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A CATEGORY THEORETIC APPROACH IN CHANGING NETWORKS OF SEMANTICS

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Abstract: Mathematical models such as sets of equations are used in engineering to analyze the behaviour of physical systems. The conventional notations in formulating engineering models do not always provide the details required in fully comprehending those equations and, therefore, artefacts like ontologies, which are the building blocks of knowledge representation models, are used to fulfil this gap. Since ontologies are the outcome of an inter-subjective agreement among a group of individuals about the same fragment of the objective world, their development and use are questions in debate with regard to their competencies and limitations to univocally conceptualize a domain of interest. A network of semantics is defined as a directed graph, consisting of vertices representing heterogeneous ontologies and edges representing alignments among them. Both its components are carriers of meaning and they undergo changes in order to be adapted to different contexts of applications. This paper aims at, firstly, defining changes occurring in networks of aligned ontologies, a difficult task, since one has to take into account that making changes based on isolated components, while ignoring the semantic interrelations among them, may result in non logical continuity, or inconsistency of the underlying semantic model and, secondly, proposing a category theoretic framework in order to overcome the obstacles emerging from the changes occurring in networks of semantics, by introducing an enriched category that can capture the overall structure of a network of aligned ontologies.

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1. Introduction

Networks of semantics, as dynamic directed graphs of interlinked ontologies describing heterogeneously local knowledge, aim at minimizing the inaccuracy of knowledge and the ambiguity regarding the interpretation of the shared information, thus enabling Information Technology applications in open, dynamic and distributed environments, such as the semantic web, involving autonomous entities, which have been designed independently, to interact successfully through a common communication channel, despite their heterogeneity in representation and manipulation.

Ontologies and alignments, semantically enriched structures, are the two main components of each network of semantics. On the one hand, ontologies convey semantics, since they are defined as the formal conceptualizations of a domain of interest [19], [38], while on the other hand, alignments are defined as the links that semantically relate two formal conceptualizations [13], [35]. As both components describe parts of the world and their interconnections, they may undergo changes, due to the dynamic nature of the describing world. These changes, despite the fact that they may occur in isolated components, they may result in an inconsistent state for the overall network of interlinked components. Thus, there is a need to provide an underlying formalism for capturing the structure of a network of aligned ontologies in order to support manipulation of changes occurring, without breaking its logical continuity.

Defining change operations for networks of semantics is not an obvious task, since one has to take into account all the possible effects that a change can provoke on all its components, as well as on its entire functionality. With this in mind, in Section 3, a categorization of the kinds of changes that may occur in networks of aligned ontologies, is proposed, while in Section 2 we emphasize on the necessity for a network of semantics and describe its modularized structure. In Sections 4 and 5, the importance of the ontology alignment process, as well as its limitations in networks of interlinked ontologies and the need for a suitable ontology alignment composition operator is underlined, respectively, through examples, while, in Section 6, justification of using Category Theory as the appropriate formalization is given. The main contribution of the paper is underlined in Section 7, by introducing an enriched category that can capture the structure of a network of semantics and by proposing a way of computing the composition of alignments as the main operation needed in such networks, in order to retain its logical continuity.

2. The necessity for a network of semantics

Applications in open, dynamic and distributed environments, such as the semantic web, face the problem of dealing with heterogeneous and vast amounts of information [6], [39]. In these applications, integration, discovery and easy access to knowledge are the most important tasks. The use of semantics to explicitly conceptualize the available scattered knowledge and to bridge the gap between the different representations that various stakeholders have is widely required. Ontologies can be used to address this issue through the semantics they convey. The problem arises when the ontologies used to model the domain of an application become too large and thus unmanageable. The idea of overcoming this obstacle is, instead of using a single ontology, to break the model in several meaningful pieces and bring them into a mutual agreement by using alignments, that is, manageable relations among multiple manageable ontologies, in order to build a network of semantics. A network of semantics is an alternative phraseology for the network of ontologies of Haase [20], who defines it as a collection of ontologies (called networked ontologies) related together through different meta-relationships, such as mapping, modularization, version and dependency relationships.

3. Changes in a network of semantics

The facts that new ontologies can be embedded in a network of already aligned ones, or can retire from the function of such a network and that ontologies and/or alignments between them have to be kept up to date in changing application environments, are some of the factors that are involved in the definition of the dynamic nature of a network of semantics. Moreover, in order to take into account the fact that making changes based on isolated entities, while ignoring the semantic interrelations among them, may result in an inconsistent state for the underlying semantic model, we consider a twofold view of such networks: a local and a global one. A local view refers to isolated entities, that is, ontologies and alignments, while the global one refers to the context in which the separate components are interconnected in a way that explicitly characterizes the semantics of a specific application. Thus, to define change operations for networks of aligned ontologies, one has to take into account, not only all the possible effects a change can have on its separate components, but also to the hypostasis of the networks themselves.

With respect to the aforementioned views of a network of semantics, we conjecture that significant improvements in managing it can be obtained, by addressing important challenges for manipulating changes in three interrelated levels:

- the ontology level, which represent changes in the ontologies, that is, the changes in their domain of usage, since most domains have a dynamic nature, the changes in their level of formality and/or their level of granularity. More precisely, Klein [29] distinguishes among three kinds of changes that may occur within an ontology, i.e., conceptual, specification and representation changes;
- the alignment level, which represents changes in the definition of alignments between the same couple of ontologies, for example by applying a different matching algorithm, or by using an alternative representation language [8] and
- the network level, which represents changes in the number and the content of the ontologies that participate in a network of semantics. For example, a new ontology must be embedded in the network of previous aligned ontologies, or must retire from the network, according to the requirements imposed by a specific application [28].

The goal is to manipulate the possible changes in a network of semantics, without breaking its logical continuity. Changes at the ontology level reflect to ontology evolution and versioning processes [41], [23], which are based on discovering semantic relations among entities of two versions of the same ontology, that is, they reflect to the ontology alignment. Changes at the alignment level reflect to the alignment versioning process [12], which aims at finding out relations among two versions of the same alignment, while, changes at the network level need the definition of an ontology alignment composition operation [44], as it will be shown in Section 5.

We are going to analyze the limitations of the ontology alignment methods available in the literature and emphasize the need for an ontology alignment composition operator in Sections 4 and 5, through examples.

4. Ontology alignment limitations

The simplest network of semantics is formed by two heterogeneous ontologies and an alignment between them. The first issue to consider is how to obtain a correct alignment, since an incorrect one may lead to false consequences through the entire network. An ontology alignment describes explicitly the relations holding between two different ontologies and it is produced from the ontologies by applying matchers.

In general, the problem of ontology alignment is described as follows: Given two ontologies O_1 and O_2, an alignment between them is defined as a set of relations (equivalence, subsumption, disjointness) between pairs of entities (classes, properties, instances), belonging to the original ontologies. More formally, in the case where ontology matching is restricted only to find out the similarity between concepts of different ontologies, the problem of ontology alignment is described as [18]: Given two ontologies O_1 and O_2, an ontology alignment is defined as the process of creating mappings in the form (c_1, c_2, s) where $c_1 \in O_1$ and $c_2 \in O_2$ are concepts from the two ontologies and $s \in [0,1]$ is the estimated similarity between the two concepts. Mappings may also have the extended form (c_1, c_2, s, r), where r is the type of the relation, such as equivalence or generalization. Several manual, semi-automatic and automatic alignment techniques [32] have been proposed to tackle the problem of discovering semantic correspondences between entities of different ontologies. They mainly focus on: (a) lexical comparison, which relies only on the labels on the ontological entities, (b) structural comparison, which relies only on the structure of the ontologies, (c) instance comparison, which compares the instances of each ontological entity, and (d) comparison based on a “background knowledge source”.

However, despite many matching algorithms and tools that have been developed so far, one of the main open issues in the alignment process is the selection of the suitable matcher [31], according to a number of factors, such as the requirements and particularities of each application. Other serious limitations of almost all existing alignment tools, are the inability to identify complex correspondences [37], the need for manual intervention [14] and the inability of deriving other than equivalence, subsumption and disjointness relations between ontology entities [17], as is described in the following subsections.

4.1. The need for complex correspondences

Most of the available ontology alignment algorithms are only able to identify simple equivalence, or subsumption statements between classes of the involved ontologies. However, the true semantic relations between elements of different ontologies are often more complex. In the following, we give an example that illustrates the need for complex correspondences that the available tools are unable to identify.

We focus on two ontologies describing user profiles. Fragments of these ontologies are presented below (Figure 1).

We refer to the ontologies as O_1 (left side of the figure) and O_2 (right side), and we use prefix $i\#$ to refer to the entities of O_i. While both ontologies share some essential concepts, they differ especially with respect to the relations expressed via the properties. These differences make the alignment process erroneous and require complex correspondences to express the true semantic relations. Firstly, we aligned the two ontologies with the Falcon-AO alignment tool [24], which generated two correspondences, namely

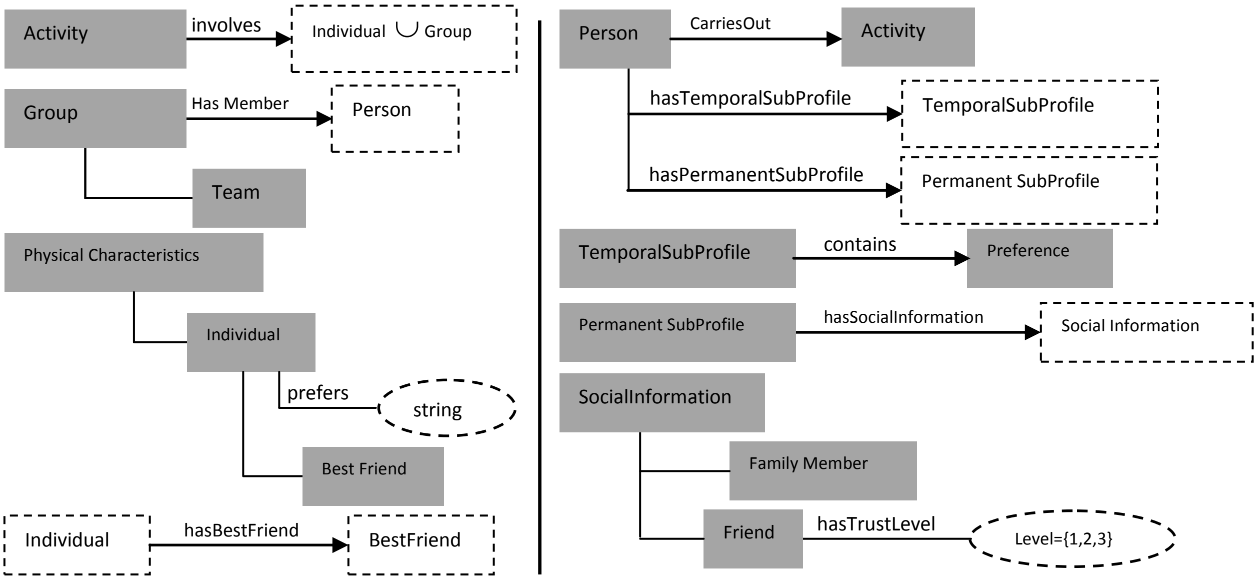

Figure 1: An example of two ontology fragments describing user profiles. A labeled square represents a class, a labeled ellipse a datatype, and a labeled arrow a property. The subsumption hierarchy of classes is represented by indentation. Domain and range of a property are restricted to be the classes connected by the accordant arrow.

$$1\#\mathtt{Activity} \equiv 2\#\mathtt{Activity} \text{ and } 1\#\mathtt{Person} \equiv 2\#\mathtt{Person}.$$

However, these correspondences are not sufficient to express the semantic relations that we are interested in. Suppose that we want to transfer instance data from O_2 to O_1. For example, which friends in O_2 have to be classified as best friends in O_1? These are friends with a high level of trust. In order to achieve this, we must use the rule

$$1\#\mathtt{BestFriend}(x) \leftarrow 2\#\mathtt{Friend}(x) \wedge 2\#\mathtt{hasTrustLevel}(x,3)$$

for migrating these friends to O_1. What about the preferences of a person according to an activity? This relation is modeled via a single datatype property in O_1 while we find a chain of properties in O_2. In order to express this dependency, we could use the rule

$$1\#\mathtt{prefers}(x,z) \leftarrow 2\#\mathtt{hasTemporalSubProfile}(x,y) \wedge 2\#\mathtt{containsPreference}(y,z).$$

When we want to know which persons are involved in which activities, things get more complicated. In this case, we have to use the rules

$$2\#\mathtt{carriesOut}(z,x) \leftarrow 1\#\mathtt{involves}(x,z) \wedge 1\#\mathtt{Individual}(z)$$

and $2\#\mathtt{carriesOut}(z,x) \leftarrow 1\#\mathtt{involves}(x,y) \wedge 1\#\mathtt{hasMember}(y,z)$ to cope with the different modeling. We conclude that ontology alignment also requires the identification of complex correspondences, which the existing tools are not able to provide them.

4.2. The need for manual intervention

Generally, the "80/20" rule applies in ontology matching [3]. Automatic ontology matching algorithms and tools can automate 80% of the work, covering common cases and producing results that are close to correct. A wide range of characteristics in ontologies to be aligned are exploited by existing approaches, such as linguistic descriptions (e.g., rdfs:label), structural information (e.g., rdfs:subClassOf) and data instances. Some of the approaches also utilize background knowledge from thesauri, or third parties' ontologies [36]. The remaining 20% requires manual contribution to verify the fine-tuning of the correspondences produced by the algorithms [22].

In general, a user interacting with an alignment tool, must examine the candidate correspondences produced by the tool, indicate which ones are correct and which ones are not, and create additional correspondences that the tool has missed. This process is a difficult task, because it requires understanding of both ontologies being aligned and how they relate to each other [16]. In the following, we give an example that illustrates the need for manual intervention.

We focus on two ontologies describing "Lamps". Fragments of these ontologies are presented in Figure 2.

Both ontologies refer to the same notion, but they differ in the labels and the properties of their concepts. This difference makes the automatic alignment process difficult and requires manual intervention to express the correct semantic relations that the algorithms have missed. Firstly, we aligned the two ontologies with some alignment tools, such as Falcon-AO, OLA [11] and Alignment API [7]. These tools generated, for example, the evident correspondence

$$1\#\mathtt{PhysicalCharacteristics} \equiv 2\#\mathtt{PhysicalCharacteristics}.$$

However, this correspondence is not sufficient to express the semantic relations that we are interested in. In this case, the user, as he is aware of the content of the ontologies to be aligned, he is able to add correspondences that the algorithms have missed, such as

$$1\#\mathtt{Item} \equiv 2\#\mathtt{Appliance} \text{ and } 1\#\mathtt{Service} \equiv 2\#\mathtt{Usage}.$$

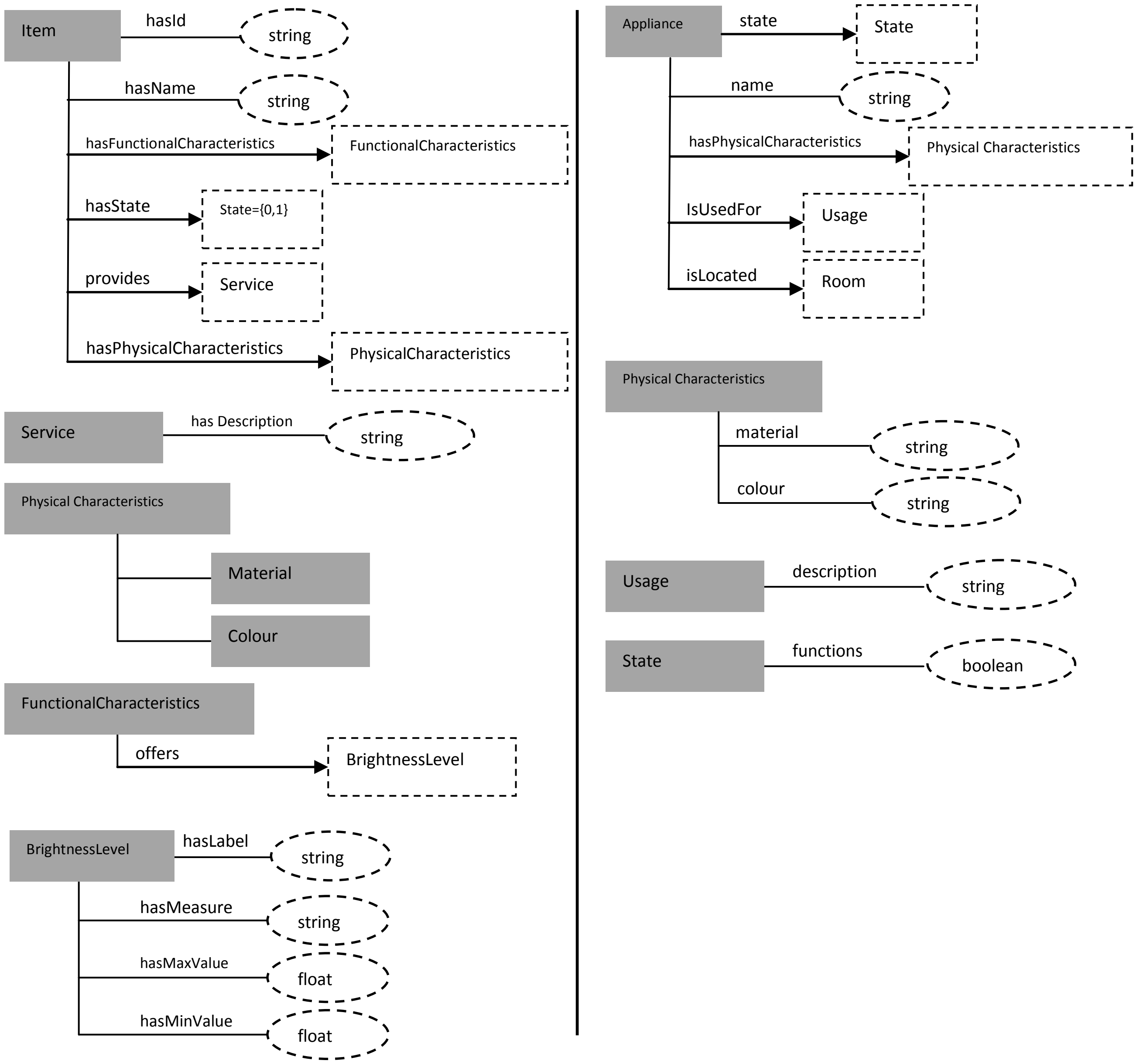

Figure 2: An example of two ontology fragments describing lamps

4.3. The need for deriving more elaborate relations between ontology entities

A serious limitation of almost all existing alignment tools, is the inability to identify relations other than equivalence, subsumption, and disjointness, between the entities of the ontologies to be aligned.

Suppose that we want to align two ontologies which describe the same object (i.e., a lamp) but in very different ways, that is, they differ with respect to the concepts used and their interrelations. These differences make the alignment of apparently dissimilar entities infeasible. In this case, in which label-based and structure-based ontology alignment approaches fail to discover important correspondences, another ontology is used as background knowledge for the matching task [1].

In our example, we focus on two ontologies that describe devices which provide light. Firstly, we aligned the two ontologies in question with existing alignment tools, which generated correspondences such as $1\#\mathtt{Light} \equiv 2\#\mathtt{Light}$, $1\#\mathtt{Device} \equiv 2\#\mathtt{Device}$, etc. But these correspondences are not sufficient to express all the semantic relations that we are interested in. For example, what is the relation between $1\#\mathtt{FloorLamp}$ and $2\#\mathtt{LightBulb}$? Is it an equivalence, a subsumption, a disjointness, or another type of relation? In order to derive a relation between these two apparently dissimilar entities, we aligned these ontologies by using a third one, as background knowledge. This background knowledge ontology describes devices that provide light by transforming electrical current into light energy. We selected the background knowledge ontology by using the Watson (http://watson.kmi.open.ac.uk/watsonWUI) ontology search engine. We queried Watson for concept labels from the ontologies to be aligned, such as light and lamp. An alternative choice for the selection of an appropriate ontology as background knowledge for the matching task could be Swoogle [34]. The algorithm used, proceeds in two steps: (1) anchoring, that is matching the entities of the two ontologies to the background knowledge ontology; and (2) deriving relations between the entities of the ontologies in question, by looking for relations between their anchored entities in the background knowledge ontology.

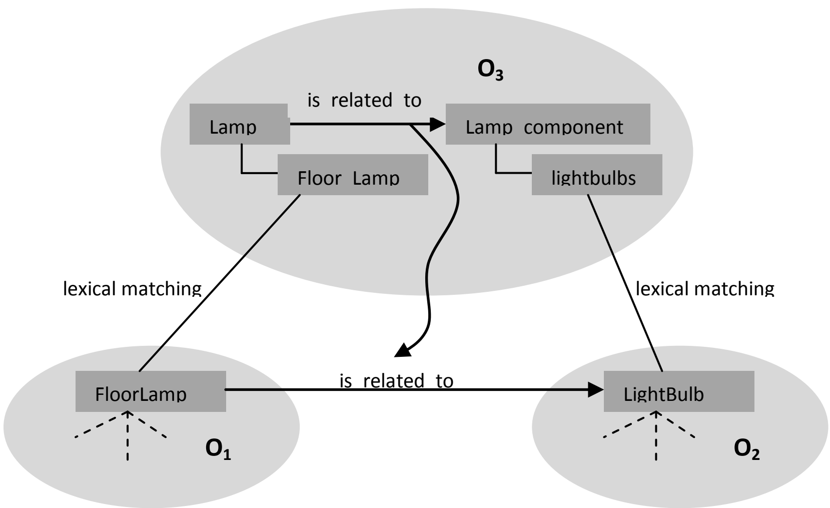

Figure 3: Example of using an ontology as background knowledge in the alignment process

As illustrated in Figure 3, 1#FloorLamp is anchored by using a simple lexical matching technique to 3#Floor_Lamp and 2#LightBulb is anchored by also using the same lexical matching technique to 3#lightbulbs. The background knowledge reveals a sequence of relations such as 3#Floor_Lamp "is_a" 3#Lamp , 3#Lamp "is_related_to" 3#Lamp_component and 3#lightbulbs "is-a" 3#Lamp_component , and thus we derived the relation that 1#FloorLamp "is-related_to" 2#LightBulb, according to the rules presented in [2]. Using background knowledge was, in this case, firstly crucial because the correspondence between 1#FloorLamp and 2#LightBulb could not be found by lexical, or structural matching, and secondly, it was essential, because this correspondence can be used to inject new information to O_1.

5. Ontology alignment composition is required

The main challenge relative to the issue of adapting to changes in networks of semantics, since they are defined as dynamic directed graphs of interlinked ontologies, emerges from the fact that making changes on isolated ontologies, while ignoring the semantic interrelations among them, may result in an inconsistent state of the underlying semantic model. On the one hand, finding correct alignments between ontologies is a very critical operation for a network of interlinked ontologies, since incorrect alignments may lead to unwanted consequences throughout the whole network and incomplete alignments may fail to provide the expected consequences [10]. Thus, once a correct and complete alignment is obtained, it is worth to be reused [42], [25]. On the other hand, considering the dynamic nature of networks of semantics, as the number of ontologies changes (increases, or decreases), the cost of obtaining alignments between ontologies is exponentially increasing. If we can compose the existing alignments, only one alignment is required, for each new ontology in order to consistently reformulate the semantic model.

Indeed, if we have an alignment between ontologies O_1 and O_2 and an alignment between ontologies O_2 and O_3, we can compose them and obtain an alignment between ontologies O_1 and O_3, thus depicting relations holding between the entities of O_1 and O_3 ontologies. This is essential in order to retain the consistency of a network of aligned ontologies in spite of changes in ontologies participating in the specific network. More precisely, as depicted in Figure 4, whenever an autonomous entity represented by an ontology, joins an already established network of already aligned ontologies, it suffices to align it to a single anchor ontology, already participating in the network. The anchor alignment produced, is then composed to already established alignments involving the anchor ontology, producing a batch of composition-generated alignments that remain, even if later on the anchor ontology leaves the network.

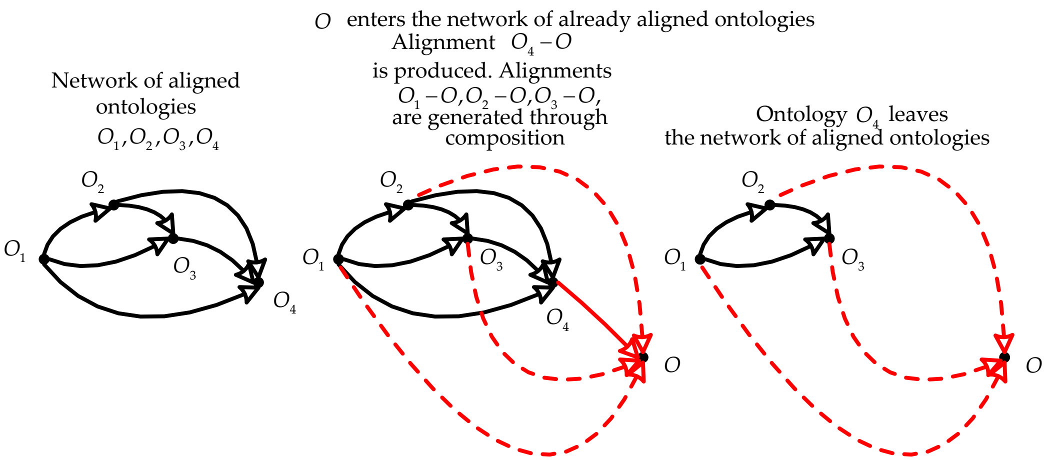

Figure 4. The significance of the composition of alignments in a network of aligned ontologies

A more illustrative example, which is depicted in Figure 5, of the necessity of an ontology alignment composition operator is given in [9], where the A5 relation algebra is proposed in order to combine correspondences of different alignments with regard to a set of base relations that are restricted to equivalence $(=)$, more specific $(<)$, more general $(>)$, disjoint $(\perp)$ and partial overlap $(\not<)$.

In the case of more elaborate relations, the composition of correspondences is not possible, in order to represent the new alignment. Thus, we focus on a category theoretical formalism in which composition of alignments will be feasible, whatever the relations involved in correspondences are.

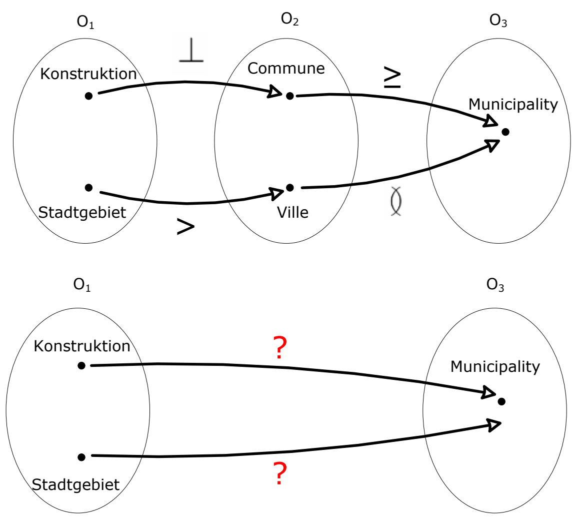

Figure 5. An example of composition of alignments

6. Why Category Theory

An appropriate formal framework on ontologies and their operations, such as ontology alignment, could contribute a lot in the direction of achieving effective interoperability among the components of a network of semantics. Some approaches followed in the literature for the formalization of ontologies and their operations consist of using Information Flow Theory [26], Goguen's work on Institution Theory [30] and Category Theory [27].

We adopt Category Theory as an appropriate formalization, because: (a) it focuses on relationships (categorical morphisms, functors, and natural transformations) and not on entities (categorical objects and categories), (b) it allows the coexistence of heterogeneous entities, since it provides the ability to define several categories, according to the kinds of entities to be described (category of ontologies, category of alignments), which can be related by the definition of special morphisms (categorical functors), (c) it offers a set of categorical constructors for creating new categories, by using predefined ones, (d) it provides a means for the combination of categorical objects (colimits can be used to compose them and limits to decompose them), and for the combination of categorical functors (natural transformations), and (e) it provides a multi-level study of its categorical notions, by defining three interrelated levels (the level of categories, of functors and of natural transformations).

7. Related work

Category Theory [4] offers several ways in combining and integrating objects and has been used as a mechanism to formalize ontology matching, providing operations to compose and decompose ontologies (alignment, merging, integration, mapping) [27], [21], [43], [5]. All these approaches are based on the assumption that only equivalence or subsumption relations hold between the entities of the two ontologies to be aligned.

In order to have a categorical view of ontologies, the category Ont of ontologies is defined in for example [5], where ontologies are considered as category objects, and pairs of functions (f,g) between a domain and a codomain ontology, are considered as the morphisms between objects, where f (and g), map concepts (respectively relations) of the domain ontology to concepts (respectively relations) of the codomain ontology. The morphisms are such that they preserve any hierarchy of concepts and any relations defined in the domain ontology, that is, if c_1 is a subconcept of c_2 in the domain ontology, $f(c_1)$ is a subconcept of $f(c_2)$ in the codomain ontology and if c_1

and c_2 are connected by the relation r in the domain ontology, $f(c_1)$ and $f(c_2)$ are connected by the relation $g(r)$ in the codomain ontology.

The alignment between two ontologies O_1 and O_2, is the task of establishing binary relations between the entities of the two ontologies. Each binary relation can be decomposed into a pair of mappings from a common intermediate source ontology, O [27]. The mappings from O to O_1 and from O to O_2, specify how the concepts and relations of O are understood in O_1 and O_2, respectively. This structure, comprising the ontologies O_1, O_2 and O, and the morphisms $(f_1,g_1),(f_2,g_2)$, is called, due to its shape, a V-alignment (Figure 6) and is also called a span, in the Category Theory terminology.

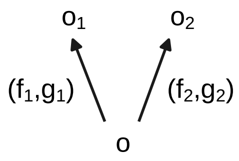

Figure 6. V-alignment

The operation of integrating two aligned ontologies into a single one, is called merging and can be accomplished with V-alignments. The ontology resulting from the unification process of merging, embodies the semantic differences of the two ontologies and collapses the semantic intersection between them. Merging of aligned ontologies can be described, in the Category Theory formalization, in terms of a Category Theory construct, called pushout. The pushout is a new object O' (an ontology in our case), together with morphisms $(f_1,g_1)'$, $(f_2,g_2)'$, such that

$$(f_1,g_1)'\circ(f_1,g_1)=(f_2,g_2)'\circ(f_2,g_2). \tag{7.1}$$

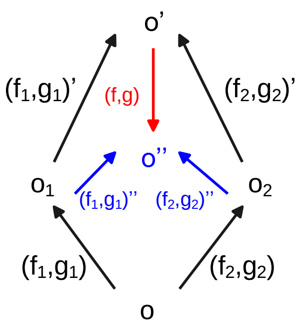

Figure 7. Merging through the pushout construct

The commutativity of the pushout diagram in Figure 7, means that components of O_1 and O_2 that are images of the same component in O (that is, the semantic intersection of O_1 and O_2), are collapsed in the resulting ontology O' (mapped to the same entity). But, this is exactly the definition of the merging operation. That is, the pushout ontology realizes the merging of O_1 and O_2. Moreover, for any other object (ontology) O'' for which the commutativity holds, i.e. for which

$$(f_1,g_1)''\circ(f_1,g_1)=(f_2,g_2)''\circ(f_2,g_2). \tag{7.2}$$

There exists a unique morphism (f,g) such that

$$(f,g)\circ(f_1,g_1)'=(f_1,g_1)'' \tag{7.3}$$

and

$$(f,g)\circ(f_2,g_2)'=(f_2,g_2)'', \tag{7.4}$$

i.e., the pushout O' is the most compact ontology that can embody the union of O_1, O_2 which possibly comprises collapsed components (i.e., embodies the semantic differences and collapses the semantic intersection).

In Category Theory, dual concepts arise by the process of reversing all the morphisms in a diagram. Thus, the dual concept of pushout is a construct called pullback, which is a particular case of another construct called limit (dual of colimit). The pullback is used in order to formalize the matching operation, by which similarities between ontologies are detected. We start with what is called a Λ-alignment, of the form depicted in Figure 8.

$$
\begin{tikzcd}
 & O & \\
O_1 \arrow[ur, "{(f_1,g_1)}"] & & O_2 \arrow[ul, "{(f_2,g_2)}"']
\end{tikzcd}
$$

Figure 8. Λ -alignment

Here, O_1 and O_2 are the ontologies to be matched and O is an intermediate ontology that guides the matching. The pullback is a new ontology O', together with morphisms $(f_1,g_1)'$, $(f_2,g_2)'$, such that

$$(f_1,g_1)\circ(f_1,g_1)' = (f_2,g_2)\circ(f_2,g_2)', \tag{7.5}$$

i.e., the pullback O' embodies all information of O_1 and O_2 that is semantically equivalent.

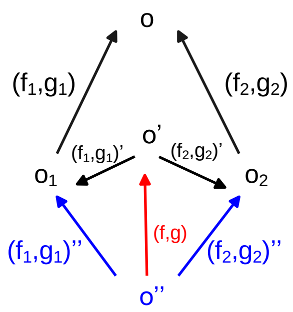

Figure 9. Matching through the pullback construct

The commutativity of the pullback diagram in Figure 9, means that components of O_1 and O_2 that have the same image in O (are semantically equivalent), are images of the same component in O'. But, this is exactly the definition of the matching operation. Thus, the pullback operation realizes the matching of O_1 and O_2. Moreover, for any other object (ontology) O'' for which the commutativity holds, i.e.

$$(f_1,g_1)\circ(f_1,g_1)'' = (f_2,g_2)\circ(f_2,g_2)'', \tag{7.6}$$

there exists a unique morphism (f,g), such that

$$(f_1,g_1)'\circ(f,g) = (f_1,g_1)'' \tag{7.7}$$

and

$$(f_2,g_2)'\circ(f,g) = (f_2,g_2)'', \tag{7.8}$$

i.e., the pullback O' is the biggest ontology that includes all the semantic intersection of O_1 and O_2.

Likewise, other operations involving manipulation of different alignments, as alignment composition, intersection and union, can be formulated in the categorical framework (Zimmermann et al. 2006). If we have alignments between ontologies O_1 and O_2, and between ontologies O_2 and O_3, we can compose them and obtain an alignment between ontologies O_1 and O_3, in the same way as composing spans in category theory, through the use of the pullback construct.

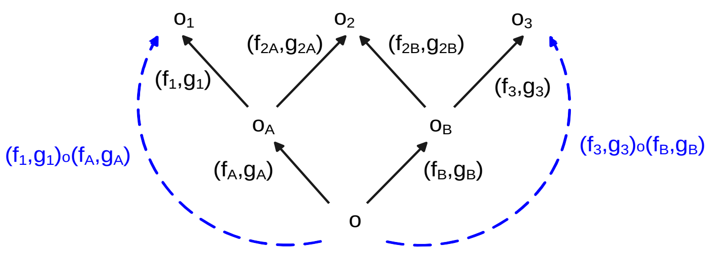

Figure 10. Composition of alignments

The ontology O (Figure 10), together with the morphisms $(f_1, g_1)\circ(f_A, g_A)$ and $(f_3, g_3)\circ(f_B, g_B)$ constitute the composition sought, where O , together with the morphisms, $(f_A, g_A), (f_B, g_B)$ is the pullback of the Λ-alignment of O_2 (O_2 with (f_{2A}, g_{2A}), (f_{2B}, g_{2B})).

In an analogous manner, the intersection between two alignments (which depicts the mutually agreed correspondences of the two alignments), is formalized by the use of a limit, while the union of two alignments (which gathers all the asserted relations specified in the two alignments), is formalized as the pushout of the intersection of the two alignments [43].
In cases where more elaborate relationships between the concepts of two ontologies is to be expressed, like for example subsumption, since this relation cannot be represented with the vocabulary of any of the two ontologies, it is externalized in an additional new ontology (called bridge ontology), as a bridge axiom. The following diagram (Figure 11) depicts the situation, with the original ontologies O_1 and O_2 containing the concepts related via subsumption and the bridge ontology b containing the bridge axioms. The fact that there exist concepts of the ontologies O_1 and O_2 occurring within the bridge ontology, is represented by the two V-alignments between the bridge ontology and the ontologies O_1 and O_2. Thus, what is called a W-alignment, is defined.

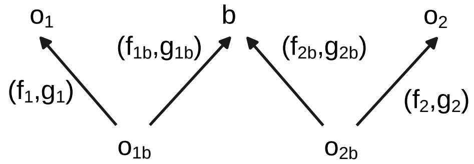

Figure 11. W-alignment

The merging operation in this case, is defined as the colimit of the alignment diagram in Figure 12 and is computed by successive pushouts [43].

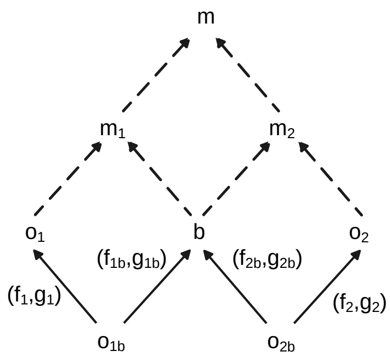

Figure 12. Merging with W-alignments

In a similar way, one can compose W-alignments. If a W-alignment exists between ontologies O_1 and O_2 with bridge ontology b_1 and if also a W-alignment exists between ontologies O_2 and O_3 with bridge ontology b_2, by composing the two W-alignments, it results that a W-alignment exists between ontologies O_1 and O_3, with bridge ontology b, which is obtained if the merging operation is applied to the bridge ontologies b_1 and b_2. The problem of this approach, consists in incorporating in the new bridge ontology b bridge axioms from the ontologies b_1, b_2 and O_2, that might be irrelative to O_1 and O_3.

Another solution to the problem of more elaborate relationships (subsumption, strict inclusion, strict containment, disjointness, overlapping with partial disjointness, temporal relations) between ontology entities, which is the one that we follow in this paper, is to enhance the category of ontologies with more elaborate morphisms that denote the relationship that holds between the syntactic entities of the two ontologies (subsumption, strict inclusion etc.). In this case, when applying the composition operation, if an entity in ontology O_2 has an elaborate relation to entities in the ontologies O_1 and O_3, there is some kind of relation between the two entities in O_1 and O_3. The latter relation depends strongly on the former one. For example, if an entity in O_1 is related to an entity in O_2 with strict inclusion and the same entity in O_2 is related to an entity in O_3 with strict containment, then the entity in O_1 can be related to the entity in O_3 by either of the following relationships: equivalence, strict inclusion, strict containment, disjointness, overlapping with partial disjointness [43].

8. A step forward

In this section, a characterization of an enriched category that describes the structure of a network of semantics is proposed, following the lines of [33], where a category-theoretic framework has been proposed for the analysis of fuzzy viewpoints. The aim is to be able to compute alignments in the case that relations more elaborate than equivalence hold between the entities of the two ontologies to be aligned and in the case the concepts inside each ontology are connected not only by equivalence or subsumption relations. For instance, if there exists an alignment between ontology O_1 and ontology O_2, which consists of the correspondence *1#Person owns 2#House*, and another one between O_2 and O_3, which consists of the correspondence *2#House has 3#Garage*, the goal is to decide which correspondence holds between *1#Person* and *3#Garage* between ontologies O_1 and O_3.

For this purpose, we first propose a slightly different definition of alignment. In this more general case, given two ontologies O_1 and O_2, a set of alignment relations Θ and a confidence structure Ξ, a correspondence is a quadruple (e, e', r, n), with e and e' entities of O_1 and O_2, respectively, $r \in \Theta$ and $n \in \Xi$ [9]. We say that the relation r holds between the ontology entities e and e' with confidence n. The confidence structure Ξ is a bounded partially ordered set, or poset $(\Xi, \leq)$ with regard to the reflexive, antisymmetric and transitive binary relation $\leq$, with a top element T and a bottom element $\perp$. Most widely, the real number unit interval $[0,1]$ is used. The set Θ of relations, expresses the relations that hold, either between concepts of an ontology, or between aligned entities of two different ontologies. These relations can be equivalence $(=)$, more specific $(<)$, more general $(>)$, disjoint $(\perp)$, partial overlap $(\not<)$ and other natural language-based descriptive relations.

One of the questions that arise, in this paper, concerns the composition of alignments, which is a way to deduce new alignments from existing ones. That is, if an alignment between ontologies O_1 and O_2 and an alignment between ontologies O_1 and O_3 with elaborate relationships exist, what are the correspondences entailed between the entities of O_1 and those of O_3 and how are they formalized by using Category Theory? To this end, we try to characterize the category that captures our structure. The notions of graph, graph homomorphism, poset, lattice and $\mathcal{L}$-valued set are needed [33].

A directed graph consists of a set of nodes and a set of edges. Each edge has a source node and a target node. A graph homomorphism is a mapping h between two graphs G and G', such that if an edge e of G has source node s and target node t and $s \xrightarrow{h} h(s)$, $t \xrightarrow{h} h(t)$, then the edge e of G is mapped to an edge e' of G' which has as source node $h(s)$ and as target node $h(t)$, as depicted in Figure 13.

A poset (partially ordered set) is a non empty set, with a binary relation called partial order, which is reflexive, antisymmetric and transitive.

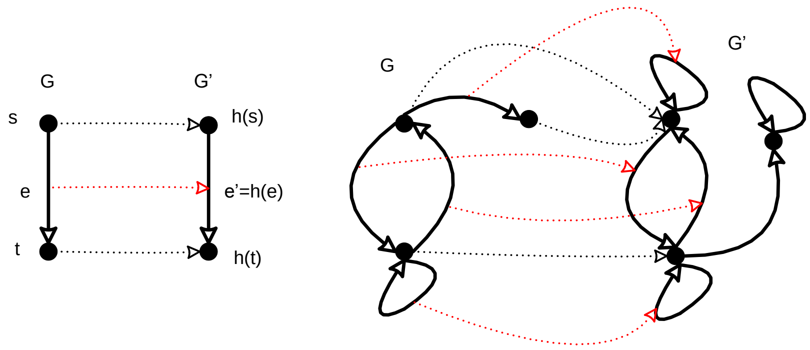

Figure 13. The definition of a graph homomorphism

$\mathcal{L}$ is a lattice, if it is a poset, it has a minimum upper bound (supremum) for any $\{a,b\} \in \mathcal{L}$ and it has a maximum lower bound (infimum) for any $\{a,b\} \in \mathcal{L}$. In order for $\mathcal{L}$ to be a complete lattice, it should have an infimum and a supremum for any $A \subseteq \mathcal{L}$. Moreover, every finite lattice is complete and every complete lattice has a top element $\top$ and a bottom element $\perp$.

An $\mathcal{L}$-valued set (S,σ) includes a complete lattice $\mathcal{L}$ (called the truth set of σ), a set S (called the carrier set of (S,σ)) and a function $\sigma : S \to \mathcal{L}$, such that $s \in S \xrightarrow{\sigma} \sigma(s) \in \mathcal{L}$ ($\sigma(s)$ is the degree of membership of s in (S,σ)).

We then define a morphism between two $\mathcal{L}$-valued sets $\mathbf{f} : (S,\sigma) \to (T,\tau)$, with $f : S \to T$ a function between sets (carrier function of $\mathbf{f}$), such that $\forall s \in S,\ \sigma(s) \leq \tau \circ f(s)$, as depicted in Figure 14.

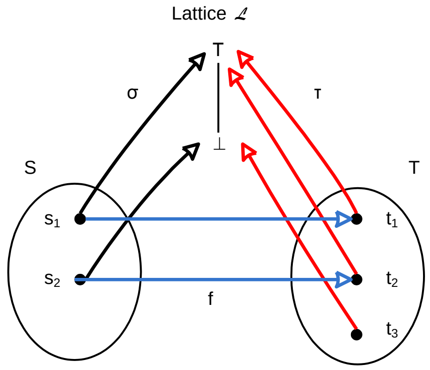

Figure 14. The definition of a morphism between two $\mathcal{L}$-valued sets

The category $Fuzz(\mathcal{L})$, which has $\mathcal{L}$-valued sets (S,σ), (T,τ),... as objects and morphisms between $\mathcal{L}$-valued sets $\mathbf{f}:(S,\sigma)\to(T,\tau)$, as morphisms is finitely complete, that is, every finite diagram in $Fuzz(\mathcal{L})$ has a limit (the limit is a generalization of the pullback). Moreover, the powerset $\mathcal{P}(Z)$ of any $Fuzz(\mathcal{L})$ object $Z=(S,\sigma)$ is a complete lattice. In the example of Figure 15, it is $S=\{a,b\}$ and the degree of membership function sends every element of S to the top element of $\mathcal{L}$.

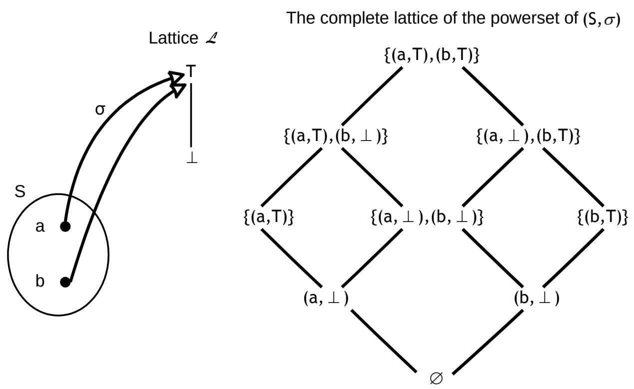

Figure 15. The complete lattice of the powerset $\mathcal{P}(Z)$ of any $Fuzz(\mathcal{L})$ object $Z=(S,\sigma)$

The category which has as objects directed graphs with a set of edges forming a $\mathcal{G}$-valued set (i.e., it is a $Fuzz(\mathcal{G})$ object), taking values in a complete lattice $\mathcal{G}$ and a set of nodes forming a $\mathcal{V}$-valued set (i.e., it is a $Fuzz(\mathcal{V})$ object), taking values in a complete lattice $\mathcal{V}$, and as morphisms graph homomorphisms, where there is a $Fuzz(\mathcal{G})$ morphism between the sets of the edges of the two graphs (which are $Fuzz(\mathcal{G})$ objects) and a $Fuzz(\mathcal{V})$ morphism between the sets of the edges of the two graphs (which are $Fuzz(\mathcal{V})$ objects), is finitely complete (i.e. has all limits). Here, $\mathcal{V}$, $\mathcal{G}$ can be the complete lattices of the powerset of some *Fuzz*($\mathcal{X}$) object, with $\mathcal{X}$ some complete lattice [33], [40].

In order to form the category that captures our structure, we finally consider:

- graph homomorphisms between the ontologies O_1 and O_2 to be aligned and the intermediate source ontology O,
- the sets of nodes of the ontologies forming *Fuzz*($\mathcal{I}$) objects, with $\mathcal{I}$ some complete lattice of relations.
- the sets of edges of the ontologies forming *Fuzz*($\mathcal{I}$) objects, with $\mathcal{I}$ some complete lattice of relations.

Then, the composition of alignments is computed as the pullback of a pair of *Fuzz*($\mathcal{I}$) or *Fuzz*($\mathcal{J}$) morphisms $\mathbf{f}:(A,\sigma)\to(C,\gamma)$ and $\mathbf{g}:(B,\tau)\to(C,\gamma)$. We first compute the pullback in the category *Set* (the category of sets and functions between sets) of the carrier functions $f:A\to C$ and $g:B\to C$. This gives us a set P together with two functions $j:P\to A$ and $k:P\to B$. Then, we compute the membership degree for every $p\in P$, as the infimum of the membership degrees of all those elements in (A,σ) and (B,τ) to which p is mapped.

Conclusion

Dealing with changes in networks of aligned ontologies requires tackling the problem of heterogeneity, which, in turn, aims at finding correspondences between different ontologies. These correspondences constitute an alignment that semantically links the underlying ontologies. Although, ontologies and alignments that are involved in a network of semantics can be considered similar at a certain level, since they both relate semantic entities, they undergo changes in a different way: ontologies undergo interior changes, as they act at a concept level, while alignments undergo exterior changes, as they act at the ontology level. In order to manipulate changes in these two levels, by considering the coexistence of heterogeneous semantic entities (ontologies and alignments), we propose an enriched category, where an ontology alignment composition operation will be able to be defined, by simultaneously retaining the logical continuity of the underlying network of semantics.

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