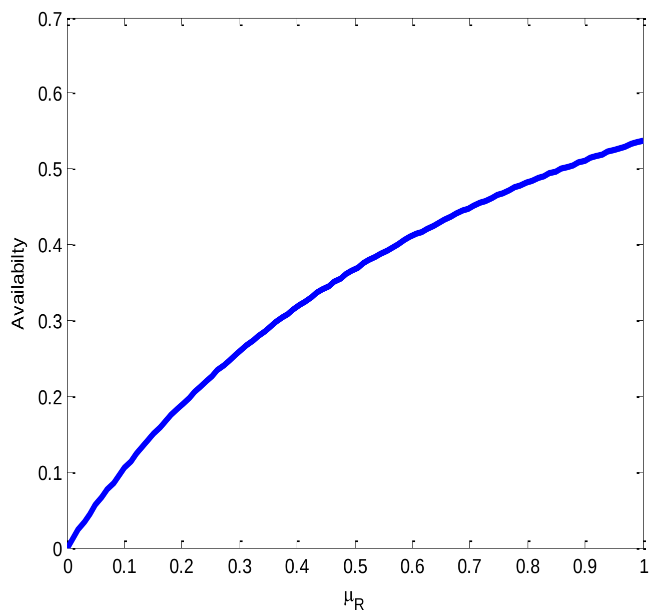

Fig. 1 effect of μ_R on system availability

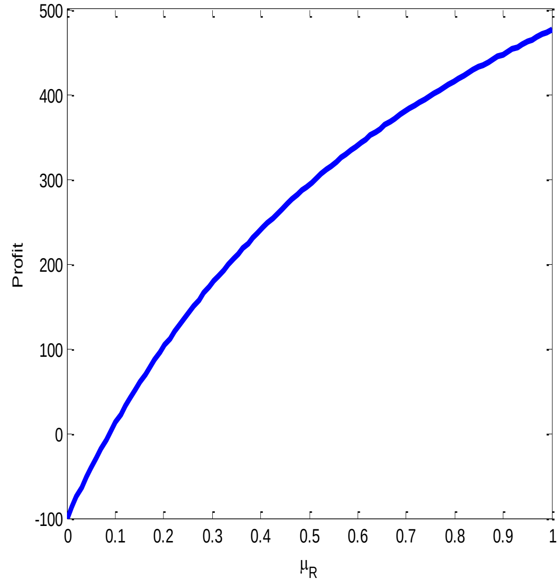

Fig. 2 effect of μ_R on profit

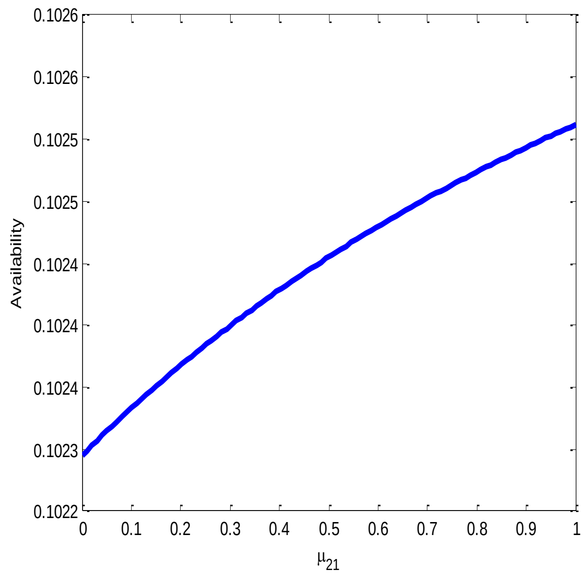

Fig. 3 effect of μ_{21} on system availability

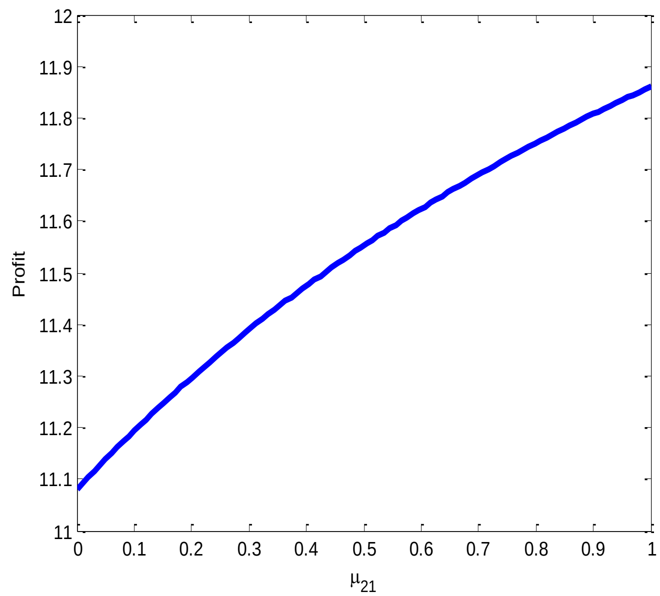

Fig. 4 effect of μ_{21} on profit

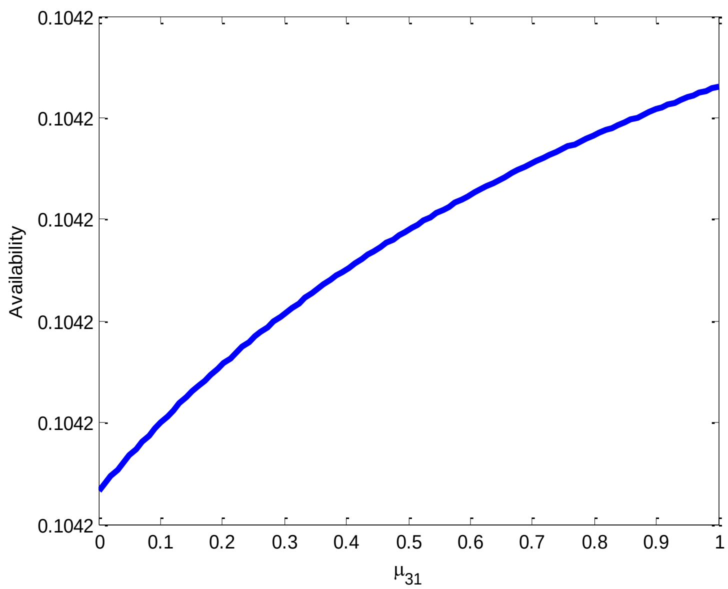

Fig. 5 effect of μ_{31} on system availability

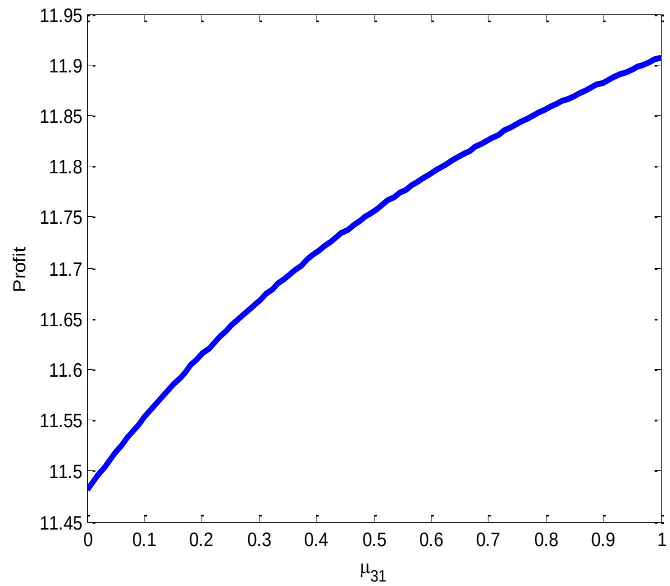

Fig. 6 effect of μ_{31} on profit

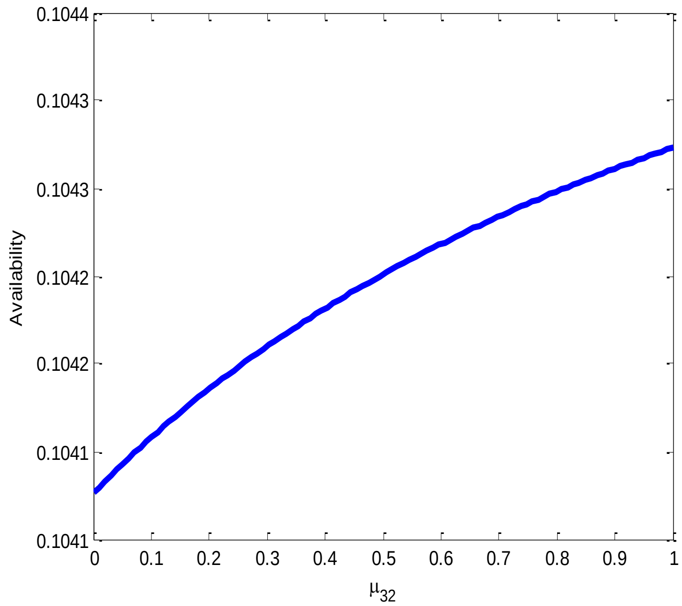

Fig. 7 effect of μ_{32} on system availability

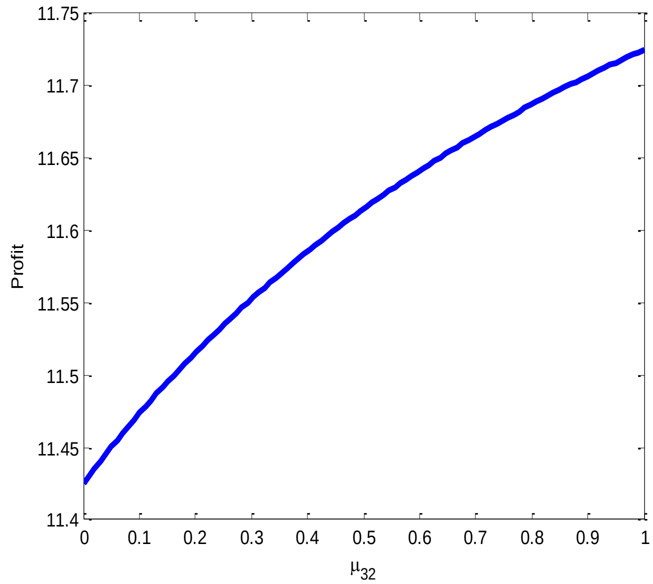

Fig. 8 effect of μ_{32} on profit

5. Conclusion

In this study, we developed explicit expressions for system availability, busy period and and perfect repair increases. It is clear from figures 1 to 8 that both minor and major deteriorate rate increase the system availability and profit with perfect repair significant results than minor and major maintenance. The developed model helps in determining the maintenance policy, which will ensure the maximum overall availability and profit of the system.

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