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COMPUTING THE CONGRUENCE CLASS OF SOME LEFT RESTRICTION SEMIGROUPS IN $\wp\Im_X$

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Abstract: In this paper, we present some examples of left restriction semigroups embedded in the partial transformation $\wp\Im_X$. Also, we compute and enumerates the congruence-class $\widetilde{R}$ and the semilattice of idempotents E_X inherent in the examples generated.

Key words: left restriction semigroups; partial transformation; Green's relation; congruence-class.
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1. INTRODUCTION

Partial transformation semigroup $\wp\Im_X$ is a weakly left E-ample semigroup otherwise known as Left restriction semigroup. The E refers to a particular semilattice refered to as partial identities on X. $\wp\Im_S$ is a (2,1)-subalgebra of $\wp\Im_X$ where the unary operation

$$+: \alpha \mapsto I_{dom\ \alpha}$$

Let X be a non empty set, $\wp\Im_X$ contains a semilattice of idempotents

$$E_X = \{I_A : A \subseteq X\}$$

E_X is the semilattice of idempotents, the partial identities.

I_A is the identity map on A. A unary operation on $\wp\Im_X$ is defined by

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$$\alpha^+ = I_{dom\,\alpha}$$

for each $\alpha \in \wp\mathfrak{I}_X$. Let S be a semigroup of $\wp\mathfrak{I}_X$ and let E be the set

$$E = \{I_Z \in E_\lambda : Z = dom\,\alpha\,, for\ some\ \alpha \in S\}$$

$\wp\mathfrak{I}_X$ is closed under $^+$, $\wp\mathfrak{I}_X$ is a left restriction semigroup. The equivalence relation $\tilde{R}$ on S is defined by the rule that

$$a\,\widetilde{R}\ b \Leftrightarrow \forall\ e \in E\ \{\ ea = a \Leftrightarrow eb = b\ \}$$

For $a, b \in S$, two elements a, b are $\widetilde{R_E}$ -related iff they have the same left identities in E. For $E = E(S)$, we denote $\widetilde{R_E}$ by $\widetilde{R}$

Note $\widetilde{R} \subseteq \widetilde{R_E}$ for any E. [3]

Equally, the relation $\widetilde{R_E}$ on S is defined by the rule that for any $a, b \in S$,

$$a\,\widetilde{R_E}\ b \text{ if f } \forall\ e \in E,\ \ ea = a \text{ iff } eb = b$$

Left restriction semigroups form a variety of unary semigroups, that is, semigroups equipped with an additional unary operation denoted by $^+$. The identities that define a left restriction semigroup S are:

$$a^+a = a, a^+b^+ = b^+a^+, (a^+b)^+ = a^+b^+, ab^+ = (ab)^+a$$

we put $E = \{a^+ : a \in S\}$

E is a semilattice known as the semilattice of projections of S. Dually, right restriction semigroups form a variety of unary semigroups. In this case, the unary operation is denoted by *. Left/Right restriction semigroup emanated from the study of Partial transformation monoids. Suppose a weakly left E-adequate semigroup satisfies the left congruence condition with respect to E, suppose that it also satisfies the left ample condition that for all $a \in S$ and $e \in S$

$$ae = (ae)^+a$$

if $E = E\,(S)$,where S is weakly left E-ample. Then, S denote a left restriction (formerly weakly left E-ample) [2].

2. PRELIMINARIES

2.1 Green's relation

Green's relation are five equivalence relations $\mathcal{R}, \mathcal{L}, \mathcal{H}, \Im\ D$,that characterise the elements of a semigroup in terms of principal ideals the generate. The relations are named for James Alexander Green, who introduced them. John Mackintosh Howie, a prominent semigroup theorist, remarked that on encountering a new semigroup, almost the first question one asks is 'What are the Greens relations like?'

For elements a and b of S, Green relations, $\mathcal{L}, \mathcal{R}, \Im, \mathcal{H}, D$, are defined by

$$a\ \mathcal{L}\ b\ iff\ \ S\,a = Sb$$

$$a\ \mathcal{R}\ b\ iff\ \ aS\ = bS$$

$$a\ \Im\ b\ iff\ \ SaS\ = SbS$$

$$a\ \mathcal{H}\ b\ iff\ \ a\ \mathcal{L}\ b \text{ and } \ a\ \mathcal{R}\ b$$

$$a\ D\ b\ iff\ \ \text{there exists a } c \text{ in } S \text{ such that } a\ \mathcal{L}\ c \text{ and } \ c\ \mathcal{R}\ b$$

That is, a and b are $\mathcal{L}$ −related if they generate the same left ideal , $\mathcal{R}$-related if they generate the same right ideal and $\Im$ −related if they generate the same two-sided ideal. These are equivalence relations on S, so each of them yields a partition of S into.[4]

2.2 **Left restriction semigroup**

A semigroup S is left restriction with respect to $E \subseteq E\ (S)$ if `

i. E is a subsemilattice of S

ii. Every element $a \in S$ is $\widetilde{R_E}$ -related to an element of E (denoted by a^+)

iii. $\widetilde{R_E}$ is a left congruence

iv. The left ample condition holds $\forall\ a \in S\ and\ e \in E,$

$$ae = \ (ae)^+a$$

Equivalently,

A semigroup S is a right restriction with respect to $E = E\ (S)$ if

i. E is a subsemilattice of S

ii. Every element $a \in S$ is $\widetilde{L_E}$ -related to an element of E (denoted by a^+)

iii. $\widetilde{L_E}$ is a left congruence

iv. The left ample condition holds $\forall\ a \in S\ and\ e \in E,$

$$ea = a(ea)^+$$

2.3 Remark

Left restriction semigroups are algebra defined by the following identities

i. $(xy)z = x(yz)$

ii. $x^+x = x$

iii. $x^+ y^+ = y^+x^+$

iv. $(x^+y)^+ = x^+ y^+$

v. $xy^+ = (xy)^+x$

vi. $(a^+)^* = a^+$ and $(a^*)^+ = a^+$

2.4 Notation

These identities imply $x^+x^+ = x^+$, $(x^+)^+ = x^+$

Proof $(x^+)^+ = (x^+x^+)^+ = (x^+x)^+ = x^+$ [2]

2.5 Raf-baduT program

Raf-baduT was designed to generate elements of partial transformation and has the features : Order of the $\wp\mathfrak{I}_{\{X\}}$, elements position dropdown, View state, Three Representations : Semigroup, notation and daga . The programme is designed for orders X= {2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} .After the examples were obtained, the algebraic properties of left congruences of $\widetilde{R_S}$ –classes and from their semilattices of idempotents, the partial identities E_X were obtained.

3. Main Results

3.1 Computing the congruence class $\widetilde{R_S}$ –class and partial identities $\widetilde{E_S}$

i. $\wp T_{\{1,2,3\}}$ given by

$$S = \{ A, B, C, D, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U , V , W\ X, Y, A1, B1, C1, E\}$$

+	A	B	C	D	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	A1	B1	C1	E
A	B	B	D	E	F	G	I	E	K	A	M	F	O	G	Q	B	D	K	I	D	G	V	A	A1	F	K	I	E
B	B	B	B	E	F	G	E	E	E	B	F	F	G	G	B	B	B	E	E	E	G	G	B	F	F	E	E	E
C	E	C	C	E	E	E	H	E	J	C	C	E	J	E	J	C	C	J	H	C	E	H	C	H	E	J	H	E
D	E	D	D	E	E	E	I	E	K	D	D	E	K	E	K	D	D	K	I	D	E	I	D	I	E	K	I	E
F	G	F	F	G	E	E	E	B	E	F	E	G	E	F	E	F	F	G	G	B	B	E	F	E	B	B	F	E
G	E	G	G	E	E	E	B	E	F	G	G	E	F	E	F	G	G	F	B	G	E	B	G	B	E	F	B	E
H	H	H	H	E	J	C	E	E	E	H	J	J	C	C	H	H	H	E	E	E	C	C	H	J	J	E	E	E
I	I	I	I	E	K	D	E	E	E	I	K	K	D	D	I	I	I	E	E	E	D	D	I	K	K	E	E	E
J	C	J	J	C	E	E	E	H	E	J	E	C	E	J	E	J	J	C	C	H	H	E	J	E	H	H	J	E
K	D	K	K	D	E	E	E	I	E	K	E	D	E	K	E	K	K	D	D	I	I	E	K	E	I	I	K	E
L	G	L	C	G	E	E	H	B	J	F	C	G	J	F	J	F	L	N	W	X	B	H	C	H	B	P	Y	E
M	G	M	D	G	E	E	I	B	K	F	D	G	K	F	K	F	M	O	V	A	B	I	D	I	B	Q	A1	E
N	C	N	G	C	E	E	B	H	F	J	G	C	F	J	F	J	N	L	X	W	H	B	G	B	H	Y	P	E
O	D	O	G	D	E	E	B	I	F	K	G	D	F	K	F	K	O	M	A	V	I	B	G	B	I	A1	Q	E
P	X	B	P	C	F	G	E	H	E	J	F	L	G	N	B	P	J	C	C	H	W	G	B	F	Y	H	J	E
Q	A	B	Q	D	F	G	E	I	E	K	F	M	G	O	B	Q	K	D	D	I	V	G	B	F	A1	I	K	E
R	D	R	C	D	E	E	H	I	J	K	C	D	J	K	J	K	R	S	T	U	I	H	C	H	I	B1	C1	E
S	C	S	D	C	E	E	I	H	K	J	D	C	K	J	K	J	S	R	U	T	H	I	D	I	H	C1	B1	E
T	H	H	D	E	J	C	I	E	K	T	S	J	R	C	C1	H	D	K	I	D	C	U	T	B1	J	K	I	E
U	I	I	C	E	K	D	H	E	J	U	R	K	S	D	B1	I	C	J	H	C	D	T	U	C1	K	J	H	E
V	I	I	G	E	K	D	B	E	F	V	O	K	M	D	A1	I	G	F	B	G	D	A	V	Q	K	F	B	E
W	H	H	G	E	J	C	B	E	F	W	N	J	L	C	Y	H	G	F	B	G	C	X	W	P	J	F	B	E
X	B	B	C	E	F	G	H	E	J	X	L	F	N	G	P	B	C	J	H	C	G	W	X	Y	F	J	H	E
Y	W	H	Y	G	J	C	E	B	E	F	J	N	C	L	H	Y+	F	G	G	B	X	C	H	J	P	B	F	E
A1	V	I	A1	G	K	D	E	B	E	F	K	O	D	M	I	A1	F	G	G	B	A	D	I	K	Q	B	F	E
B1	U	I	B1	C	K	D	E	H	E	J	K	R	D	S	I	B1	J	C	C	H	T	D	I	K	C1	H	J	E
C1	T	H	C1	D	J	C	E	I	E	K	J	S	C	R	H	C1	K	D	D	I	U	C	H	J	B1	I	K	E
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

Where

A= $\begin{pmatrix}1 & 2 & 3\\ 1 & x & 2\end{pmatrix}$, $B = (1)$, $C = \begin{pmatrix}1 & 2 & 3\\ x & 2 & x\end{pmatrix}$ $D = \begin{pmatrix}1 & 2 & 3\\ x & x & 2\end{pmatrix}$, $F = \begin{pmatrix}1 & 2 & 3\\ 3 & x & x\end{pmatrix}$,

$G = \begin{pmatrix}1 & 2 & 3\\ 2 & x & x\end{pmatrix}$, $H = \begin{pmatrix}1 & 2 & 3\\ x & 1 & x\end{pmatrix}$, $I = \begin{pmatrix}1 & 2 & 3\\ x & x & 1\end{pmatrix}$,J= $\begin{pmatrix}1 & 2 & 3\\ x & 3 & x\end{pmatrix}$, $K = \begin{pmatrix}1 & 2 & 3\\ x & x & 3\end{pmatrix}$,

$L = \begin{pmatrix}1 & 2 & 3\\ 3 & 2 & x\end{pmatrix}$, $M = \begin{pmatrix}1 & 2 & 3\\ 3 & x & 2\end{pmatrix}$, $N = \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & x\end{pmatrix}$,$O = \begin{pmatrix}1 & 2 & 3\\ 2 & x & 3\end{pmatrix}$, P= $\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & x\end{pmatrix}$,

$Q = \begin{pmatrix}1 & 2 & 3\\ 1 & x & 3\end{pmatrix}$, $R = \begin{pmatrix}1 & 2 & 3\\ x & 2 & 3\end{pmatrix}$, $S' = \begin{pmatrix}1 & 2 & 3\\ x & 3 & 2\end{pmatrix}$, $T = \begin{pmatrix}1 & 2 & 3\\ x & 1 & 2\end{pmatrix}$, $U = \begin{pmatrix}1 & 2 & 3\\ x & 2 & 1\end{pmatrix}$,

$V = \begin{pmatrix}1 & 2 & 3\\ 2 & x & 1\end{pmatrix}$, $W = \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & x\end{pmatrix}$, $X = \begin{pmatrix}1 & 2 & 3\\ 1 & 2 & x\end{pmatrix}$, $Y = \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & x\end{pmatrix}$, $A1 = \begin{pmatrix}1 & 2 & 3\\ 3 & x & 1\end{pmatrix}$,

B1= $\begin{pmatrix}1 & 2 & 3\\ x & 3 & 1\end{pmatrix}$, $C1 = \begin{pmatrix}1 & 2 & 3\\ x & 1 & 3\end{pmatrix}$, $\varepsilon = \begin{pmatrix}1 & 2 & 3\\ x & x & x\end{pmatrix}$,

The $\widetilde{R_S}$ –classes are :

$\widetilde{R_B} = \{A, P, Q, X\}$, $\widetilde{R_C} = \{L, R, U, X\}$, $\widetilde{R_D} = \{K, O, Q, R, S, C1\}$, $\widetilde{R_G} = \{A, B, P, Q, X\}$, $\widetilde{R_H} = \{C, L, R, U, X\}$, $\widetilde{R_I} = \{K, O, Q, C1\}$ $\widetilde{R_J} = \{ C, L, R, U, X\}$, $\widetilde{R_K} = \{K, O, Q, R, C1\}$, and $\widetilde{R_E} =$ $\{ A, B, C, D, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U , V , W\ X, Y, A1, B1, C1\}$

II. $\wp T_{\{1,2,3,4\}}$: $S = \{\alpha, \alpha^+, \beta, \gamma, \gamma^+, \sigma, \sigma^+, \varepsilon\}$

$\alpha = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & x & x\end{pmatrix}$, $\beta = \begin{pmatrix}1 & 2 & 3 & 4\\ x & 2 & 3 & 4\end{pmatrix}$, $\gamma = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & x & x & x\end{pmatrix}$ $\sigma = \begin{pmatrix}1 & 2 & 3 & 4\\ x & 1 & x & x\end{pmatrix}$ $\alpha^+ = \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & x & x\end{pmatrix}$, $\gamma^+ = \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & x & x & x\end{pmatrix}$, $\sigma^+ = \begin{pmatrix}1 & 2 & 3 & 4\\ x & 2 & x & x\end{pmatrix}$ $\varepsilon = \begin{pmatrix}1 & 2 & 3 & 4\\ x & x & x & x\end{pmatrix}$

+	α	β	α^+	γ	γ^+	σ	σ^+	ε
α	α^+	γ	α	σ^+	σ	γ^+	γ	ε
β	σ	β	σ^+	ε	ε	σ	σ^+	ε
α^+	α	σ^+	α^+	γ	γ^+	σ	σ^+	ε
γ	γ^+	γ	γ	ε	ε	σ^+	σ	ε
γ^+	γ	ε	γ^+	γ	γ^+	ε	ε	ε
σ	σ^+	ε	σ	σ^+	σ	ε	ε	ε
σ^+	σ	σ^+	σ^+	ε	ε	σ	σ^+	ε
ε	ε	ε	ε	ε	ε	ε	ε	ε

As S is closed under composition and $^+$, S is a (2,1)-subalgebra of $\wp\mathfrak{I}_{\{1,2,3,4\}}$ and so S is left restriction semigroup with distinguished semilattice.

$E_s = \{\alpha^+, \beta, \gamma^+, \sigma^+, \varepsilon\}$ [2]

The $\widetilde{R_S}$ –classes are : $\widetilde{R_\gamma} = \{\alpha^+, \gamma^+\}$, $\widetilde{R_\sigma} = \{\alpha^+, \beta, \sigma^+\}$, $\widetilde{R_{\sigma^+}} = \{\alpha^+, \beta\}$ and $\widetilde{R_E} = \{\alpha^+, \beta, \gamma^+, \sigma^+\}$

iii. $\wp T_{\{1,2,3,4\}}$: $S = \{A, B, C, D, F, G, H, I, J, K, L, M, N, O, P, E\}$

where $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & x & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & x & x & x \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 3 & x & x \end{pmatrix}$ $D = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & x & x & x \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & x & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & x & x & x \end{pmatrix}$, $H = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 1 & x & x \end{pmatrix}$ $I = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 4 & x & x \end{pmatrix}$,

$J = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & 4 & x \end{pmatrix}$, $K = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & x & 4 \end{pmatrix}$, $L = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & x & x \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 2 & x & x \end{pmatrix}$,

$N = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & x & x \end{pmatrix}$, $O = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & 3 & x \end{pmatrix}$, $P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & x & 4 & x \end{pmatrix}$, $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & x & x \end{pmatrix}$

The $\widetilde{R_S}$ –classes are:

$\widetilde{R_A} = \widetilde{R_D} = \widetilde{R_F} = \widetilde{R_G} = \{A, B\}$, $\widetilde{R_E} = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P\}$

iv. $\wp T_{\{1,2,3,4\}}$: $S = \{A, B, C, D, F, G, H, I, J, K, L, M, N, O, E\}$

where

$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & x & x & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 3 & x & x \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & x & x & x \end{pmatrix}$ $D = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & x & x & x \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & x & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 1 & x & x \end{pmatrix}$, $H = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 4 & x & x \end{pmatrix}$ $I = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & 4 & x \end{pmatrix}$,

$J = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & x & x \end{pmatrix}$, $K = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & 3 & x \end{pmatrix}$, $L = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & x & x \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 2 & x & x \end{pmatrix}$,

$N = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & x & 4 & x \end{pmatrix}$, $O = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & 4 & 3 & x \end{pmatrix}$, $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 \\ x & x & x & x \end{pmatrix}$ $E_s = \{A, J, L, N, E\}$

The $\widetilde{R_S}$ –class is : $\widetilde{R_E} = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O\}$

v. $\wp T_{\{1,2,3,4,5\}}$: $S = \{A, B, C, D, E, F, G, H, I, J, K, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & x & x & 5 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & x & x & 4 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & x & x & 5 & 4 \end{pmatrix}$,

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & x & x & 4 & 5 \end{pmatrix}$, $F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & x & x & 5 & 4 \end{pmatrix}$ $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & 3 & x & 5 & 4 \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & 2 & x & 4 & 5 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & x & x & 4 & 5 \end{pmatrix}$, $J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & 3 & x & 5 & 4 \end{pmatrix}$,

$K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & 2 & x & 4 & 5 \end{pmatrix}$ $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & x & x & x & x \end{pmatrix}$ $E_s = \{B, H, K, \varepsilon\}$

The $\widetilde{R_S}$ –classes are:

$\widetilde{R_B} = \{D, G, I, K\}$, $\widetilde{R_C} = \{B, D, G, I, K\}$, $\widetilde{R_J} = \widetilde{R_G} = \{H, K\}$ and $\widetilde{R_E} = \{A, B, C, D, E, F, G, H, I, J, K\}$

vi. $\wp\mathfrak{I}_{\{1,2,3,4,5\}}$: $S = \{\alpha, \beta, \alpha^+, \tau, \tau^+\}$

$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & x & x & 5 & 4 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & x & x & 5 & 4 \end{pmatrix}$, $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & x & x & 5 & 4 \end{pmatrix}$,

$\alpha^+ = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & x & x & 4 & 5 \end{pmatrix}$, $\tau^+ = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ x & x & x & 4 & 5 \end{pmatrix}$

+	α^+	α	β	τ^+	τ
α^+	α^+	α	β	τ^+	τ
α	τ	τ^+	τ^+	τ	τ^+
β	τ	τ^+	τ^+	τ	τ^+
τ^+	τ^+	τ	τ	τ^+	τ
τ	τ	τ^+	τ^+	τ	τ^+

As S is closed under composition and $^+$, S is a (2,1)-subalgebra of $\wp\mathfrak{I}_{\{1,2,3,4,5\}}$ and so S is left restriction semigroup with distinguished semilattice. $E_s = \{\tau^+, \alpha^+\}$

The $\widetilde{R_S}$ –classes are: $\widetilde{R_S} = \{\alpha^+, \alpha, \beta, \}$& $\{\tau^+, \tau\}$ [3]

vii. $\wp T_{\{1,2,3,4,5,6\}}$: $S = \{A, B, C, D, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & x & 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & x & 3 & x & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 3 & x & 5 & 4 \end{pmatrix}$,

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 3 & x & 5 & x \end{pmatrix}$, $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & x & x \end{pmatrix}$ $E_s = \{C, D, E\}$

The $\widetilde{R_S}$ –classes are : $\widetilde{R_C} = \{A, B\}$, $\widetilde{R_D} = \{A, B, C\}$ and $\widetilde{R_E} = \{A, B, C, D\}$

viii. $\wp T_{\{1,2,3,4,5,6\}}$

$S = \{A, B, C, D, F, G, H, I, J, K, L, M, N, O, P, Q, R, S^*, T, U, V, W, X, Y, A1, B1, C1, D1, F1, G1,$
$H1, I1, J1, K1, L1, M1, N1, O1, P1, Q1, R1, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & x & 4 & 5 & x & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & x & 5 & x & x & x \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & x & x & x & x & x \end{pmatrix}$,

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & x & x & x & x & x \end{pmatrix}$ $F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 6 & x & x & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 6 & x & x & x \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 5 & x & x & x & x \end{pmatrix}$ $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & 6 & x & x \end{pmatrix}$ $J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & x & 6 & x & x & x \end{pmatrix}$,

$K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 5 & x & x & x \end{pmatrix}, L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & x & x & x & x & x \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & 6 & x \end{pmatrix}$,

$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & x & x & 6 & x & x \end{pmatrix}$, $O = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 5 & 6 & x & x & x \end{pmatrix}$, $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & 5 & x & x \end{pmatrix}$,

$Q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & x & 5 & x & x & x \end{pmatrix}$,$R = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 4 & x & x & x & x \end{pmatrix}, S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & x & 6 \end{pmatrix}$,

$T = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & x & x & x & x \end{pmatrix}$ $U = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & x & x & x & x \end{pmatrix}$, $V = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & x & x & x & 6 & x \end{pmatrix}$,

$W = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 5 & x & 6 & x & x \end{pmatrix}$, $X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 6 & 5 & x & x & x \end{pmatrix}$ $Y = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 4 & x & x & x \end{pmatrix}$,

$A1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & x & x & x & x & x \end{pmatrix}$, $B1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & x & x & x & x & 6 \end{pmatrix}$, $C1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & x & x & x \end{pmatrix}$

$D1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 3 & x & x & x & x \end{pmatrix}$, $F1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & x & x & x & x \end{pmatrix}$, $G1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 3 & x & x & x \end{pmatrix}$,

$H1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & x & x & x & x & x \end{pmatrix}$ $I1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 6 & x & x & x \end{pmatrix}$, $J1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & 5 & x \end{pmatrix}$,

$K1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & x & x & 5 & x & x \end{pmatrix}$, $L1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 5 & x & x & 6 & x \end{pmatrix}$ $M1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & 5 & 6 & x & x \end{pmatrix}$,

$N1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & x & 5 & x & x & x \end{pmatrix}$ $O1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & x & 5 \end{pmatrix}$ $P1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & x & 6 & x & x \end{pmatrix}$

$Q1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 6 & x & 5 & x & x \end{pmatrix}$ $R1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & 5 & x & x & x & 6 \end{pmatrix}$ $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ x & x & x & x & x & x \end{pmatrix}$

$E_s = \{S, G1, K1, E\}$

The $\widetilde{R_S}$ –classes are : $\widetilde{R_S} = \{B1, R1\}$ and $\widetilde{R_{O1}} = \{ R1, B1, S^*\}$

ix. $\wp T_{\{1,2,3,4,5,6,7\}}$: $S = \{ A, B, C, D, E, F, G, H, I, J, K, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 5 & 6 & 3 & 1 & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 3 & 1 & 5 & x & x \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 5 & x & 3 & x & x \end{pmatrix}$,

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 3 & x & 5 & x & x \end{pmatrix}$, $F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 2 & 5 & x & 3 & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 3 & x & 5 & 7 & x \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 5 & 7 & 3 & x & x \end{pmatrix}$, $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 2 & 3 & x & 5 & x & x \end{pmatrix}$, $J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 5 & x & 3 & 7 & x \end{pmatrix}$,

$K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & 2 & 3 & 7 & 5 & x & x \end{pmatrix}$, $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ x & x & x & x & x & x & x \end{pmatrix}, E_s = \{D, E\}$

The $\widetilde{R_S}$ –classes are:

$\widetilde{R_C} = \{B, D, G, I, K\}$,$\widetilde{R_D} = \{ B.G, I, K\}$ and $\widetilde{R_E} = \{ A, B, C, D, E, F, G, H, I, J, K\}$

x. $\wp T_{\{1,2,3,4,5,6,7,8\}}$: $S = \{ A, B, C, D, F, G, H, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & x & x & 5 & 6 & 1 & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & 5 & 6 & 3 & x \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & 5 & 6 & x & x \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & x & x & x & 5 & 6 & x & x \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & 5 & 6 & 8 & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & 8 & x & x & 5 & 6 & x & x \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & 8 & x & x & 5 & 6 & x & x \end{pmatrix}$, $\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & x & x & x & x \end{pmatrix}$, $E_s = \{C, E\}$

The $\widetilde{R_S}$ –classes are : $\widetilde{R_C} = \{A, B, D, F, G, H\}$ and $\widetilde{R_E} = \{A, B, C, D, F, G, H, \}$

xi. $\wp T_{\{1,2,3,4,5,6,7,8\}}$: $S = \{A, B, C, D, F, G, H, I, J, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 4 & 2 & x & 5 & 6 & x & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & x & 4 & x & 5 & 6 & x & x \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & x & x & x & 5 & 6 & x & x \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & 5 & 6 & x & x \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & x & x & x & 5 & 6 & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & x & x & x & 5 & 6 & x & x \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & x & x & x & 5 & 6 & x & x \end{pmatrix}$ $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & x & x & x & 5 & 6 & x & x \end{pmatrix}$,

$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & x & x & x & 5 & 6 & x & x \end{pmatrix}$

$\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & x & x & x & x \end{pmatrix}$, $E_s = \{A, C, D, \varepsilon\}$

The $\widetilde{R_S}$ –classes are : $\widetilde{R_C} = \{A, B\}$, $\widetilde{R_F} = \widetilde{R_I} = \widetilde{R_G} = \widetilde{R_J} = \widetilde{R_H} = \{A, B, C\}$ and $\widetilde{R_E} = \{A, B, C, D, F, G, H, I, J\}$

xii. $\wp T_{\{1,2,3,4,5,6,7,8\}}$: $S = \{A, B, C, D, F, G, H, I, J, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & x & 4 & 5 & 7 & 1 & x & x \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & x & 5 & 7 & x & 3 & x & x \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & x & 7 & x & x & 5 & x & x \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & x & x & x & x & 5 & x & x \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & x & 7 & x & x \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & x & x & x & 5 & 6 & x & x \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & x & 8 & x & x \end{pmatrix}$, $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & x & 4 & 5 & 7 & 1 & x & x \end{pmatrix}$,

$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & 8 & x & x & x & x & x \end{pmatrix}$, $K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & x & x & x & x & x & x & x \end{pmatrix}$,

$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & x & x & x & x & 7 & x & x \end{pmatrix}$, $M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & x & x & x & x & x & 8 & x \end{pmatrix}$,

$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & 8 & x & x & x & 7 & x & x \end{pmatrix}, O = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & 8 & x & x & x & x \end{pmatrix} P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & x & 8 & x & x & x & x & x \end{pmatrix}$

$\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ x & x & x & x & x & x & x & x \end{pmatrix} \quad E_s = \{\varepsilon\}$

The $\widetilde{R_S}$ –class is : $\widetilde{R_E} = \{ A, B, C, D, F, G, H, I, J, K, L, M, N, O, P\}$

xiii. $\wp T_{\{1,2,3,4,5,6,7,8,9\}}$: $S = \{ A, B, C, D, F, G, H, I, J, E\}$ 10 elements

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & x & 6 & 1 & 5 & x & 7 & 8 & x \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & x & x & 2 & 5 & x & 7 & 8 & x \end{pmatrix},$

$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & x & x & x & 5 & x & 7 & 8 & x \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & x & x & x & 5 & x & 7 & 8 & x \end{pmatrix},$

$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & x & x & 9 & 5 & x & 7 & 8 & x \end{pmatrix}, G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & 9 & x & x & 5 & x & 7 & 8 & x \end{pmatrix},$

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & 9 & x & x & 5 & x & 7 & 8 & x \end{pmatrix}, I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & x & x & x & 5 & x & 7 & 8 & x \end{pmatrix},$

$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x & x & x & 6 & 5 & x & 7 & 8 & x \end{pmatrix}$

$E_s = \{C\}$

The $\widetilde{R_S}$ –class is: $\widetilde{R_C} = \{ A, B, C, D, F, G, H, I, J\}$

$xiv.$ $\wp T_{\{1,2,3,4,5,6,7,8,9,10\}}$: $S = \{ A, B, C, D, F, G, H, I, J, K, L, E\}$

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & x & 4 & 5 & 1 & 7 & 8 & x & 2 & x \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & x & 5 & 1 & 9 & 8 & x & x & x & x \end{pmatrix},$

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & 9 & 2 & x & x & x & x & x & x \end{pmatrix}, F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & 2 & x & x & x & x & x & x & x \end{pmatrix}$

$G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & x & x & x & x & x & x & x & x & x \end{pmatrix}, H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & x & x & 10 & x & x & x & x & x \end{pmatrix},$

$I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & x & 10 & x & x & x & x & x & x \end{pmatrix} J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & 10 & x & x & x & x & x & x & x \end{pmatrix},$

$K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & 10 & x & x & x & x & x & x & x & x \end{pmatrix}, L =$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & x & x & x & x & x & x & 10 & x \end{pmatrix}$

$\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ x & x & x & x & x & x & x & x & x & x \end{pmatrix}$

$E_s = \{E\}$

The $\widetilde{R_S}$ –class is : $\widetilde{R_E} = \{ A, B, C, D, F, G, H, I, J, K, L\}$

CONCLUSION

This work extends [1] where some left restriction semigroups embedded in partial transformation $\wp\mathfrak{I}_X$ $\{X = 1,2,3,45,6,7,8,9,10\}$ were computed ,in here, their left congruence- classes in the form of $\widetilde{R_s}$ –class and partial identities E_s have been computed.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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